3.4 COMPUTER PROGRAMS IN SHELL STABILITY ANALYSIS.

The names of various digital computer programs are listed in Table 3.4-1, which indicates their scope for a shell stability analysis. Three classes of problems are specified: cylindrical shells, shells of revolution, and general shells.

Table 3.4-1. Computer Programs for Shell Stability Analysis

| Types of Shells | Symmetric System Nonsymmetric Displacements | Nonsymmetric System |
|-------------------------|---|---|
| Cylindrical Shells | CORCYL ^a DBSTAB ^a SCAR MARK IV | INTACT STAGS |
| Shells of Revolution | BOSOR ^a BOSOR3 SABOR3-F | |
| General Shell | | BERK3 NASTRAN ^a REXBAT |

a. Programs available for use at Marshall Space Flight Center.

Often the stability analysis of cylindrical shells can be solved in closed form. Those concerning shells of revolution can frequently be simplified by separation of variables. Variations in the circumferential direction are assumed to be periodic, and the method of superposition is used for the linear analysis of shells of revolution subjected to nonsymmetric loads. The meridional variation is determined by series expansion, the method of finite

differences, numerical integration, or the method of finite elements. For the analysis of general shells, however, a two-dimensional numerical analysis is required, since the variables cannot be separated. The core storage required and the computer time per case increase very rapidly as the number of mesh points or terms in a double series expansion increases.

Tables 3.4-2, 3.4-3, and 3.4-4 list the programs by name, cite References 1 through 12 in which they are documented, specify the method of numerical analysis used, and briefly describe the major features of the analysis.

In general, CORCYL, DBSTAB, SCAR, MARK IV, INTACT, BOSOR, BOSOR3, and SABOR3-F might be expected to have a higher "confidence index" than the other programs. This is not because of defects in the programs but because of the relative ease of proving that convergence has been obtained. In STAGS, BERK3, NASTRAN, AND REXBAT, core storage is often too small to ascertain conclusively that the stresses obtained are accurate to within a percent or so. In addition, convergence checks are generally very expensive in terms of computer time. Most of the computer programs requiring a two-dimensional numerical analysis are harder to use than those requiring a one-dimensional numerical analysis since more input data must be specified.

A simplified input and output explanation (along with example problems) is given in an MSFC internal document¹ for the programs CORCYL, DBSTAB, BOSOR, and NASTRAN.

^{1.} Structural Analysis Computer Utilization Manual, Astronautics Laboratory, NASA/MSFC (to be published).

Table 3.4-2. Computer Programs for Stability Analysis of Cylindrical Shells

| Program Name | Reference No. | Method of Analysis ^a | Comments |
|-----------------|------------------|---------------------------------------|--|
| CORCYL | 1 | 1 | Linear small-deflection theory. Ring-stiffened corrugated cylinder under axial compression. Rings are distributed along the cylinder. Eccentricity of rings with respect to corrugation centerline is considered. |
| DBSTAB | 2 | 1 | Small-deflection theory. Orthogonally stiffened cylindrical shell under axial compression and lateral pressure. Restricted to moderately or heavily stiffened cylinders. Rings and stringers are considered eccentric with respect to the skin's middle surface. Local buckling of the skin between adjacent stringers before general instability is allowed, and the resulting reduction in skin stiffness is determined. |
| SCAR | 3 | 1 | Membrane prebuckled theory and simply supported edges. Various types of wall construction permitted, as well as combined pressure and axial compression. Ring stiffeners and longitudinal stringers permitted. |
| MARK IV | 4 | 1 | SCAR-type analysis for optimiza- tion of integrally stiffened cylinders with respect to weight. |

Table 3.4-2. (Concluded)

| Program Name | Reference No. | Method of Analysis ^a | Comments |
|-----------------|------------------|---------------------------------------|--|
| INTACT | 5 | 2 | Buckling of cylinders under bend- ing, axial compression, and pres- sure. Interaction curves calculated. Otherwise, same as SCAR. |
| STAGS | 6 | 4 | Nonlinear analysis. Large deflections and elastic-plastic behavior permitted. Discrete rings and stringers included. Maximum number of unknowns is 4300. |

a. Method of Analysis:

- i = Closed form
- 2 = Series expansion
- 3 = Numerical integration
- 4 = Finite difference
- 5 = Finite element

Table 3.4-3. Computer Programs for Stability Analysis of Shells of Revolution

| Program Name | Reference No. | Method of Analysis | Comments |
|-----------------|------------------|--------------------------|--|
| BOSOR | 7 | 4 | Nonlinear prebuckling effects. General with respect to geometry of meridian, shell wall design, edge conditions, and loading. Axisymmetric loading. |
| BOSOR3 | 8 | 4 | Rings can be treated as discrete elastic structures. Option of non-linear prebuckling effects or linear bending theory for prebuckling analysis. Segmented shells can be analyzed with each segment independent of other segments. Otherwise, same as BOSOR. |
| SABOR3-F | $9^{\mathbf{a}}$ | 5 | Calculation of vibration frequencies of stacked and branched shells. |

a. Also, W. A. Loden: SABOR3-F/EIGSYS Instructions, unpublished report, Lockheed Missiles and Space Company, August 1967.

Table 3.4-4. Computer Programs for Stability Analysis of General Shells

| Program Name | Reference No. | Method of Analysis | Comments |
|-----------------|------------------|--------------------------|---|
| BERK3 | 10 | 5 | Flat triangular elements with extensional and bending stiffness are used for the calculation of stresses and vibration frequencies of general shells or shells of revolution with cutouts. Up to 6000 unknowns can be handled. Discrete stiffeners permitted. |
| NASTRAN | 11 | 5 | General-purpose program for elastic structural analysis. Not restricted to shells. Contains library of elements including rods, beams, shear panels, plates, and shells. |
| REXBAT | 12 | 5 | General-purpose program for linear structural analysis with respect to static stresses and vibration frequencies. Up to 6000 unknowns can be handled. |

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