SECTION C

STABILITY ANALYSIS

SECTION C1

COLUMNS

TABLE OF CONTENTS

C 1.0.0 COLUMNS

C 1.1.0 Introduction

In general, column failure may be classed under two headings:

(1) Primary failure (general instability)

(2) Secondary failure (local instability)

Primary or general instability failure is any type of column failure, whether elastic or inelastic, in which the cross-sections are translated and/or rotated but not distorted in their own planes. Secondary or local instability failure of a column is defined as any type of failure in which cross-sections are distorted in their own planes but not translated or rotated. However, the distinction between primary and secondary failure is largely theoretical because most column failures are a combination of the two types.

Fig. C 1.1.0-1 illustrates the curves for several types of column failure.

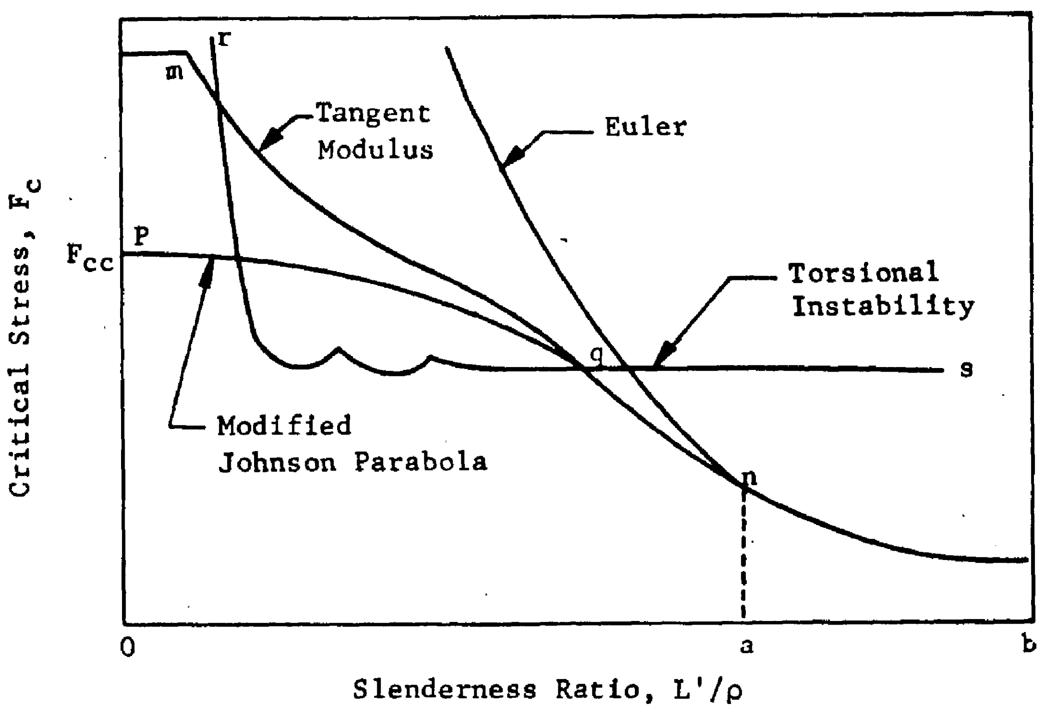


Fig. C 1.1.0-1

L' represents the effective length of the column and is dependent upon the manner in which the column is constrained, and ρ is the minimum radius of gyration of the cross-sectional area of the column.

For a value of L'/ρ in the range "a" to "b", the column buckles in the classical Euler manner. If the slenderness ratio, L'/ρ, is in

C 1.1.0 Introduction (Cont'd)

the range of "0" to "a", a column may fail in one of the three following ways:

(1) Inelastic bending failure. This is a primary failure described by the Tangent Modulus equation, curve mn. This type of failure depends only on the mechanical properties of the material.

(2) Combined inelastic bending and local instability. The elements of a column section may buckle, but the column can continue to carry load until complete failure occurs. This failure is predicted by a modified Johnson Parabola, "pq", a curve defined by the crippling strength of the section. At low values of L'/ρ the tendency to cripple predominates; while at L'/ρ approaching the point "q", the failure is primarily inelastic bending. Geometry of the section, as well as material properties, influences this combined type of failure.

(3) Torsional instability. This failure is characterized by twisting of the column and depends on both material and section properties. The curve "rs" is superimposed on Fig. C 1.1.0-1 for illustration. Torsional instability is presented in Section C 1.5.0.

These curves are discussed in detail in Sections C 1.3.0, C1.3.2, and C 1.5.0. For a given value of L'/ρ between (0) and (a), the critical column stress is the minimum stress predicted by these three failure curves.

Each of the Tangent Modulus curves has a cutoff stress at low L'/ρ values (point "m"). This cutoff stress has been chosen as $F_{0.2}$ for ductile material, and is the stress for which $E_t/E = 0.2$. E_t is the Tangent Modulus and E is the Modulus of Elasticity.

C 1.2.0 Long Columns (elastic buckling)

A column with a slenderness ratio (L'/ρ) greater than the critical slenderness ratio (point "a" Fig. C 1.1.0-1) is called a long column. This type of column fails through lack of stiffness instead of a lack of strength.

The critical column load, P_c, as given by the Euler formula for a pin ended column ($L' = L$) of constant cross section, is

$$P_c = \frac{\pi^2 EI}{(L)^2} \qquad (1)$$

where

E = Young's Modulus
I = Least moment of inertia
L = Length of the column

End conditions - The strength of a column is in part dependent on the end conditions; in other words, the degree of end-fixity or constraint. A column supported at the ends so that there can be neither lateral displacement nor change of slope at either end is called fixed-ended. A column, one end of which is fixed and the other end neither laterally supported nor otherwise constrained, is called free-ended. A column, both end-surfaces of which are flat and normal to the axis, and bear evenly against rigid loading surfaces, is called flat-ended. A column, the ends of which bear against transverse pins, is called pin-ended.

The critical load for long columns with various end conditions as shown in Fig. C 1.2.0-1 are:

$$\text{(a)} \quad P_c = \frac{\pi^2 EI}{4{L_1}^2} \qquad (2)$$

$$\text{(b)} \quad P_c = \frac{2.05\pi^2 EI}{{L_2}^2} \qquad (3)$$

$$\text{(c)} \quad P_c = \frac{4\pi^2 EI}{{L_3}^2} \qquad (4)$$

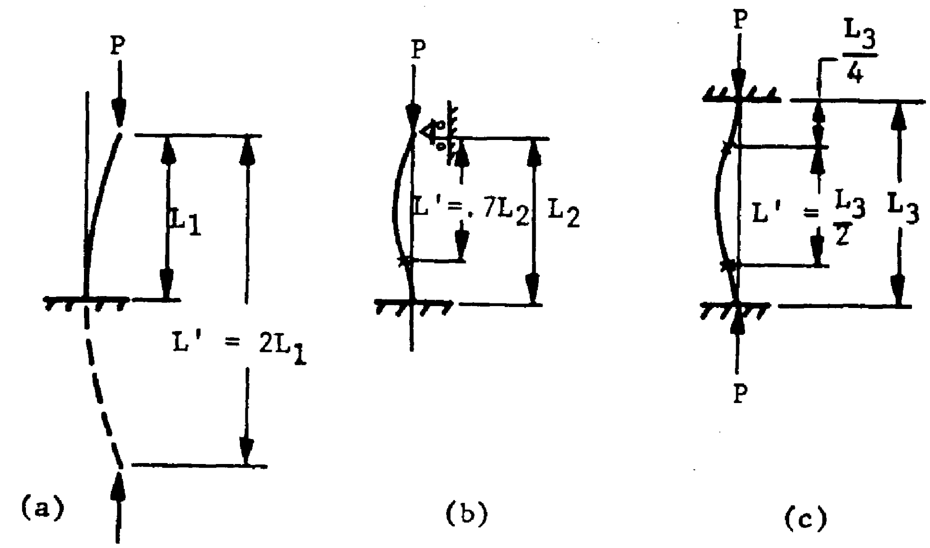


Fig. C 1.2.0-1

The effective column lengths L' for Fig. C 1.2.0-1 are $2L_1$, 0.7 L_2, and 0.5 L_3 respectively. For the general case, $L' = L/\sqrt{c}$, where c is a constant dependent on end restraints.

Fixity coefficients (c) for several types of elastically restrained columns are given in Figs. C 1.2.0-2 through C 1.2.0-4.

Limitations of the Euler Formulas. The elastic modulus (E) was used in the derivation of the Euler formulas. Therefore, all the reasoning is applicable while the material behaves elastically. To bring out this significant limitation, Eq. 1 will be written in a different form. By definition, $I = A\rho^2$, where A is the cross-sectional area and ρ is its radius of gyration. Substitution of this relation into Eq. 1 gives

$$P_c = \frac{\pi^2 EI}{(L')^2} = \frac{\pi^2 EA\rho^2}{(L')^2} \quad \text{---------------------------} \quad (5)$$

$$F_c = \frac{P_c}{A} = \frac{\pi^2 E}{\left(\frac{L'}{\rho}\right)^2} \quad \text{---------------------------------} \quad (6)$$

The critical stress (F_c) for a column is defined as an average stress over the cross-sectional area of a column at the critical load (P_c).

C 1.2.0 Long Columns (Cont'd)

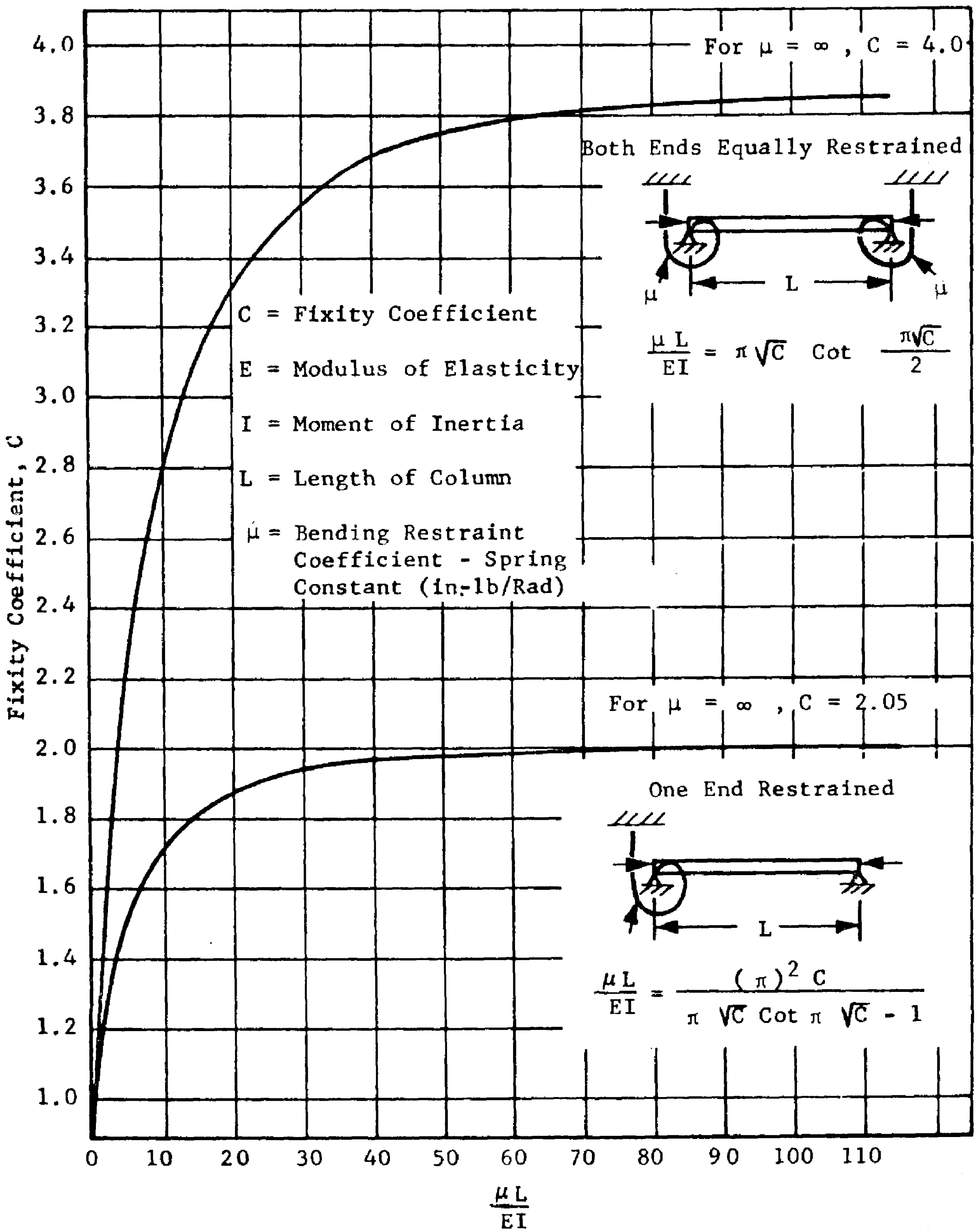


Fig. C1.2.0-2 Fixity Coefficient for a Column with End Supports Having a Known Bending Restraint

C 1.2.0 Long Columns (Cont'd)

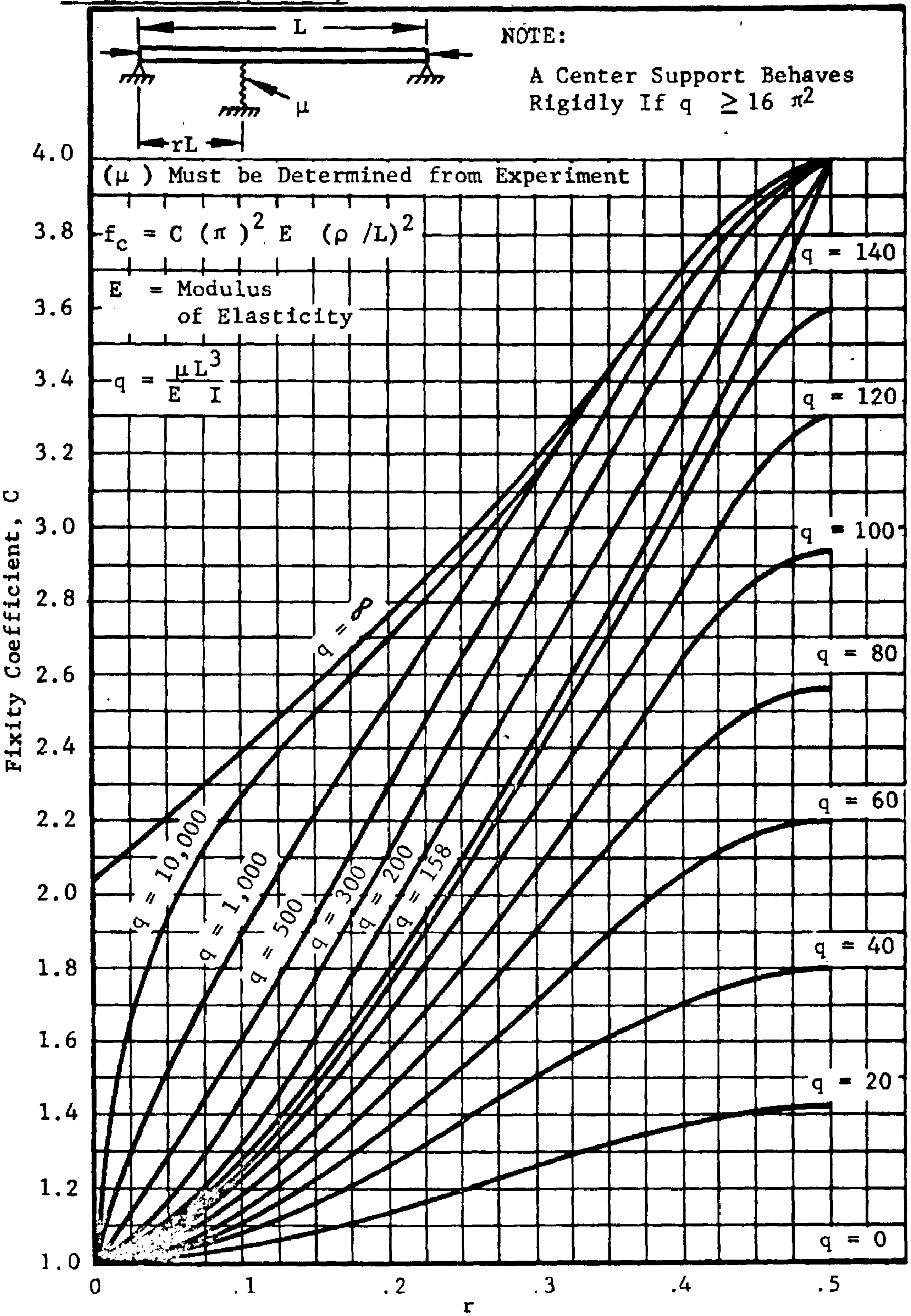
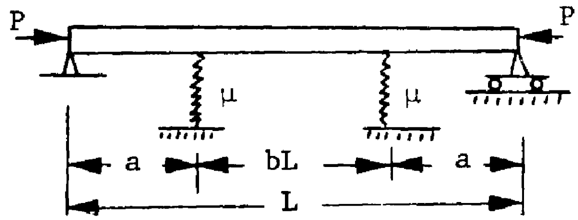


Fig. C1.2.0-3 Fixity Coefficient for a Column with Simply Supported Ends and an Intermediate Support of Spring Constant, μ

C 1.2.0 Long Columns (Cont'd)



$$q = \frac{\mu L^3}{8EI}$$

Where (μ) = Spring Constant Which Is Equal To The Number of Pounds Necessary To Deflect The Spring One Inch Extrapolated to Zero Deflection.

$$P_c = \frac{C\pi^2 EI}{L^2} \text{ or } f_c = \frac{C\pi^2 E}{(L/\rho)^2}$$

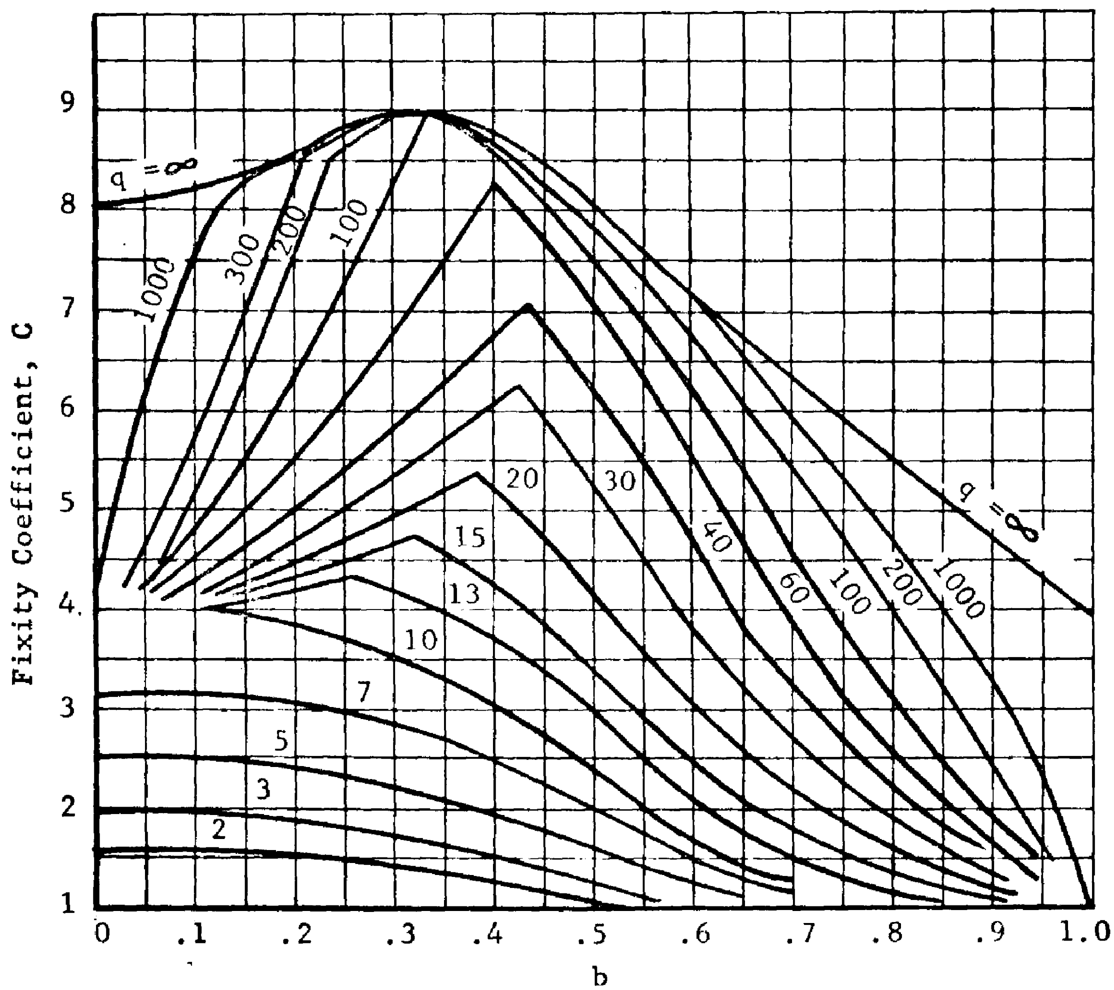


Fig. C1.2.0-4 Fixity of a Column with Two Elastic, Symmetrically Placed Supports Having Spring Constants, μ

C 1.3.0 Short Columns

Most columns fall into the range generally described as the short column range. With reference to Fig. C 1.1.0-1 of Section C 1.1.0, this may be described by $0 < L'/\rho < a$. This distinction is made on the basis that column behavior departs from that described by the classical Euler equation, Eq. (6). The average stress on the cross-section at buckling exceeds the stress defined by the proportional limit of the material. The slenderness ratio corresponding to the stress at the proportional limit defines the transition.

In the short column range a torsionally stable column may fail by crippling or inelastic bending, or a combination of both, as described in Sections C 1.3.1 and C 1.3.2.

C 1.3.1 Crippling Stress

When the corners of a thin-walled section in compression are restrained against any lateral movement, the corner material can continue to be loaded even after buckling has occurred in the section. When the stress in the corners exceeds its critical stress, the section loses its ability to support any additional load and fails. The average stress on the section at the failure load is called the crippling stress F_{cc}. Fig. C 1.3.1-1a shows the cross-sectional distortion occurring over one wave length in a typical thin-walled section. Fig. C 1.3.1-1b shows the stress distribution over the cross-section just before crippling.

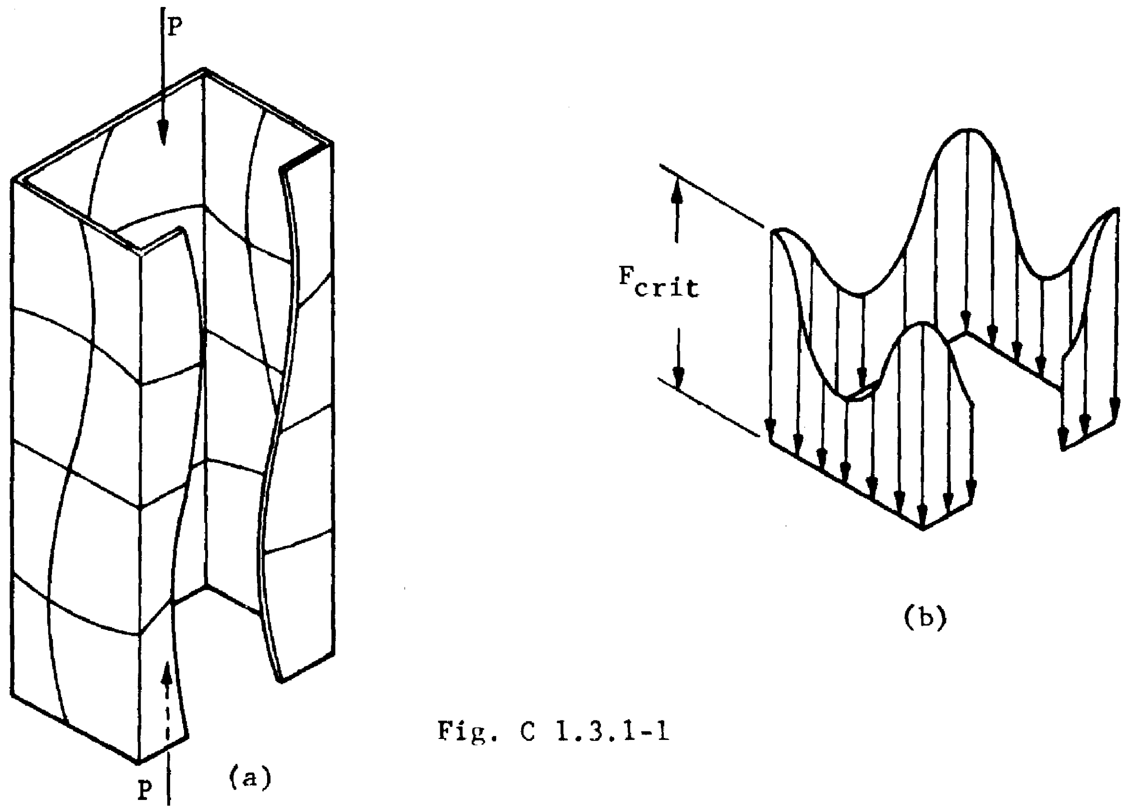


Fig. C 1.3.1-1

The empirical method for predicting the crippling stress of extruded and sheet metal elements is presented in this section. This crippling stress, F_{cc}, applies to extremely short column lengths and indicates the beginning of short column failure. It constitutes the column cut-off stress for sections composed of thin elements.

The crippling load of a member is equal to the product of the crippling stress and the actual area of the member; however, in calculating the crippling stress, the summation of the element areas is not equal to the actual area of the member.

A common structural component is composed of an angle, tee, zee, etc. attached to a thin skin. The buckling stress of the skin panel is less than the crippling stress of the stiffener. Taking a thin

C 1.3.1 Crippling Stress (Cont'd)

panel plus angle stiffeners at spacing, b, as shown on Fig. C 1.3.1-2, apply a compressive load. Up to the critical buckling load for the skin, the direct compressive stress is uniformly distributed. After the skin buckles, the central portion of the plate can carry little or no additional load; however, the edges of the plate, being restrained by the stiffeners, can and do carry an increasing amount of load. The stress distribution is shown in Fig. C 1.3.1-2.

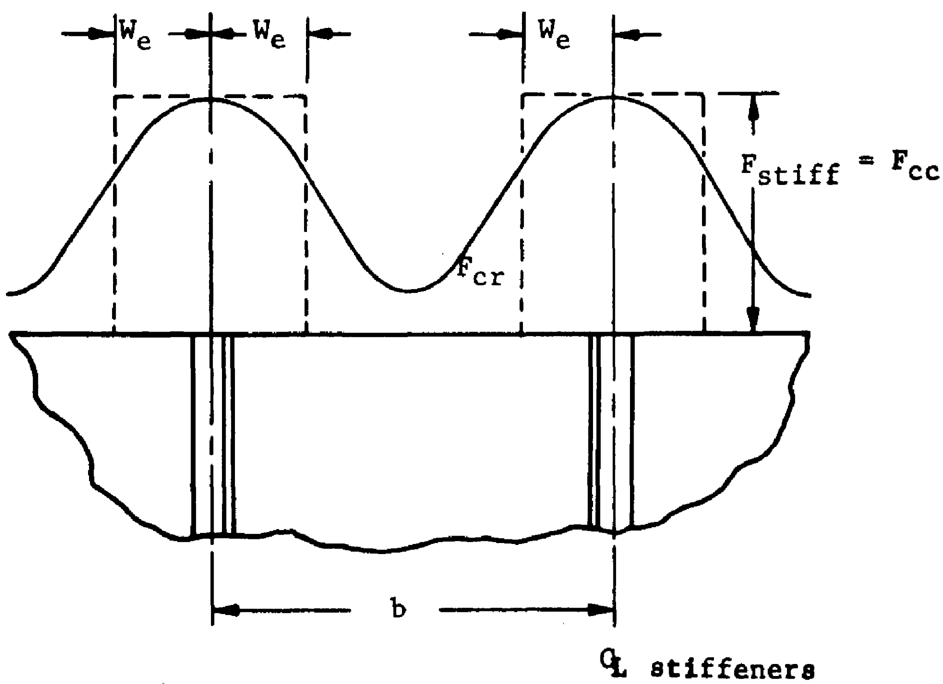


Fig. C 1.3.1-2

For the purpose of analysis, the true stress distribution shown by the solid line in Fig. C 1.3.1-2 is replaced by a uniform distribution as shown by the dotted lines. Essentially, an averaging process is used to determine the effective width, W_e, in which the stress, F_{cc}, is held constant.

Notation

F_{cc} = the crippling stress of a section.
f_{ccn} = the crippling stress of an element.
b_n = effective width of an element.
b_{fn} = flat portion of effective width of an element.
t_n = thickness of an element.
R = bend radius of formed stiffeners measured to the centerline.
R_b = extruded bulb radius.
W_e = effective width of skin.
E = Modulus of Elasticity.

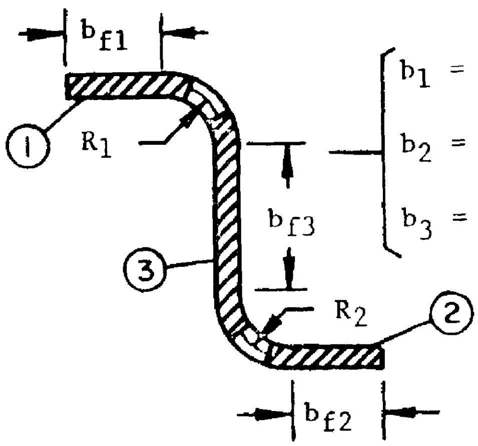
C 1.3.1 Crippling Stress (Cont'd)

Use of the crippling curves

I. The crippling stress, F_{cc}, at a stiffener is computed by the following expression

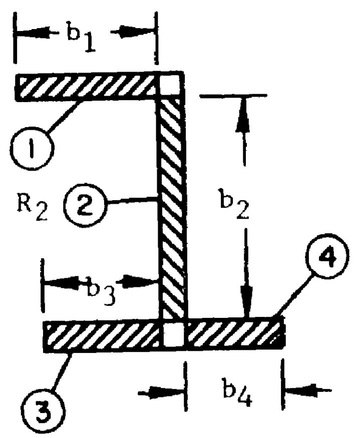
$$F_{cc} = \frac{\Sigma b_n t_n f_{ccn}}{\Sigma b_n t_n} \qquad \text{(1)}$$

II. The method for dividing formed sheet and extruded stiffeners into elements is shown in Fig. C 1.3.1-3.



Formed

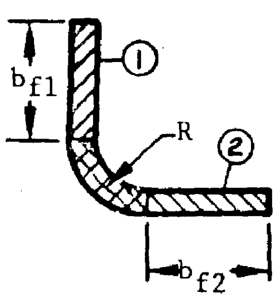
$$b_1 = b_{f1} + .535\ R_1$$
$$b_2 = b_{f2} + .535\ R_2$$
$$b_3 = b_{f3} + .535\ R_1 + .535\ R_2$$



Extruded

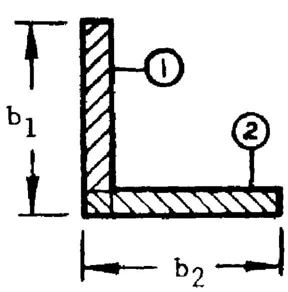
Fig. C 1.3.1-3

III. Angle stiffeners have low crippling stresses as each leg is in the one-edge-free condition and offers little support to the other leg. The method of dividing such stiffeners into effective elements is shown in Fig. C 1.3.1-4.



Formed

$$b_1 = b_{f1} + 1.57\ R$$
$$b_2 = b_{f2} + 1.57\ R$$



Extruded

Fig. C 1.3.1-4

C 1.3.1 <u>Crippling Stress (Cont'd)</u>

IV. Certain types of formed stiffeners, as shown in **Figures** C 1.3.1-5, C 1.3.1-6, and C 1.3.1-7, whose radii are equal and whose centers are on the same side of the sheet, require special consideration. Table C 1.3.1-1 explains the handling of these cases.

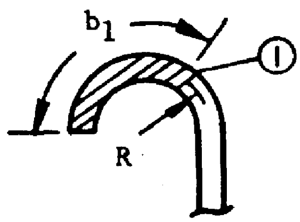


Fig. C 1.3.1-5

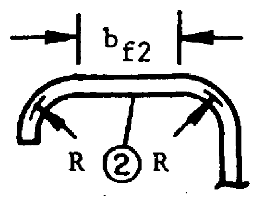


Fig. C 1.3.1-6

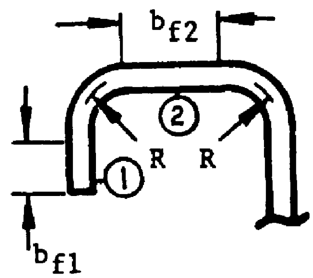
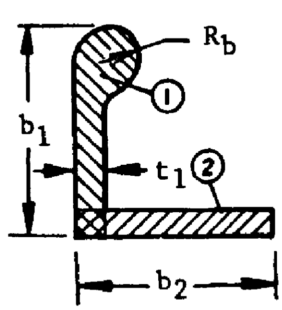


Fig. C 1.3.1-7

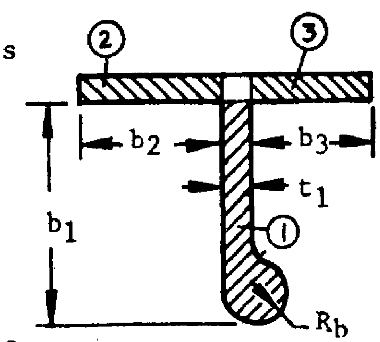
When b_{f2}	And b_{f1}	As shown in Fig.	Use
= 0	= 0	C 1.3.1-5	$b_1 = 2.10R$ (one edge free)
< R	= 0	C 1.3.1-6	$b_2 = 2.10R$ (one edge free)
≥ R	= 0	C 1.3.1-6	$b_2 = b_{f2} + 1.07R$ (Avg. one & no edge free)
< R	< R	C 1.3.1-7	$b_2 = 2.10R$ (one edge free, neglect b_1)
≥ R	< R	C 1.3.1-7	$b_2 = b_{f2} + 1.07R$ (Avg. one & no edge free, neglect b_1)
≥ 0	≥ R	C 1.3.1-7	$b_1 = b_{f1} + 1.07R$ (one edge free) $b_2 = b_{f2} + 1.07R$ (no edge free)

Table C 1.3.1-1

V. Special conditions for extrusions



(a)

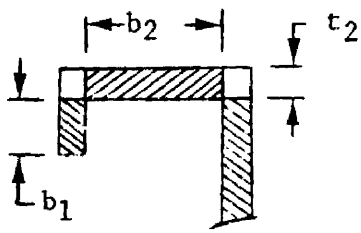


(b)

Fig. C 1.3.1-8

The crippling stress of an outstanding leg with bulb is 0.7 of the value for the no-edge-free condition if R_b is greater than or equal to the thickness of the adjacent leg (t_1 Fig. C 1.3.1-8). When $R_b < t_1$, the outstanding leg shall be considered as having one edge free.

C 1.3.1 Crippling Stress (Cont'd)



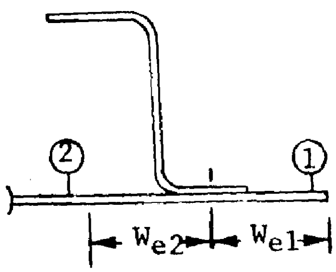
for $b_1 > t_2$ and $\leq 3t_2$ neglect b_1; and F_{cc2} = Avg. of no edge free and one edge free.

for $b_1 > 3t_2$ Regular method.

Fig. C 1.3.1-9

VI. The effective width of sheet, in a sheet-stiffener combination under compression, is determined from the plot of $2W_e/t$ versus f_{stiff} (Fig. C 1.3.1-12).

Note the following special cases



One-Edge-Free Sheet

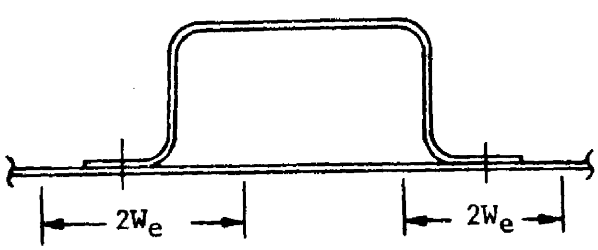
Fig. C 1.3.1-10

one edge free

$$\frac{W_{e1}}{t} = .382\left(\frac{2W_e}{t}\right)_{\text{chart (no edge free)}}$$

no edge free

$$\frac{2W_{e2}}{t} = \left(\frac{2W_e}{t}\right)_{\text{chart}}$$



Effective Sheet with Large Hat Stiffener

Fig. C 1.3.1-11

(a) If effective widths overlap, reduce accordingly.

(b) Calculate as one or no edge free as necessary.

C 1.3.1 Crippling Stress (Cont'd)

Effective width of stiffened sheet.

The effective width, (W_e), is the width of skin on either or both sides of the stiffener acting at the stiffener stress level. This stress level for the skin is obtainable only if there is no inter spot-weld, or inter rivet buckling.

For calculating the effective width of sheet acting with the stiffener the following equation is graphed on Fig. C 1.3.1-12

$$\frac{2W_e}{t} \approx \frac{K\,(E_s)\ \text{skin}}{\sqrt{(E_s)\ \text{stiff}}} \sqrt{\frac{1}{f_{stiff}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Where
- W_e = effective width of skin (in.)
- t = thickness of skin (in.)
- E_s = secant modulus at stress level of stiffener (ksi)
- f = stress (KSI)
- K = 1.7 for simply supported case (no edge free)
- K = 1.3 for one edge free.

For a sheet-stiffener combination of the same material, Eq. 2 becomes

$$\frac{2W_e}{t} = K\sqrt{\frac{E_s}{f_{stiff}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

The procedure for determining the crippling stress for a sheet-stiffener compression panel is

(1) Determine approximate stress level of stiffener.

$$f_{stiff} = \frac{\text{Load}}{\text{Area}} = \frac{P}{A_{stiff}}$$

(2) Determine (W_e) by using f_{stiff} and Fig. C 1.3.1-12. This procedure is not applicable if the sheet is subjected to inter spot-weld or inter rivet buckling.

(3) The crippling stress for the composite section is then calculated by Eq. 1.

C 1.3.1 Crippling Stress (Cont'd)

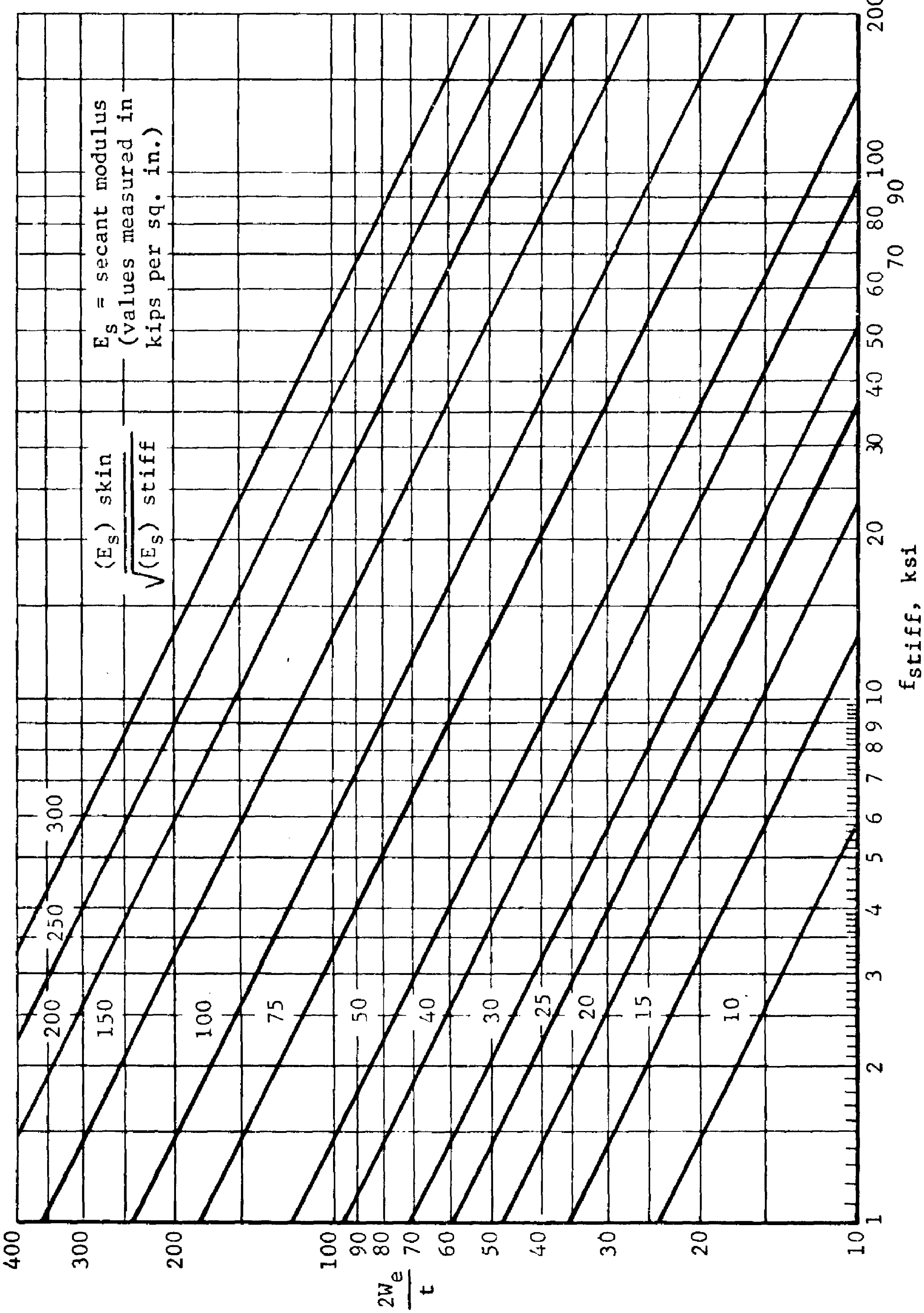


Fig. C1.3.1-12a Effective Width of Stiffened Sheet

C 1.3.1 Crippling Stress (Cont'd)

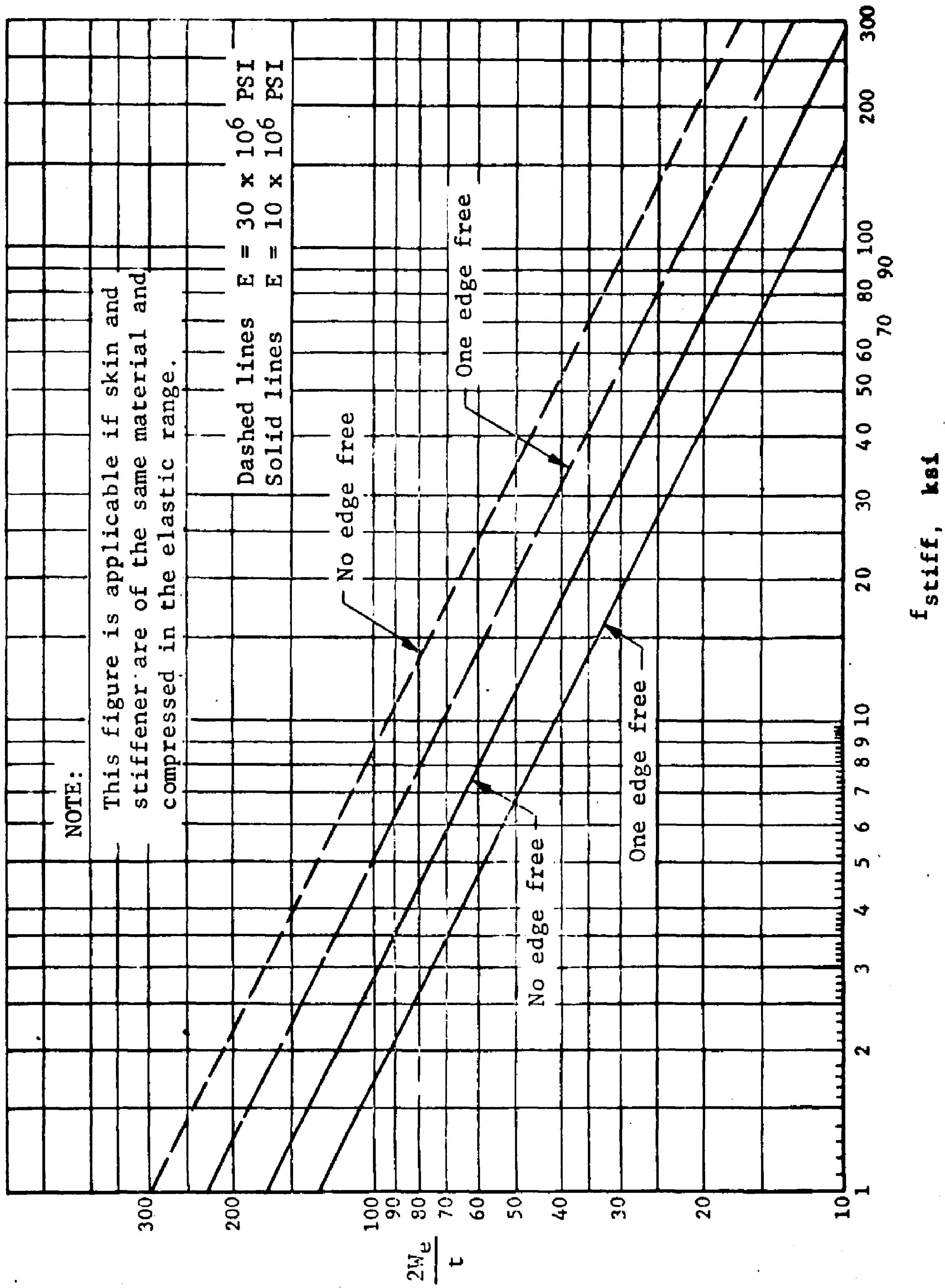


Fig. C 1.3.1-12b Effective Width of Stiffened Sheet

C 1.3.1 Crippling Stress (Cont'd)

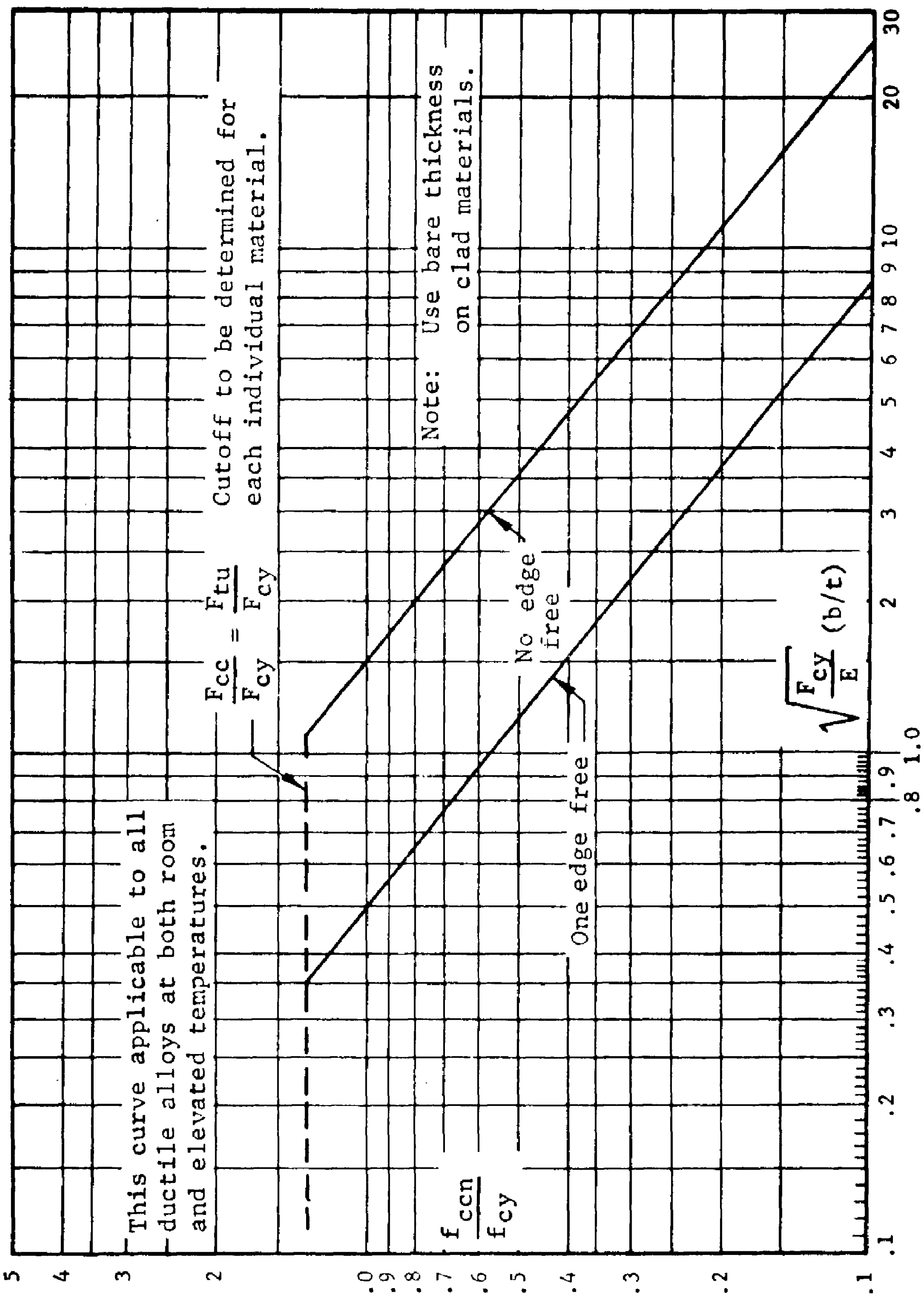


Fig. C 1.3.1-13 Nondimensional Crippling Curves

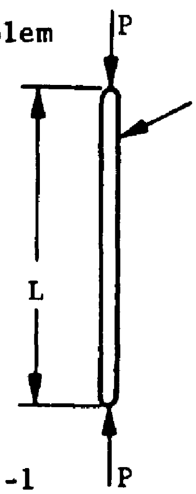
C 1.3.2 Column Curve for Torsionally Stable Sections

The column curves in Fig. C 1.3.2-3 are presented for the determination of the critical column stress for torsionally stable sections. The modes of failure are discussed in sections C 1.2.0 and C 1.3.0.

These curves are Euler's long column curve and Johnson's modified 2.0 parabolas. They are to be used to determine the critical stress, F_c, for columns at both room and elevated temperatures. It is noted that the modulus of elasticity, E, corresponds to the temperature at which the critical stress is desired.

The following sample problem is used to illustrate the use of Fig. C 1.3.2-3 to determine the critical stress, F_c.

Illustrative problem



6061-T6 Square tubing 3.00 x .065

L = 60 inches (pin ended)
Temperature, T = 500° F (exposed ½ hr)

$E = 7.9 \times 10^6$ psi @ 500° F
F_{cy} = 26.6 ksi @ 500° F
ρ = 1.730 in.

Determine the critical stress for the column.

Fig. C 1.3.2-1

Solution

Determine the crippling stress of the section by the method outlined in Section C 1.3.1.

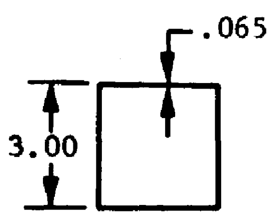


Fig. C 1.3.2-2

b = 3 - .065 = 2.935 (Center line values used here)

t = .065

$$\frac{b}{t}\sqrt{\frac{F_{cy}}{E}} = \left(\frac{2.935}{.065}\right)\sqrt{\frac{26.6 \times 10^3}{7.9 \times 10^6}} = 2.62$$

From Fig. C 1.3.1-13

$$\frac{f_{ccn}}{f_{cy}} = .64$$

C 1.3.2 Column Curve for Torsionally Stable Sections (Cont'd)

Use Eq. 1 Section C 1.3.1

$$\frac{F_{cc}}{F_{cy}} = \frac{\Sigma f_{ccn} b_n t_n}{F_{cy} \Sigma b_n t_n} = \frac{4(.64)\ (2.935)\ (.065)}{4(2.935)\ (.065)} = .64$$

$$F_{cc} = .64(26.6) = 17{,}030 \text{ psi}$$

The critical stress for the column is obtained from Fig. C 1.3.2-3.

$$\frac{F_{cc}}{E} = \frac{17{,}030}{7.9 \times 10^6} = 2.16 \times 10^{-3}$$

For pin-ended column

$$L = L' = 60 \text{ in.}$$

$$\frac{L'}{\rho} = \frac{60}{1.73} = 34.7$$

Then from Fig. C 1.3.2-3

$$\frac{F_c}{E} = 2.02 \times 10^{-3}$$

Giving a critical stress of

$$F_c = 2.02 \times 10^{-3}\ (7.9 \times 10^6) = 15{,}960 \text{ psi}$$

C 1.3.2 Column Curves for Torsionally Stable Columns (Cont'd)

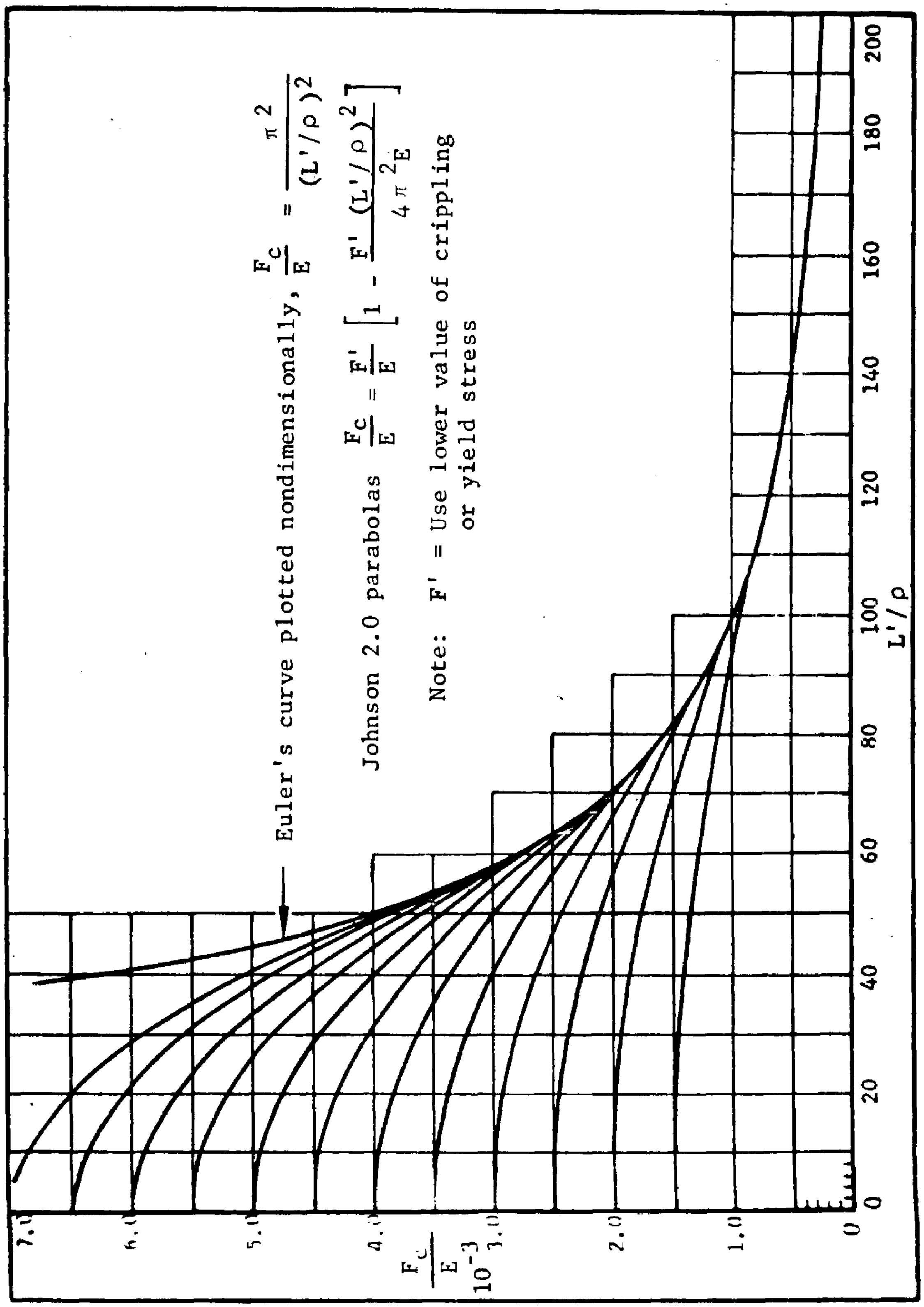


Fig. C 1.3.2-3 Critical Stress for Torsionally Stable Columns

C 1.3.3 Sheet Stiffener Combinations

Flat panel

Sheet-stiffener combinations of flat compression panels may be analyzed as columns. Each stiffener of the panel plus an effective width of sheet acting at the stiffener stress constitutes an individual column that is free to bend about an axis parallel to the panel sheet. The sheet between stiffeners is continuous and offers considerable restraint against stiffener failure about an axis perpendicular to the sheet even though the sheet itself has buckled between stiffeners.

The stress distribution over the panel section after the sheet has buckled is shown by the solid curve in Fig. C 1.3.3-1. The dotted curve is the assumed stress distribution using the concept of effective widths. The effective widths (W_e) for torsionally stable and unstable sections are given in Section C 1.3.1.

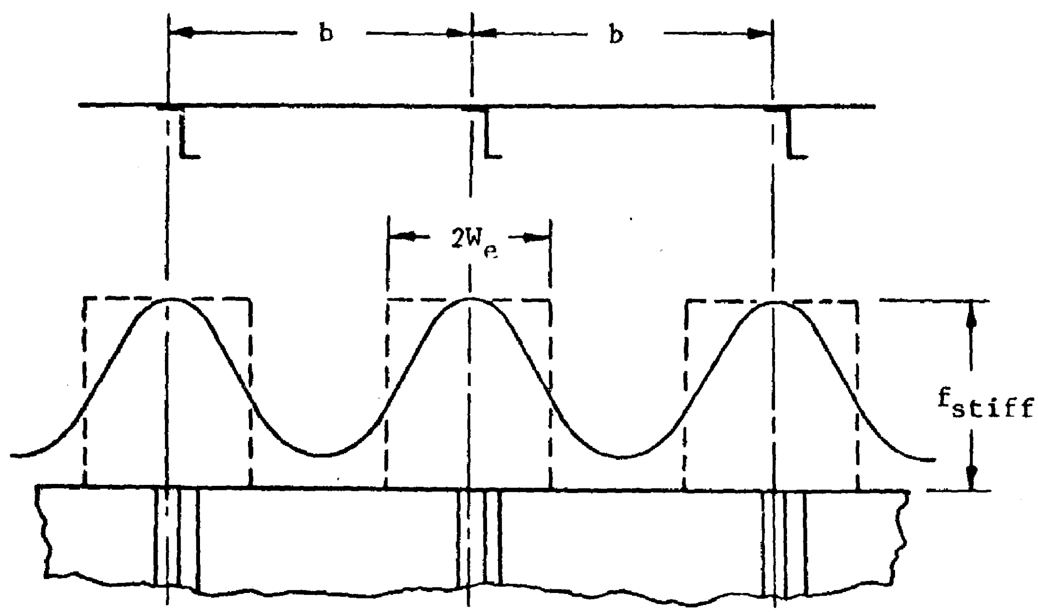


Fig. C 1.3.3-1

The procedure for determination of the critical stress and load on a sheet-stiffener compression panel is

(1) Determine the slenderness ratio L'/ρ of the stiffener alone where ρ is the radius of gyration of the stiffener cross-section about a centroidal axis parallel to the sheet.

(2) From the crippling curve (Fig. C 1.3.1-13) determine F_{cc} of the stiffener cross-section. The value F_{cc}/E is given by F_c/E at $L'/\rho = 0$ in Fig. C 1.3.2-3.

C 1.3.3 Sheet Stiffener Combinations (Cont'd)

(3) Using the column curves (Fig. 1.3.2-3), with L'/ρ and F_{cc} determined in steps (1) and (2), record the value of F_c. (Interpolate between curves as required)

(4) Determine the effective widths of sheet by using Fig. C 1.3.1-12 where $f_{stiff} = F_c$.

(5) Use Fig. C 1.3.3-3 to compute ρ of the stiffener plus effective sheet.

(6) Re-enter the column curve (Fig. C 1.3.2-3) with new L'/ρ and record the value of F_c.

(7) Repeat steps (4), (5), and (6) until satisfactory convergence to a final stress, F_c, is obtained. Convergence generally occurs after two trials. F_c is the critical stress of the stiffened sheet.

(8) The critical load, P_c, is

$$P_c = F_c \left[A_{st} + t_s \Sigma W_e \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where A_{st} is the cross-sectional area of the stiffener.

Curved panels

Analysis of curved stiffened panels requires but a slight extension in procedure beyond that described for flat panels. Fig. C 1.3.3-2 shows a curved panel with the effective widths of sheet that act with the stiffeners.

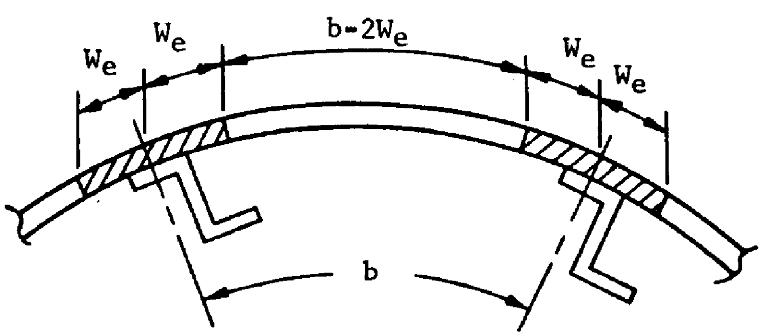


Fig. C 1.3.3-2

The load-carrying capacity of such a panel is equal to that of a flat panel plus an additional load attributable to the effect of the curvature of the sheet between the stiffeners. The critical load is

$$P_c = P_{flat} + P_{curved}$$

C 1.3.3 Sheet Stiffener Combinations (Cont'd)

or for Fig. C 1.3.3-2

$$P_c = (F_c)_{\text{column}} (A_{st} + 4t_s W_e) + (F_{cr})_{\substack{\text{curved} \\ \text{panel}}} (b - 2W_e)\, t_s$$

The critical stress (F_{cr}) of the curved panel is calculated by the equations of section C 3.0.0. Note that in computing this stress the entire width "b" of the curved panel is used. Only the reduced width, $b - 2W_e$, is used in calculating the load that is contributed by the curved panel.

C 1.3.3 Sheet Stiffener Combinations (Cont'd)

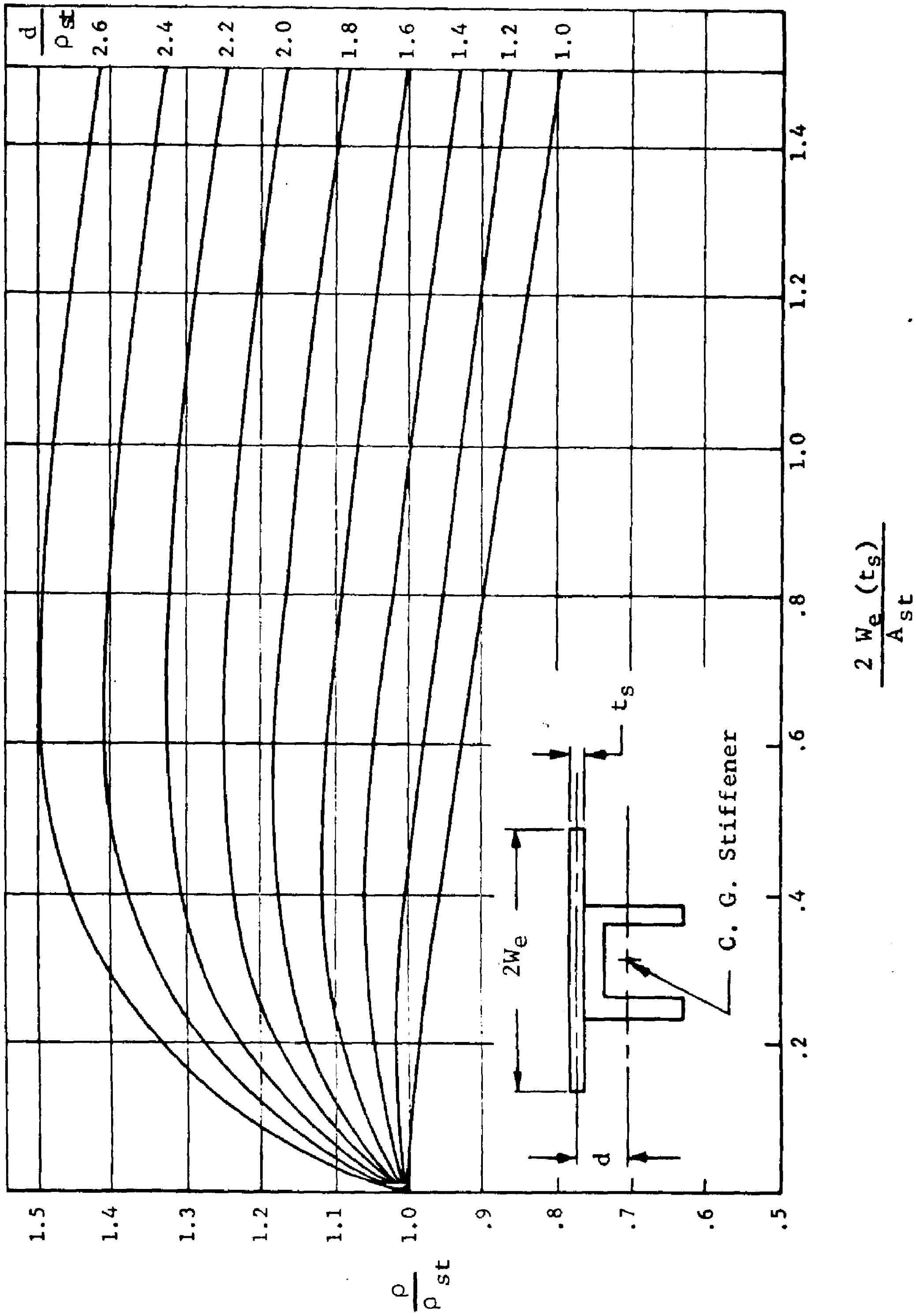


Fig. C 1.3.3-3 Variation of Radius of Gyration for Sheet Stiffener Combinations

C 1.4.0 Columns with Variable Cross Sections

The modified Euler equation (tangent modulus) used to determine the critical load of a prismatic, torsionally stable column not subjected to crippling failure is

$$P_c = \frac{c\pi^2 E_t I}{(L')^2} \quad \text{.......................................} \quad (1)$$

This section gives appropriate column buckling coefficients (m) and formulas for computing the Euler loads for varying cross-section columns. Where $m = \frac{1}{c}$ in Eq. 1.

The following example is typical for calculating the critical load of a stepped column.

Example

Given:

$E_1 = 10 \times 10^6$ **psi** (aluminum)
$I_1 = .30 \text{ in}^4$
$E_2 = 30 \times 10^6$ **psi** (steel)
$I_2 = .50 \text{ in}^4$
$A_1 = 1.94 \text{ in}^2$

Determine critical buckling load, P_c

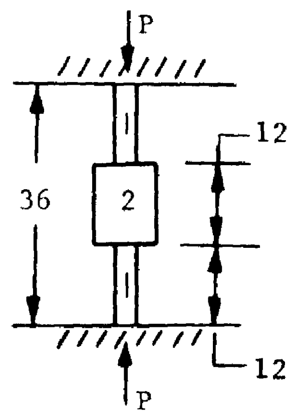


Fig. C 1.4.0-1

Solution:

$$\frac{a}{L} = \frac{12}{36} = .33 \qquad \frac{E_1 I_1}{E_2 I_2} = \frac{10 \times 10^6 \ (.30)}{30 \times 10^6 \ (.50)} = .20$$

From Fig. C 1.4.0-2, m = .545

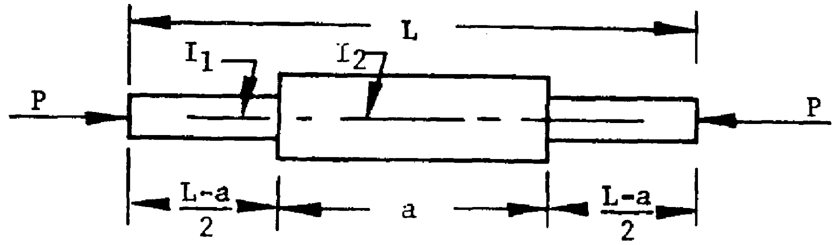
$$P_c = \frac{\pi^2 (E_t I)_1}{mL^2} = \frac{(3.14)^2}{.545} \quad \frac{(10)10^6 \ (.30)}{(36)^2} = 41,800 \text{ lb}$$

Stress level of aluminum section (max. of column)

$$f_1 = \frac{P_c}{A_1} = \frac{41800}{1.94} = 21,600 \ \textbf{psi}$$

If f_1 is below the proportional limit of the material in question, then P_c is the critical load of the column. However, if f_1 is above the stress at the proportional limit, the tangent modulus (E_t) at the stress level must be used. This leads to a trial process to determine the critical load of the column.

C 1.4.0 Columns with Variable Cross Sections (Cont'd)



$$P_c = \frac{\pi^2}{m} \frac{(E_t I)_1}{L^2}$$

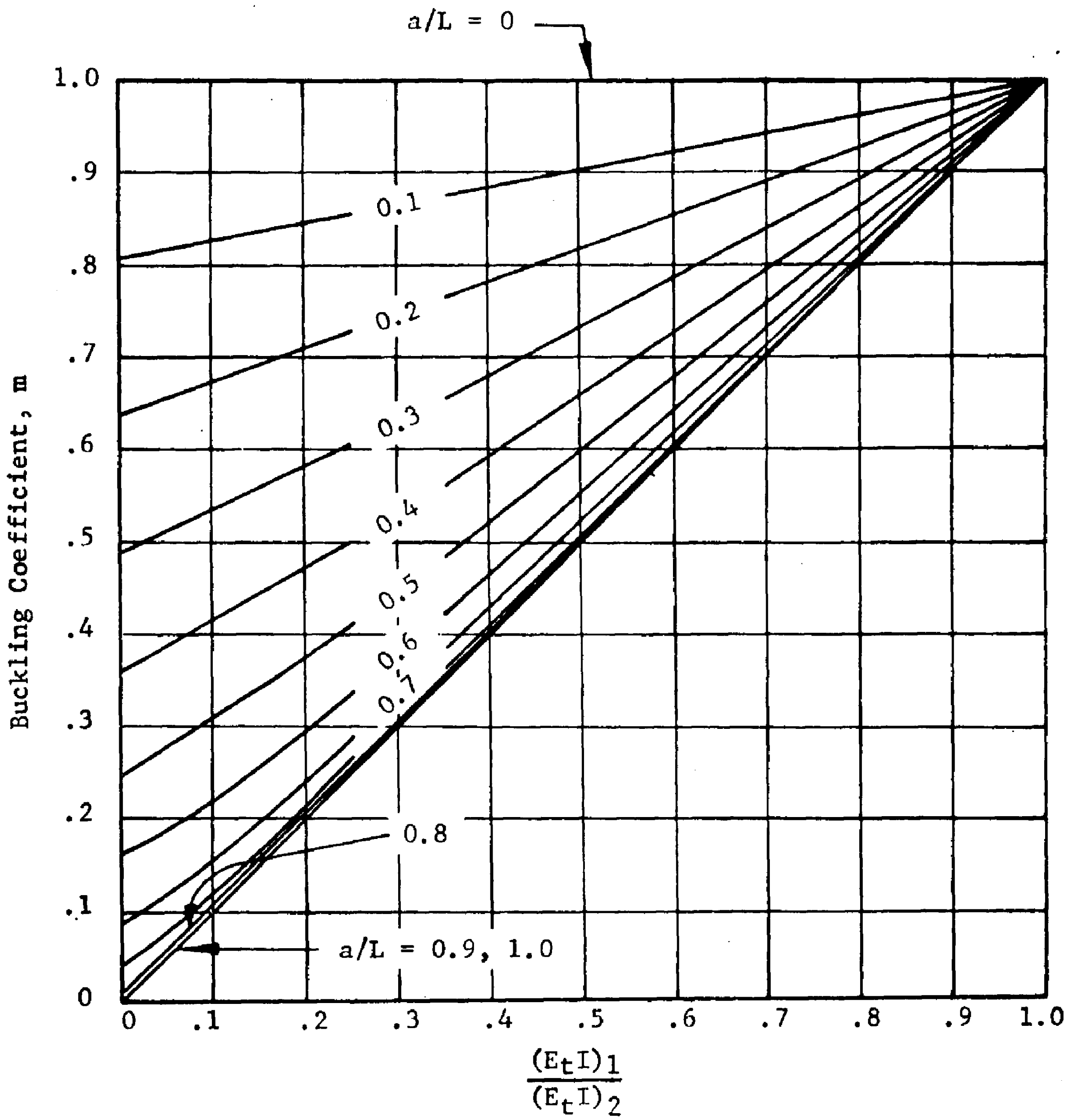
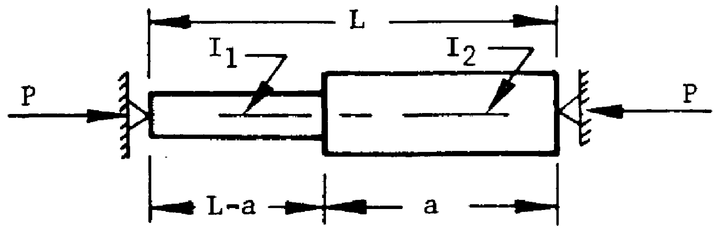


Fig. C1.4.0-2 Buckling Coefficient

C 1.4.0 Columns With Variable Cross Sections (Cont'd)



$$P_c = \frac{\pi^2}{m}\frac{(E_tI)_1}{L^2}$$

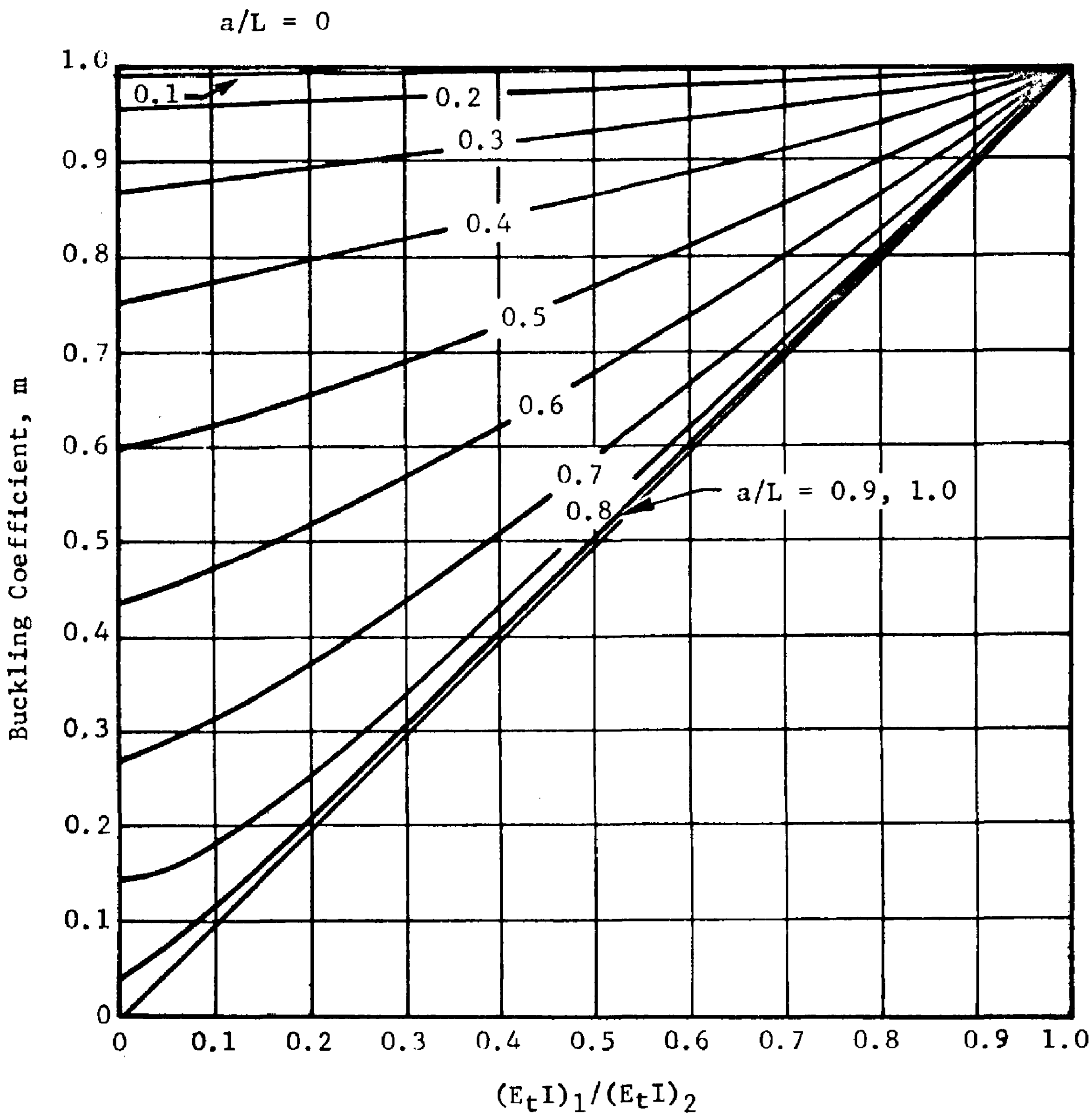
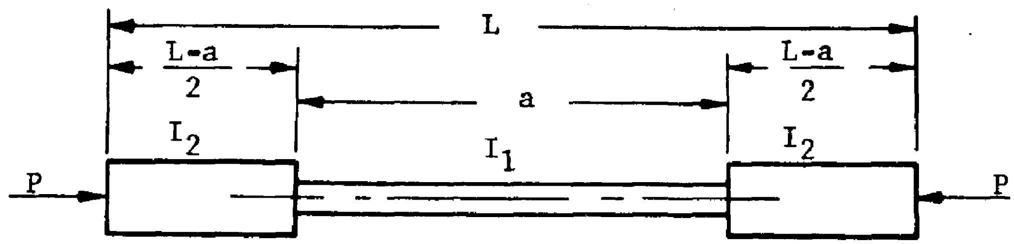


Fig. C1.4.0-3 Buckling Coefficient

C 1.4.0 Columns with Variable Cross Sections (Cont'd)



$$P_c = \frac{\pi^2}{m} \frac{(E_t I)_1}{L^2}$$

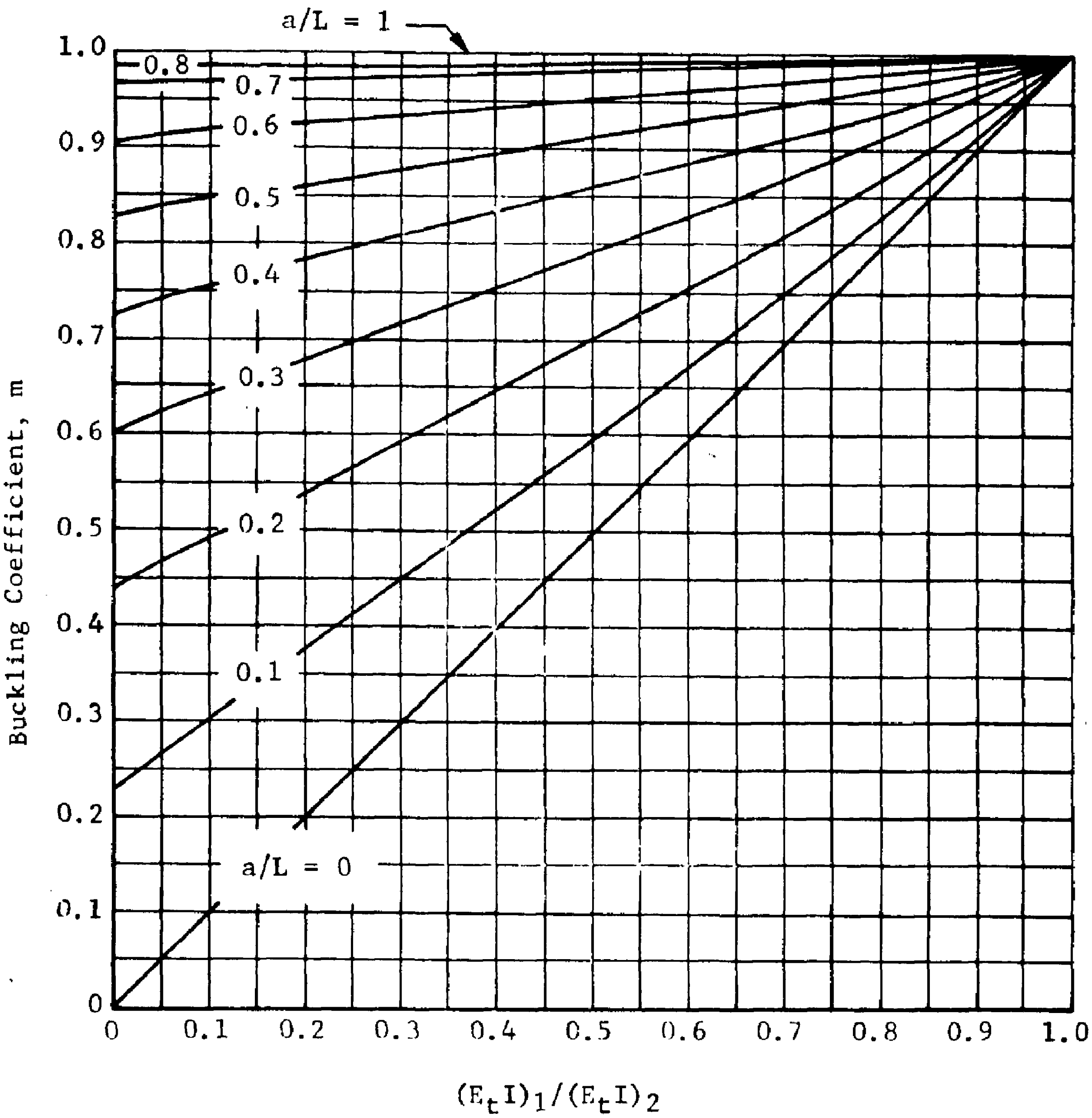


Fig. C1.4.0-4 Buckling Coefficient