SECTION C3

STABILITY OF SHELLS

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DEFINITION OF SYMBOLS

Symbol	Definition
A_s, A_r	Stiffener area, and ring area, respectively
A, B	Lengths of semiaxes of ellipsoidal shells
a	Radius of curvature of circular toroidal-shell cross section
B_1	Extensional stiffness of isotropic sandwich wall
b	Stiffener spacing; also, distance from center of circular cross section of circular toroidal shell cross section to axis of revolution
b_e	Effective width of skin between stiffeners
C_{ij}	Coefficients of constitutive equations
$\overline{C}_x$, $\overline{C}_y$, $\overline{C}_{xy}$, $\overline{K}_{xy}$	Coupling constants for orthotropic cylinders
c	Coefficient of fixity in Euler column formula
D	Wall flexural stiffness per unit width, $\frac{Et^3}{12(1 - \mu^2)}$
D_q	Transverse shear-stiffness parameter for isotropic sandwich wall, $G_{xz} \frac{h^2}{h - \frac{1}{2}(t_1 + t_2)}$
$\overline{D}_x$, $\overline{D}_y$	Bending stiffness per unit width of wall in x- and y-directions, respectively; $\overline{D}_x = \overline{D}_y = D$ for isotropic cylinder

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
$\overline{D}_{xy}$	Modified twisting stiffness of wall; $\overline{D}_{xy} = 2D$ for isotropic cylinder
D_1	Flexural stiffness of isotropic sandwich wall, $\frac{E_s th^2}{2(1 - \mu^2)}$
d	Ring spacing
E	Young's modulus
E_R	Reduced modulus
E_S , E_f	Young's moduli: face sheet; sandwich, respectively
E_c	Young's modulus of elastic core
E_r , E_s	Young's moduli: rings; stiffeners, respectively
E_s , E_θ	Young's moduli of orthotropic material in the s- and θ-directions, respectively
E_{sec}	Secant modulus for uniaxial stress-strain curve
E_{tan}	Tangent modulus for uniaxial stress-strain curve
E_x , E_y	Young's modulus of orthotropic material in x- and y-directions, respectively
E_z	Young's modulus of sandwich core in direction perpendicular to face sheet of sandwich
E_1 , E_2	Young's moduli of the face sheets for isotropic sandwich shell

Symbol	Definition
$\overline{E}$	Equivalent Young's modulus for isotropic sandwich shell
$\overline{E}_s$, $\overline{E}_\theta$	Equivalent Young's moduli of orthotropic material in the s- and θ-directions, respectively
$\overline{E}_x$, $\overline{E}_y$, $\overline{E}_{xy}$	Extensional stiffness of wall in x- and y-directions, respectively; $\overline{E}_x = \overline{E}_y = \frac{Et}{1 - \mu^2}$, $\overline{E}_{xy} = \frac{\mu Et}{1 - \mu^2}$ for isotropic cylinder
e_r	Distance of the centroid of the ring-shell combination from the middle surface
f	Ratio of minimum to maximum principal compressive stress in face sheets
G	Shear modulus
G_s , G_r	Shear moduli: stiffeners; rings, respectively
G_{sz}	Shear modulus of core of sandwich wall in s-z plane
G_{xy}	Inplane shear modulus of orthotropic material
G_{xz} , G_{yz}	Shear moduli of core of sandwich wall in x-z and y-z planes, respectively
$\overline{G}$	Equivalent shear modulus
$\overline{G}_{xy}$	Shear stiffness of orthotropic or sandwich wall in x-y plane; $\overline{G}_{xy} = Gt$ for isotropic cylinder

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
h	Depth of sandwich wall measured between centroids of two face sheets
$\bar{I}$	Moment of inertia per unit width of corrugated cylinder
I_r , I_s	Moments of inertia of rings and stiffeners, respectively, about their centroid
J_r , J_s	Beam torsion constants of rings and stiffeners, respectively
k_p	Buckling coefficient of cylinder subject to hydrostatic pressure, $pr\ell^2/\pi^2 D$
k_{pc}	Buckling coefficient of cylinder with an elastic core subject to lateral pressure, pr^3/D
k_x	Buckling coefficient of cylinder subject to axial compression, $N_x \ell^2/\pi^2 D$ or $N_x \ell^2/\pi^2 D_1$
k_{xy}	Buckling coefficient of cylinder subjected to torsion, $N_{xy} \ell^2/\pi^2 D$ or $N_{xy} \ell^2/\pi^2 D_1$
k_y	Buckling coefficient of cylinder subject to lateral pressure, $N_y \ell^2/\pi^2 D$ or $N_y \ell^2/\pi^2 D_1$
L	Slant length of cone
L_o	Ring spacing measured along cone generator
ℓ	Length of cylinder, axial length of cone, or length of toroidal-shell segment

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
M	Bending moment on cylinder or cone
M_{cr}	Critical bending moment on cone or cylinder
M_{press}	Bending moment at collapse of a pressurized cylinder or cone
$M_{p=0}$	Bending moment at collapse of a nonpressurized cylinder
M_t	Twisting moment on cylinder
M_1 , M_2 , M_{12}	Moment resultants per unit of middle surface length
m	Number of buckle half-waves in the axial direction
N	Axial tension force per unit circumference applied to a toroidal segment
N_o	$\frac{2\gamma E}{\sqrt{1-\mu^2}} \frac{h}{r} \sqrt{t_1 t_2}$
N_x	Axial load per unit width of circumference for cylinder subjected to axial compression
N_{xy}	Shear load per unit width of circumference for cylinder subjected to torsion
N_y	Circumferential load per unit width of circumference for cylinder subjected to lateral pressure
N_1 , N_2 , N_{12}	Force resultants per unit of middle surface length
n	Number of buckle waves in the circumferential direction

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
P	Axial load on cylinder or cone; concentrated load at apex of spherical cap
P_{cr}	Critical axial load on cone; critical concentrated load at apex of spherical cap
$P_{p=0}$	Axial load on nonpressurized cylinder at buckling
P_{press}	Axial load on pressurized cylinder at buckling
p	Applied uniform internal or external hydrostatic pressure
$p_{c\ell}$	Classical uniform buckling pressure for a complete spherical shell
p_{cr}	Critical hydrostatic (uniform) pressure
R	Shear flexibility coefficient, $\frac{\pi^2 D}{\ell^2 D_q}$
R_A	Effective radius of a thin-walled oblate spheroid, $\frac{B}{A}$
R_b	Ratio of bending moment on cylinder or cone subjected to more than one type of loading to the allowable bending moment for the cylinder or cone when subjected only to bending
R_c	Ratio of axial load in cylinder or cone subjected to more than one type of loading to the allowable axial load for the cylinder or cone when subjected only to axial compression

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
R_m	Maximum radius of torispherical shell
R_p	Ratio of external pressure on cylinder or cone subjected to more than one type of loading to the allowable external pressure for the cylinder or cone when subjected only to external pressure
R_s	Radius of spherical shell
R_t	Ratio of torsional moment on cylinder or cone subjected to more than one type of loading to the allowable torsional moment for the cylinder or cone when subjected only to torsion
R_{tr}	Toroidal radius of torispherical shell
r	Radius of cylinder, equivalent cylindrical shell or equator of toroidal shell segment
r_1	Radius of small end of the cone
r_2	Radius of large end of the cone
S	Cell size of honeycomb core
s	Distance along cone generator measured from vertex of cone
s_1	Distance along cone generator measured from vertex of cone to small end of cone
T	Torsional moment on cone

Symbol	Definition
T_{cr}	Critical torsional moment on cone
t	Skin thickness of isotropic cylinder or cone; thickness of corrugated cylinder
$\bar{t}$	Effective thickness of corrugated cylinder; area per unit width of circumference; effective skin thickness of isotropic sandwich cone
t_f	Face thickness of sandwich cylinder having faces of equal thickness
t_k	Skin thickness of k^{th} layer of layered cylinder
t_1 , t_2	Facing-sheet thicknesses for sandwich construction having faces of unequal thickness
x, y, z	Coordinates in the axial, circumferential, and radial directions, respectively
Z	Curvature parameter: $\frac{\ell^2}{rt}\sqrt{1-\mu^2}$ for isotropic cylinder and toroidal-shell segment; $2\frac{\ell^2}{rh}\sqrt{1-\mu^2}$ for isotropic sandwich cylinder
$\tilde{z}_k$	Distance of center of k^{th} layer of layered cylinder from reference surface (positive outward)
$\tilde{z}_s$, $\tilde{z}_r$	Distance of centroid of stiffeners and rings, respectively, from reference surface (positive when stiffeners or rings are on outside)

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
α	Semivertex angle of cone
β	Buckle aspect ratio $\left(\frac{n\ell}{\pi r m}\right)$
γ	Correlation factor to account for difference between classical theory and predicted instability loads
Δ	Distance of reference surface from inner surface of layered wall
$\Delta\gamma$	Increase in buckling correlation factor resulting from internal pressure
δ	Ratio of core density of honeycomb sandwich plate to density of face sheet of sandwich plate
δ_k	Distance of centroid of k^{th} layer of layered cylinder from inner surface
ϵ_1 , ϵ_2 , ϵ_{12}	Reference-surface strains
η	Plasticity reduction factor
η_o	Ring-geometry parameter
θ	Coordinate in the circumferential direction
λ	Spherical-cap geometry parameter
μ	Poisson's ratio
μ_c	Poisson's ratio of core material
μ_s , μ_θ	Poisson's ratios associated with stretching of an orthotropic material in the s- and θ-directions, respectively

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
μ_x , μ_y	Poisson's ratios associated with stretching of an orthotropic material in x- and y-directions, respectively
$\bar{\mu}_s$, $\bar{\mu}_\theta$	Equivalent Poisson's ratios in the s- and θ-directions, respectively
$\bar{\rho}$	Average radius of curvature of cone, $\frac{r_1 + r_2}{2 \cos \alpha}$
ρ_1	Radius of curvature at small end of cone, $\frac{r_1}{\cos \alpha}$
ρ_2	Radius of curvature at large end of cone, $\frac{r_2}{\cos \alpha}$
σ_N	Normal stress
σ_{max}	Maximum membrane compressive stress
σ_p	Critical axial stress for a cylinder with an elastic core
σ_s	Local failure stress
$\sigma_{x_{cr}}$	Axial stress, critical
σ_y	Circumferential stress
τ	Shear stress
τ_{cr}	Torsional buckling stress of an unfilled cylinder; critical shear stress
$\tau_{xy_{cr}}$	Shear stress in the x-y plane, critical
ϕ	Half the included angle of spherical cap

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
$\bar{\phi}_2$	Half the included angle of spherical cap portion of torispherical closure
$\kappa_1, \kappa_2, \kappa_3$	Reference-surface curvature changes
ψ_1 , ψ_2	[See eq. (17).]

C3.0 STABILITY OF SHELLS.

The buckling load of a shell structure is defined as the applied load at which an infinitesimal increase in that load results in a large change in the equilibrium configuration of the shell. The change in equilibrium configuration is usually a large increase in the deflections of the shell, which may or may not be accompanied by a change in the basic shape of the shell from the prebuckled shape. For most types of shells and loading conditions, the buckling load is quite pronounced and easily identified.

The load-carrying capability of the shell may or may not decrease after buckling depending on the type of loading, the geometry of the shell, the stress levels of the buckled shell, etc. Only the buckling load will be discussed in this section because the information available on collapse load is very limited. In general, the buckling load and collapse load are nearly the same, but if they are different the deformations before the collapse are often very large.

The magnitude of the critical static load of a structure generally depends on its geometric proportions, the manner in which it is stiffened and supported, the bending and extensional stiffnesses of its various components, or other reasonably well-defined characteristics. For thin-walled shell structures less certain characteristics, such as small deviations of the structure from its nominal unloaded shape, may also have quite important effects on the load at which buckling will occur. Other factors that affect buckling, such as nonuniform stiffnesses, and variation of loading with time are not considered here.

For columns and flat plates, the classical small deflection theory predicts the buckling load quite well; in general, the theoretical buckling load is used as the design allowable buckling load. However, this method of analysis cannot be used generally for shell structures. The buckling load for some types of shells and loadings may be much less than the load predicted by classical small deflection theory and, in addition, the scatter of the test data may be quite large. Explanations for these discrepancies are discussed in References 1 through 5. When sufficient data exist, a statistical reduction of the test data may be useful in determining a design allowable buckling load. This method has been used to determine most of the design curves presented in this section.

One of the primary shortcomings of this method of obtaining design curves is that the test specimens and boundary conditions used to obtain the design curves may not be typical of the particular structure which the design curves are being used to analyze. However, until additional information on shell stability is obtained, a statistical analysis has been used whenever possible to obtain design curves.

Whenever sufficient data do not exist to obtain a statistical design allowable buckling load, design recommendations are made on available information. Usually this involves recommending correction factors or "knockdown" factors to reduce the theoretical buckling loads. Such a recommendation may be too conservative in some cases; nevertheless, further theoretical and experimental investigations are necessary to justify raising the design curves.

Most analysis procedures presented in this section are for shells with simply supported edges. For most applications simply supported edges should be assumed unless test results are obtained which indicate the effects of the actual boundary condition of the design.

An attempt has been made to simplify the analysis procedures so that the design allowable buckling loads may be obtained from hand computations and graphs. The analyses which have been presented are sometimes quite long (orthotropic cylinders, for instance) but, in general, results can be obtained quickly with a few simple computations. In many cases, computer programs are available for a more sophisticated analysis. The applicable programs are described, and their limitations and availability are noted. They can generally be obtained from COSMIC or from the Computer Utilization Manual.

As more information on shell stability becomes available, revisions to this section will be made. However, the analyst should attempt to keep abreast of the changes in current technology because of recent theoretical and experimental investigations.