1.5 CUMULATIVE FATIGUE DAMAGE.

1.5.1 Theory.

One important problem in fatigue analysis is how to calculate fatigue life. From the S-N diagram, we know that the higher the alternating stress, the lower the number of cycles a part will endure before failure. Also, at stresses below the fatigue limit an infinite number of cycles can be sustained.

In most cases the weight penalty imposed by using the fatigue limit as an allowable stress cannot be tolerated. If stress cycling is at a known level, an S-N curve can be used to determine the number of cycles to failure and thus the life of the part can be predicted. Most structural components or parts are subjected to irregular fluctuating stresses in which the maximum and minimum stresses are constantly changing. Thus, it is necessary to use a fatigue theory which will account for the damage caused by different magnitudes of stress cycles.

Of the several cumulative damage fatigue theories known, the one most widely used and best known is the one suggested by Palmgren and later, independently, by Miner. The Palmgren-Miner hypothesis is that the fatigue damage incurred at a given stress level is proportional to the number of cycles applied at that stress level divided by the total number of cycles required to cause failure at the same level. This damage is usually referred to as the cycle ratio or cumulative damage ratio. If the repeated loads are continued at the same level until failure occurs, the cycle ratio will be equal to one. When fatigue loading involves many levels of stress amplitude, the total damage is a sum of the different cycle ratios and failure should still occur when the cycle ratio sum equals one.

$$\sum_i \frac{n_i}{N_i} = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \ldots . = 1.0 \quad .$$

This equation has been used by designers for many years, but at the same time it is under criticism by researchers. It is found that:

1. In many test results the summation of n/N is far from one.

2. The fatigue damage is not linearly proportional to the number of cycles or the cycle ratio n_1/N_1.

3. There is interaction in the fatigue damages between various stress levels which Miner neglected. In the interaction there is also a sequence effect which means that the fatigue damage resulting from the high load first, with the low load next, will be different from the damage resulting from the low load first, with the high load next.

Aware of its limitations, designers still use Miner's equation as a preliminary guide because of its simplicity, versatility, and sufficient accuracy, commensurate with the data currently available for this type of analysis.

What are the consequences when Miner's equation is used for random-loading conditions? Random-load fatigue testing is time-consuming and costly, and the scant data available do not permit a general clear-cut answer to this question. According to Freudenthal (Ref. 21), the sum $\Sigma n/N$ is always less than one. In some cases the sum is as low as 0.13 but it is mostly between 0.20 and 0.60. The primary reason for this discrepancy, according to Freudenthal, is the interaction of fatigue damages between various load amplitudes.

On the other hand, it has been found that notched parts generally give a summation value greater than one. Since practically all of the structural fatigue failures originate in some form of notch, the question of what value to use in place of 1.0 has been asked.

The National Aeronautics and Space Administration conducted full-scale fatigue tests on C-46 transport airplane wings and their summation of cycle ratios was 1.4 (Ref. 22).

However, until more information becomes available, Miner's equation is recommended for preliminary analysis.

1.5.2 Analysis of Data.

Numerical simplifying assumptions are often needed to speed up the analyzing of data. Mainly, these assumptions have to do with primary and secondary cycles. The following four conventions serve to establish and define primary and secondary cycles:

1. A mean value of stress, S_{mean} for the entire record is approximated. Although a time average and a peak point average generally yield different average values, the two may be assumed to be equal.

2. The maximum stress occurring between a positive S_{mean} crossing (positive in slope) and a negative (negative slope) S_{mean} crossing is called S_{max}. All other maximum stress values are designated as S_{ma}.

3. The minimum stress occurring between a negative S_{mean} crossing and a corresponding positive crossing is designated as S_{min}. All other minimum stress values are S_{mi} points. See Fig. E1-34.

4. The combination of an S_{min} and the next S_{max} form a primary cycle. The combination of an S_{mi} and the next S_{ma} form a secondary cycle.

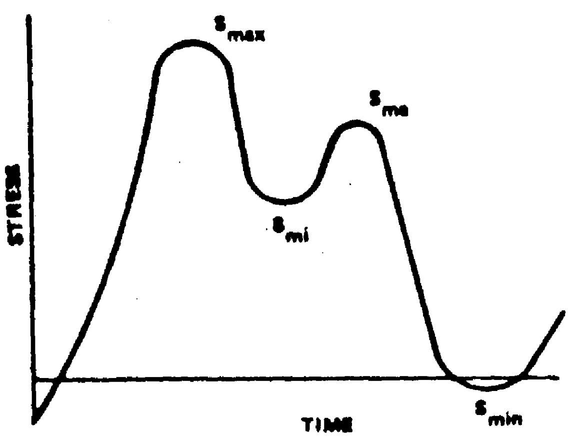


FIGURE E1-34. A PART OF A STRESS RECORD

Accurate evaluation of fatigue damage usually requires that all primary cycles within a given period be considered, but it rarely requires that all secondary cycles be considered. The effect of neglecting secondary stresses in random data will be investigated in terms of the following propositions:

Proposition I — If peaks (maximums) and troughs (minimums) are normally distributed about a mean stress, the damage per cycle caused by all cycles which do not cross the mean stress line can be neglected.

Proposition II — If peaks and troughs are normally distributed about a positive mean stress, the damage per cycle caused by all cycles which neither cross nor exceed the mean stress line can be neglected.

Proposition III — If peaks and troughs are normally distributed about a positive mean stress, the only time a secondary cycle needs to be considered is when it lies above the mean stress, when it occurs just after S_{max} has been established, and when it reaches the lowest S_{min} established since the last mean stress crossing.

Usually for random data, one of these propositions can be considered valid. Fig. E1-35 illustrates several more or less useful approximations, some of which are generally valid, and all of which are usually more valid than the half-cycle rearrangement described above. In each case, a heavy base line is used to indicate a mean stress line.

1.5.2.1 Peak Counting Techniques.

Figure E1-36 shows the three most commonly used cycle counting methods among a considerably greater number which have been proposed and used. Some of the others are modifications of the ones shown. For example, the first one is frequently modified so that positive and negative half cycles are counted and tabulated separately. The second method pairs off succeeding positive and negative amplitudes so that each cycle mean can be calculated.

The third method, without refinement, is not very accurate unless only one frequency is present. If a low and a high frequency appear with equal amplitudes, the high-frequency, low-range (amplitude) activity will mask the low-frequency, high-range component; the result will be indistinguishable from a high-frequency, high-amplitude loading which is very damaging. See Case 3 of Fig. E1-36.

However, an important refinement to the third method of Fig. E1-36 is the use of zones and the counting of passages from one zone to another, particularly to a nonadjacent zone. This method, which is listed as method 3a in the figure, is equivalent to method 2 if a large number of zones and zone separations are used and properly recorded.

1.5.2.2 Statistical Methods of Random Load Analysis.

Most of the fundamental concepts of fatigue indicate that some type of cycle-counting technique is needed to evaluate the fatigue damage caused by random loading. Direct techniques of this type, such as peak counts and range counts are simple and accurate but often quite cumbersome to apply. Other, less direct methods are sometimes sought to estimate the same information less laboriously. Two such indirect methods are presented in Refs. 2 and 3, in which sampling or alternative measurements are used and converted into a useful form by statistical means. In general, these methods have not been as accurate in evaluating cumulative fatigue damage as the more direct methods. However, their use with large quantities of data makes the loss of accuracy acceptable.

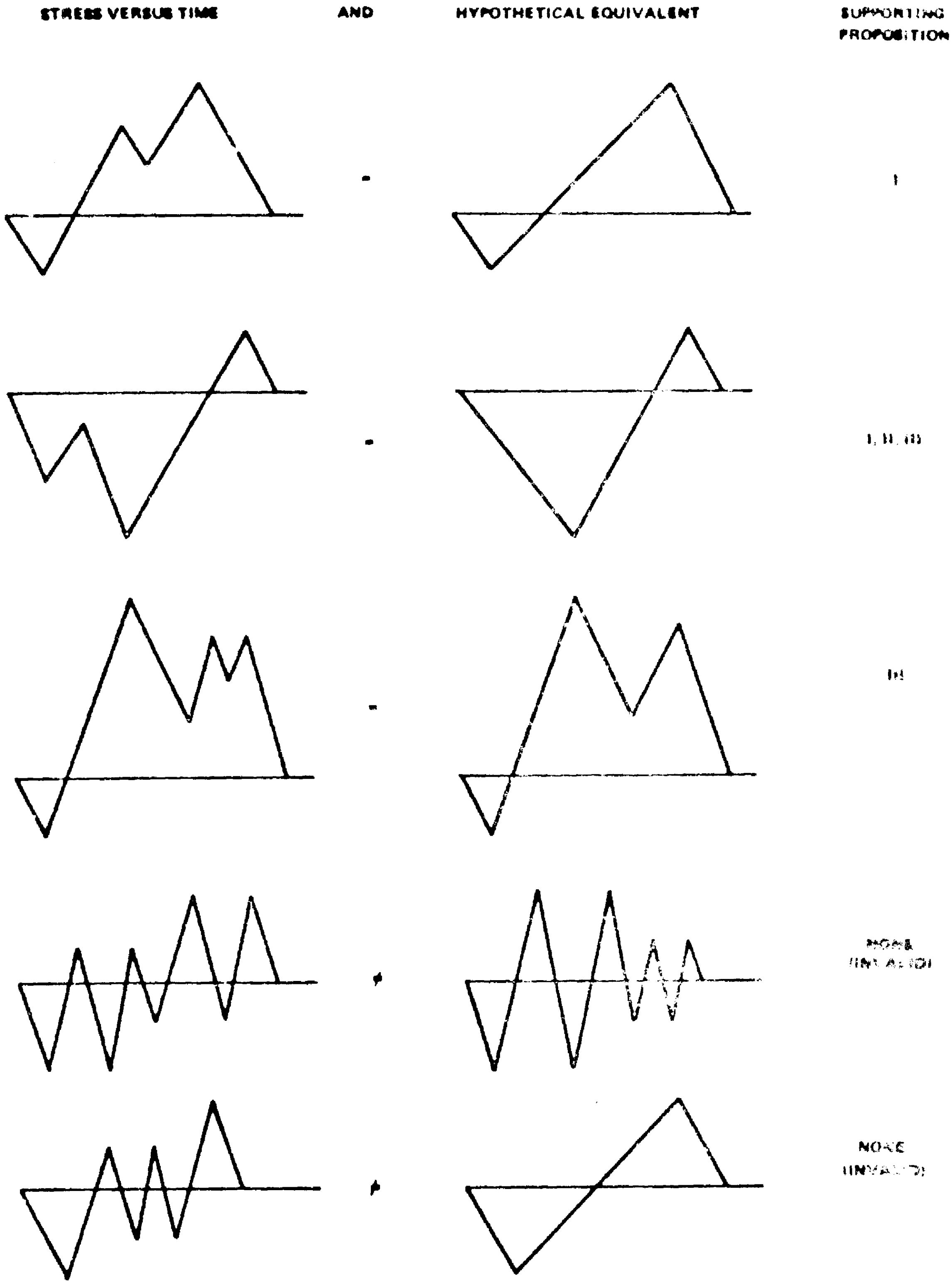
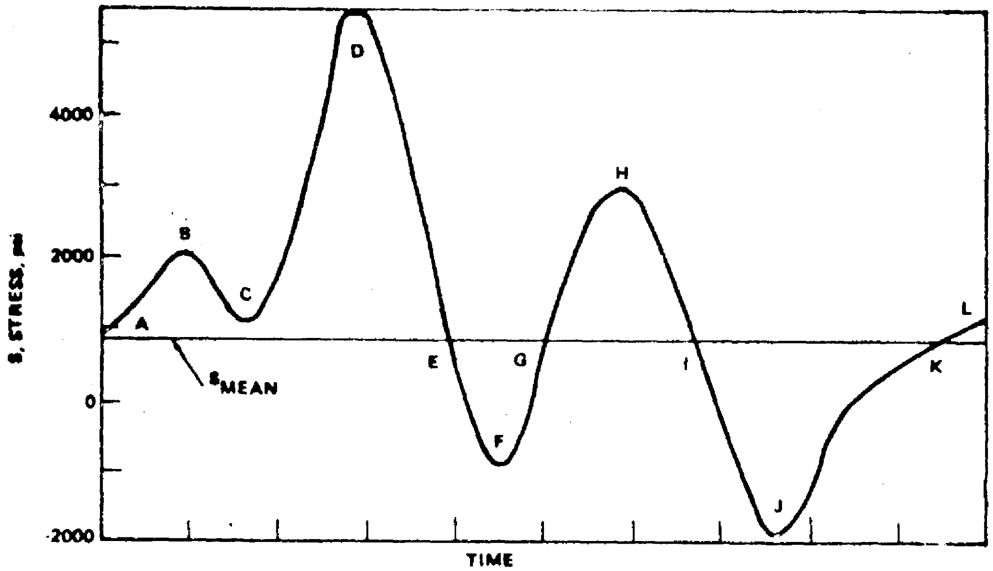
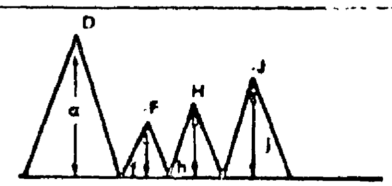
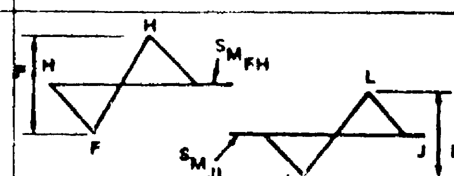
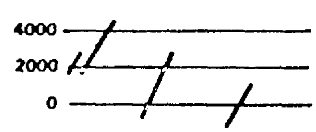


FIGURE E1-35. FIVE DATA-REDUCING APPROXIMATIONS OF VARYING DEGREES OF VALIDITY



COUNTING METHOD	QUANTITIES MEASURED OR COUNTED	EQUIVALENT WAVEFORM
1. MEAN CROSSING PEAK COUNT	n, d, f, h, j, etc.	
2. PAIRED RANGE COUNT	n, $S_{M_{FH}}$ AND FH $S_{M_{JL}}$ AND JL	
3. LEVEL CROSSING COUNT	$n_{4000} = 1$ $n_{2000} = 3$ $n_0 = 2$ etc.	
3a. ZONE PASSING COUNT	SAME AS METHOD 3 EXCEPT EACH CROSSING IS A COUNT ONLY WHEN PRECEDED BY A CROSSING AT EACH OF SEVERAL VARIOUSLY SEPARATED LEVELS AND IS CATEGORIZED ACCORDING TO SEPARATION	

NOTE: n IS A NUMBER OF POSITIVE SLOPE DESIGNATED LEVEL CROSSINGS (IF WITHOUT SUBSCRIPT, THE DESIGNATED LEVEL IS S_{mean})

FIGURE E1-36. APPLICATION OF SOME CYCLE COUNTING METHODS TO A TYPICAL STRESS RECORD (Lower Table Illustrates Each Method in Terms of an Equivalent Record)

1.5.3 Example Problem (Paired Range Count Method).

To evaluate the damage caused by complex stress-time cycles, it is necessary to correct for the difference in mean stress, S_m, for the various stress alternations, S_a. Four constant amplitude S-N curves for different mean stresses are shown in Fig. E1-37. A complex stress-time hisotry for an identical specimen is shown in Fig. E1-38.

The first step is to determine the damage caused by the smaller variations which are crosshatched in Fig. E1-38(a). The largest mean stress is approximately 20 ksi and, as shown in Fig E1-37, for this mean stress an alternating stress of 4 ksi or less causes no damage. At any mean stress less than 20 ksi, the alternating stress which causes no damage increases so that any alternating stress of 4 ksi or less can be omitted. Thus, variations BC, DE, GH, JK, NO, PQ, and ST cause no fatigue damage. The stress-time history is replotted in Fig. E1-38(b), with the previously evaluated variations removed.

The second step is to evaluate the smallest remaining stress variations IL and RU. For IL, the alternating stress is 5 ksi and the mean stress is 10 ksi. Referring to Fig. E1-37, under these conditions failure would result in 10^7 cycles. Hence the damage is $n/N = 1/10^7 = 10^{-7}$. Calculating the damage caused by RU in the same manner $n/N = 1/2.2 \times 10^4 = 4.55 \times 10^{-5}$. The stress-time history is again replotted in Fig. E1-38(c), with the evaluated variations removed.

The third step is to evaluate the remaining cycle, FM, with $S_a = 20$ ksi and $S_m = 5$ ksi. Interpolating between $S_m = 0$ and $S_m = 10$, failure would result in 3.5×10^3 cycles. Hence, the damage caused is $1/3.5 \times 10^3 = 2.86 \times 10^{-4}$. The last step is to sum the damage caused by all the variations. In this case,

$$10^{-7} + 4.5 \times 10^{-5} + 2.86 \times 10^{-4} = 3.316 \times 10^{-4} \quad .$$

Using the cumulative damage theory with $\Sigma\, n/N = 1.0$, the stress-time history can be repeated 3000 times $(1.0/3.316 \times 10^{-4} = 3000)$.

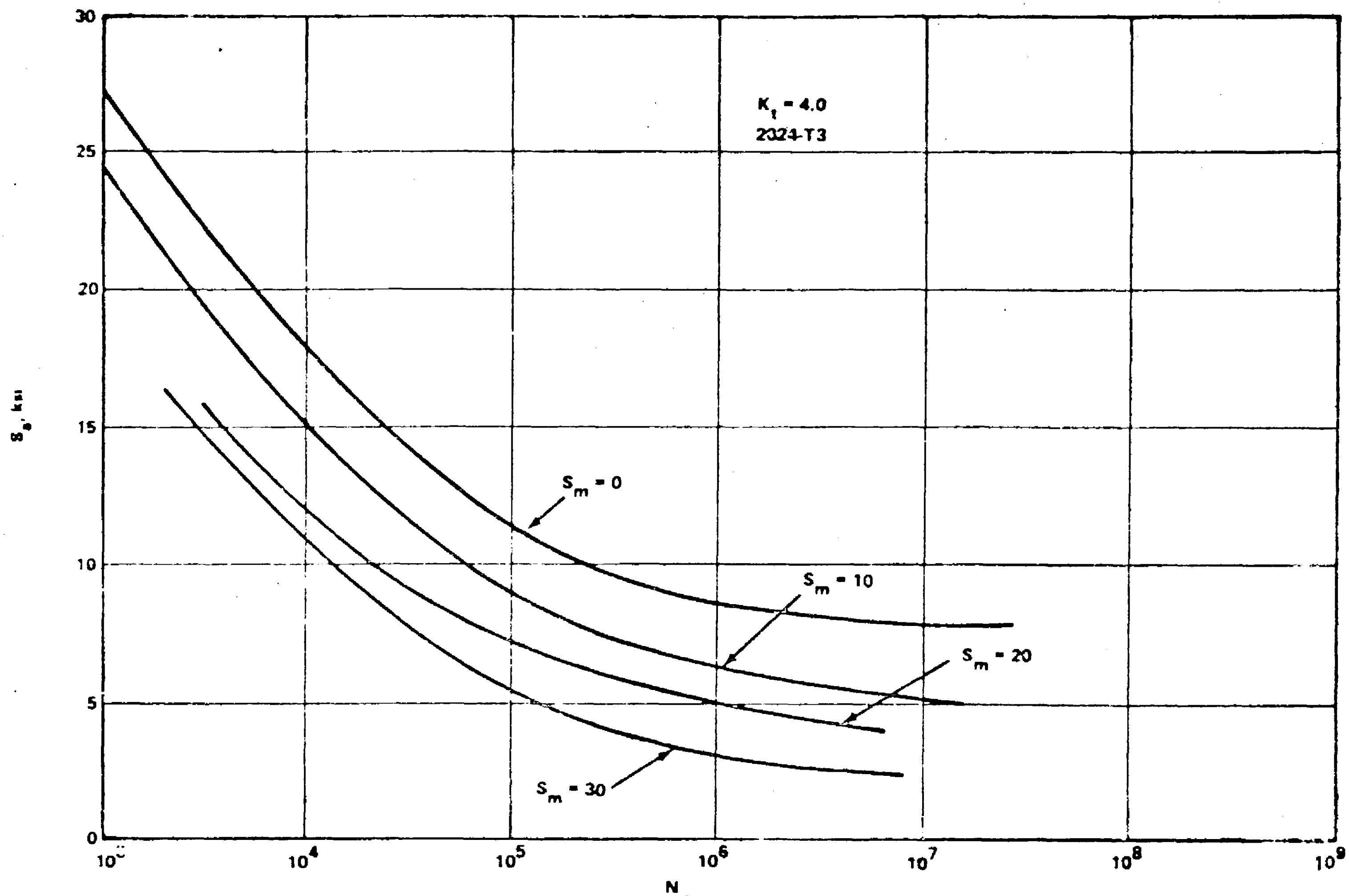


FIGURE E1-37. ALTERNATING STRESS VERSUS CYCLES TO FAILURE FOR 2024-T3.

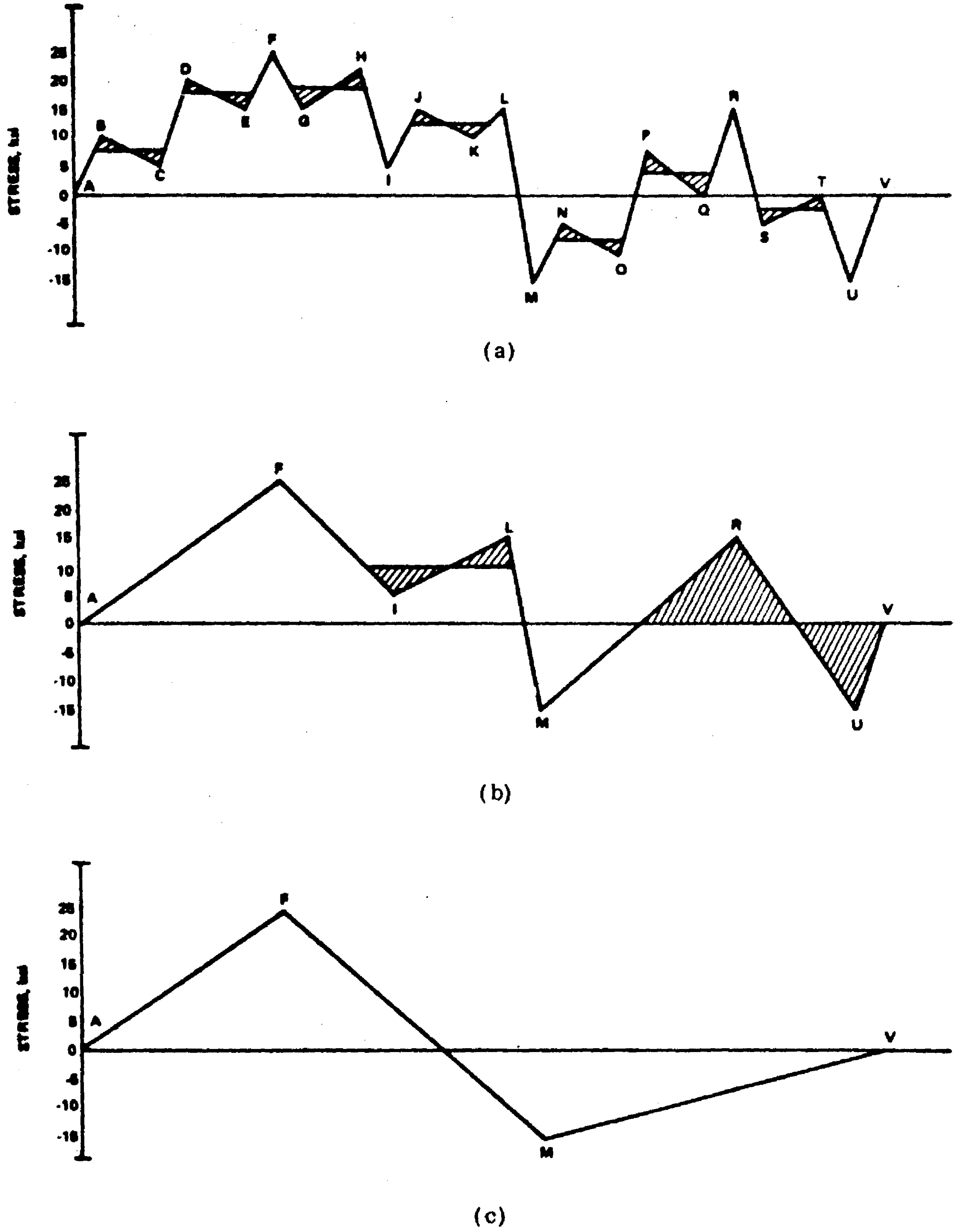


FIGURE E1-38. STRESS-TIME HISTORY CURVES

1.6 MATERIAL SELECTION TO RESIST FATIGUE.

1.6.1 High Cycle.

The first step to be considered in fatigue design is selection of the structural material. The four most commonly used materials are steel, titanium, aluminum, and magnesium. Unnotched rotating-beam fatigue data are presented in Fig. E1-39 and notched data are shown in Fig. E1-40. Each alloy was selected because it has the highest fatigue strength in its class, based on available data, i.e., 4340 heat-treated 260-280 ksi has the greatest fatigue strength of all the steels. For the unnotched specimen, Ti-155A, titanium is, by far, the best material. The three other materials, 7075-T6 aluminum, AZ80A-F magnesium, and 4340 steel heat-treated 260-280 ksi are approximately equal. It should be noted that titanium and steel have well-defined fatigue limits.

For the notched specimen also, Ti-155A, titanium is the best material, as shown in Fig. E1-40. The three other materials, 2024-T4 aluminum, AZ80A-F magnesium, and 4340 steel heat-treated 260-280 ksi, are approximately equal. For the notched specimen, all four have well-defined fatigue limits. Only one class of structural material, aluminum, changed alloys as a result of selecting the best fatigue strength in unnotched and notched specimens. The notch used had a stress concentration factor of approximately 2.8.

Rotating-bending specimens usually give different results than axial load specimens due to the large stress gradients produced in the bending tests and which are not present in axial tests. Different forms (e.g., sheet, bar, rod, etc.) sometimes cause different fatigue strengths. Therefore, an accurate selection of alloy for fatigue strength should be based on fatigue data with similar loading, form, and stress concentration factor as the particular structure in question.

1.6.2 Low Cycle.

Different materials may show substantially different abilities to resist repeated straining and repeated stressing; also, they have various abilities to retain their properties when subjected to large cyclic plastic strains. Because of these differences, the relative low-cycle fatigue resistance of several candidate materials should be rated, preferably without going through the arduous task of extensive fatigue testing (Ref. 23).

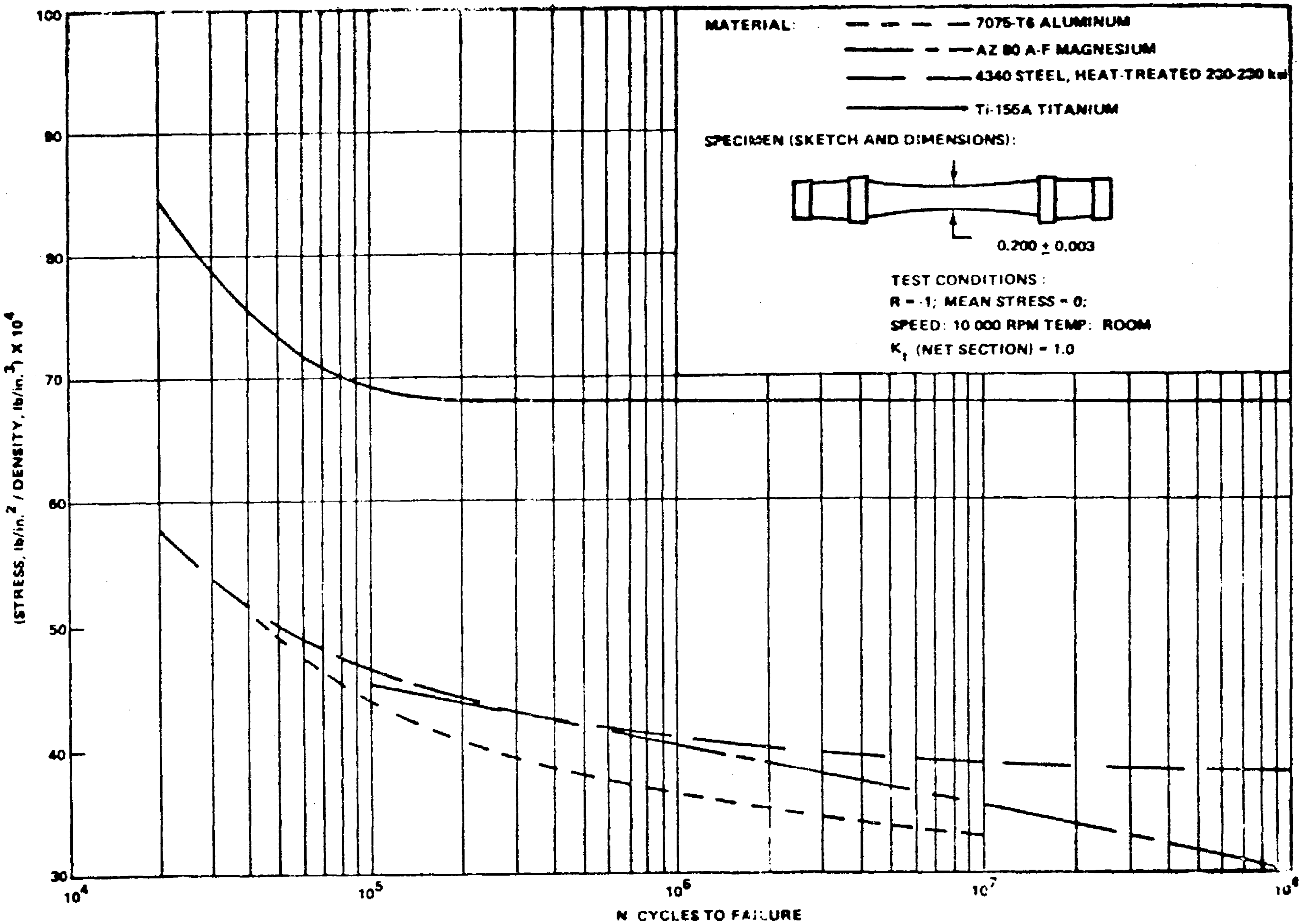


FIGURE E1-39. UNNOTCHED FATIGUE DATA FOR FOUR COMMON MATERIALS

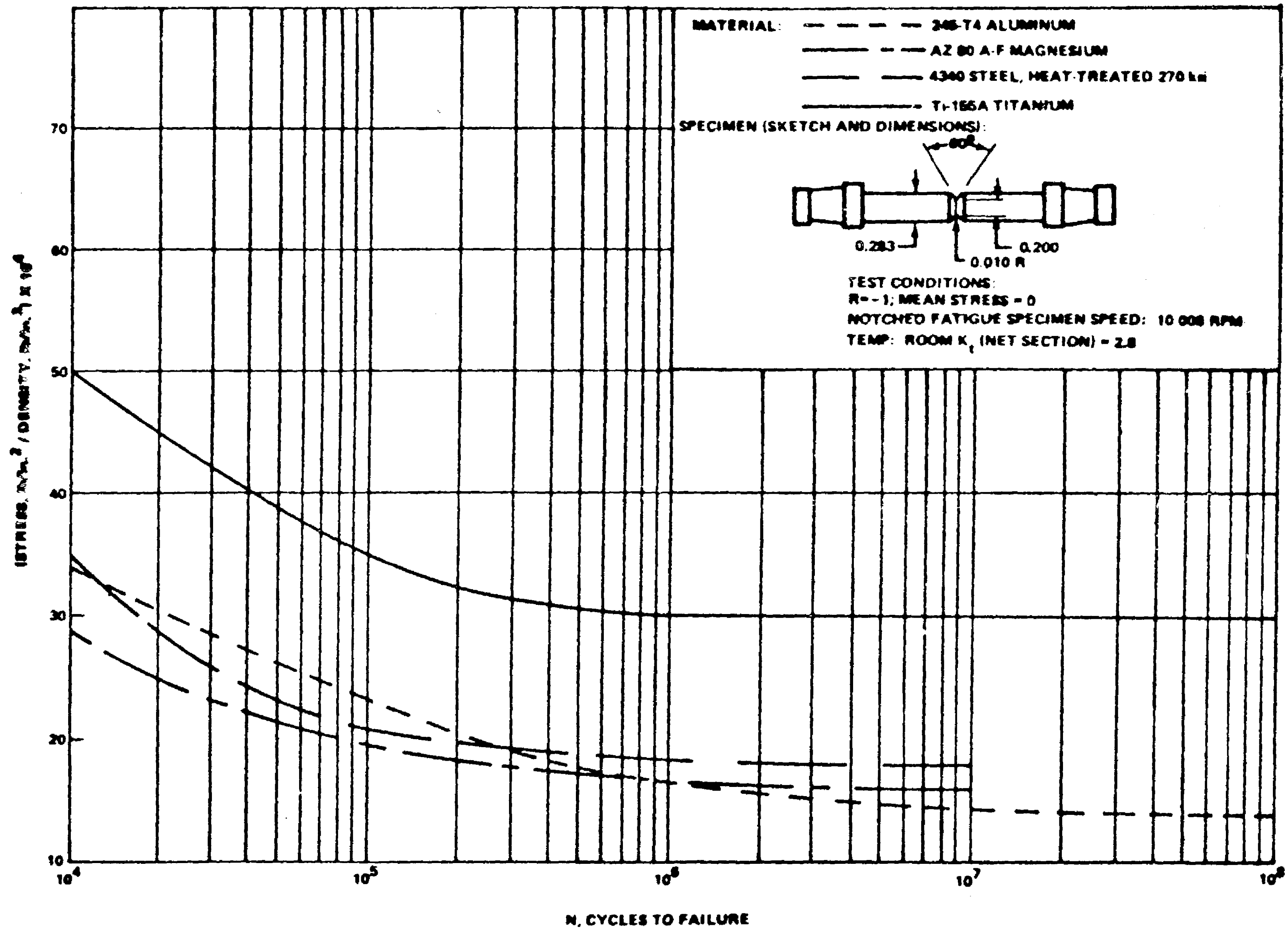


FIGURE E1-40. NOTCHED FATIGUE DATA FOR FOUR COMMON MATERIALS

A distinguishing difference between high-cycle and low-cycle fatigue is that in the latter the plastic strain component is considerably larger. These large plastic strains can induce significant changes in the stress-strain response of most materials. The most suitable test for studying these changes in deformation resistance is one in which the specimen is cycled between fixed limits of strain, rather than stress. During cyclic straining, a mechanical hysteresis loop develops (Fig. E1-41). The controlled parameter is total strain range ΔE_t, which is composed of an elastic and plastic component. Plastic component ΔE_p is the width of the hysteresis loop; whereas, the height of the hysteresis loop is $2\sigma_a$, where σ_a is the stress amplitude.

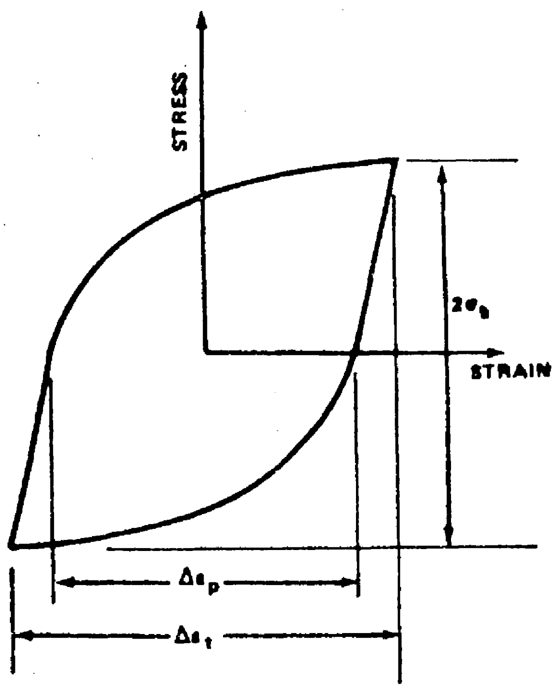
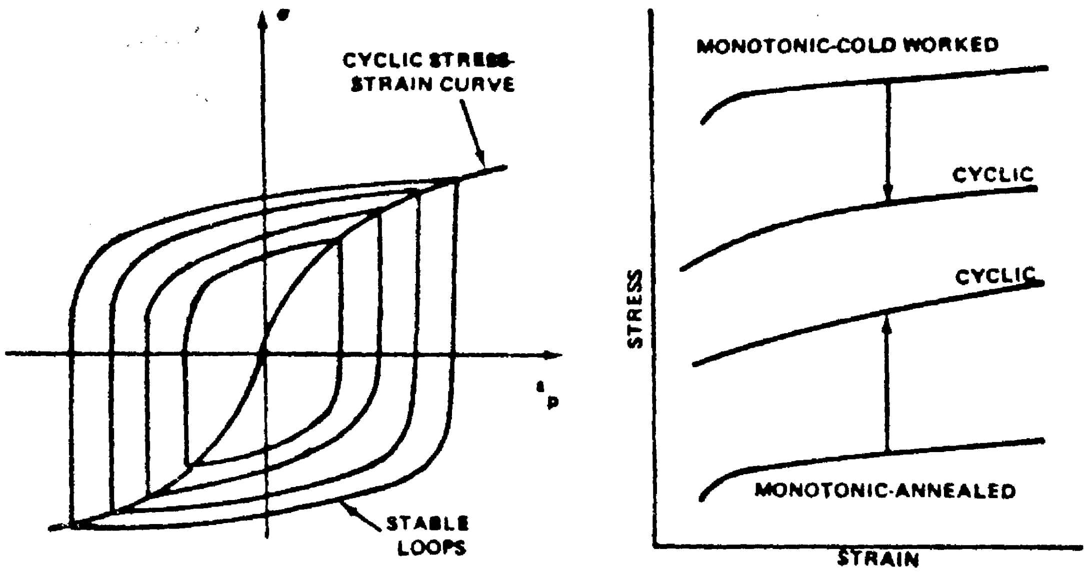


FIGURE E1-41. SCHEMATIC OF MECHANICAL HYSTERESIS LOOP

During cyclic straining, materials either harden or soften, depending upon their previous history. For example, annealed materials will generally undergo a cyclic hardening process, which is indicated by an increase in the stress required to enforce the strain limit on successive cycles. On the other hand, cold-worked materials generally soften. Changes in the stress response occur rapidly in the early portion of the life but reach a reasonably stable level or steady-state condition after about 10 to 20 percent of the life.

After this transient stage, a steady-state or saturation condition is attained during which the hysteresis loops maintain an essentially constant shape until prior to complete fracture. The curve drawn through the tips of these stabilized hysteresis loops (obtained from specimens tested at different amplitudes) is called the cyclic stress-strain curve, Fig. E1-42(a). It provides a convenient description of the steady-state cyclic stress-strain response of a material. Thus, monotonic (static) and cyclic stress-strain curves may be displayed on the same diagram, Fig. E1-42(b), so that the effect of different variables can be represented in a concise way.



(a) (b)

FIGURE E1-42. CYCLIC STRESS-STRAIN CURVE, AS DETERMINED FROM SEVERAL HYSTERESIS LOOPS, (a). COMPARISON WITH MONOTONIC CURVE, (b)

Although metals may undergo cyclic hardening or softening, the problem of softening is more important, since the strength properties of the material during low-cycle fatigue may be appreciably less than one would expect from the static stress-strain curve. Two questions, therefore, seem to be important: Will the material cyclically soften? If so, how much will it soften?

A component, or a critical location in a structure, may undergo either repeated straining or repeated stressing, depending upon the geometry and load environment. Some materials show good strain resistance, whereas others show good stress resistance. It is, therefore, necessary to decide whether a design requires a strain-resistant or stress-resistant material.

The relation between the total strain amplitude, $\Delta\epsilon_t/2$, and the cycles to failure, N_f, is given by

$$\frac{\Delta\epsilon_t}{2} \approx \frac{\Delta\epsilon_p}{2} = \epsilon_f{}' \, (2N_f)^c \quad ,$$

where c is the fatigue ductility exponent.

Thus, the short-life strain resistance of materials depends mainly upon the fatigue ductility coefficient, ϵ_f', and c. Using typical experimental values for the constants, it is apparent that a material with a large value of ϵ_f' and a small value of c has the best resistance to repeated straining.

The fact that a material shows good strain resistance does not necessarily imply that it has a good cyclic stress resistance. To point out the importance of deciding whether the material to be selected must resist repeated strains or repeated stresses, seven materials were tested for both stress and strain resistance in Ref. 23. The material with the highest stress resistance was fourth in ability to resist cyclic strain. Likewise, the material third best in cyclic strain resistance dropped to sixth in stress resistance.

1.7 DESIGN GUIDES.

At the present level of knowledge, it is impractical to establish quantitatively the limiting fatigue design rules for specific structural configurations. The diversity in missions, loads, stresses, materials, and environments certainly suggests this to be an impracticable, if not impossible, task. Qualitatively, however, the practices to be followed in the design of fatigue resistant structure can be defined, and if strict adherence to these established practices is maintained, potential fatigue problems can be reduced in the initial stage of structural design.

In general, the basic rules used in the airframe industry will apply in the design of space vehicle systems. Past experience and learning from the one field can be carried over to the other, although some precautions should be taken. Old designs may be unsuited to a new environment.

When procedures are proposed, they often become general rules which regulate structural design, even though they may be inappropriate. Recommendations used in the sense of "design rules" are far more appropriate. With this interpretation, a few of the more pertinent guides for the design of space vehicle systems may be listed:

1. Keep the design simple.

2. Provide for multiple load paths when feasible.

3. Give extra consideration to tension-loaded fittings and components.

4. Apply fitting factors of safety to net stresses around holes and cutouts.

5. Laboratory test all newly designed joints and compare with "time-tried" structures.

6. Utilize longitudinal grain direction of materials whenever possible (particularly for aluminum and steel alloys).

7. Provide generous fillets and radii.

8. Break all sharp edges; polish critical regions if it is considered necessary.

9. Reduce bearing stresses in riveted and bolted members to design minimums.

10. Take precautions to protect parts from corrosion.

11. Whenever possible, reduce eccentricity of joints and fittings.

12. Ensure that doublers and structural reinforcements result in gradual, rather than abrupt, changes in cross section.

13. Provide easy access for service inspection of structure.

14. Provide inspection procedures during fabrication and assembly of structure.

15. When practical, produce parts and fittings from forged material rather than from extrusions or machined plate stock.

16. Design parts for minimum mismatch or installation; this results in lower residual and preload tensile strains.

17. Avoid superposition of "notches" in design.

18. Make a proper selection of materials with cost, strength allowables, fabricability, and environmental effects in mind.

19. Pay close attention to fabrication techniques for optimum forming of components.

20. Establish reliable welding techniques for reproducibility of joints strengths.

21. Construct rigid and precision tooling for the manufacturer of production parts.

Undoubtedly, there are additional useful fatigue guides which could be added to the list above. Many such guides are unwritten and only intuitively known by the most experienced design specialists. Excellence in design, however, is not accomplished by the designer alone. It requires the close cooperation of specialists able to perform complex dynamic stress analyses: acousticians, vibration engineers, metallurgists, specialists in structural testing and reliability analysis, as well as those experienced in tooling and manufacturing.

These guides and recommended practices for designers are suggested for the sole purpose of reducing the overall development time from preliminary design layout to assembly in production.

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