B9.4 ISOTROPIC THIN PLATES — LARGE DEFLECTION ANALYSIS

Large deflection theory of plates was discussed in Paragraph B9.2.3. It was determined that the region covered by the classification of large deflection analysis was approximately $\frac{1}{3} t < w/t > 10t$. In this region, the load resistance of plates is a combination of bending and direct tensile stress. Solutions for available plate geometries and loads will be given in this subsection. Figure B9-7 gave a guide as to the regimes of membrane plates, medium-thick plates, and thin plates. Curves are given in Fig. B9-12 for membrane plates and for medium-thick plates. Between these two regimes is the regime of thin plates, which generally includes most of the plate dimensions and pressures encountered in aerospace design.

B9.4.1 Circular Plates — Uniformly Distributed Load

A circular plate whose edge is clamped so that rotation and radial displacement are prevented at the edge is shown in Fig. B9-13. The plate, loaded by a uniformly distributed load, has a maximum deflection which is large relative to the thickness of the plate as shown in Fig. B9-13c. In Fig. B9-13d a diametral strip of one unit width cut from the plate shows the bending moments per unit of width and the direct tensile forces which act in this strip at the edge and at the center of the plate. The direct tensile forces arise from two sources. First, the fixed support at the edge prevents the edge at opposite ends of a diametral strip from moving radially, thereby causing the strip to stretch as it deflects. Second, if the plate is not clamped at its edge but is simply supported

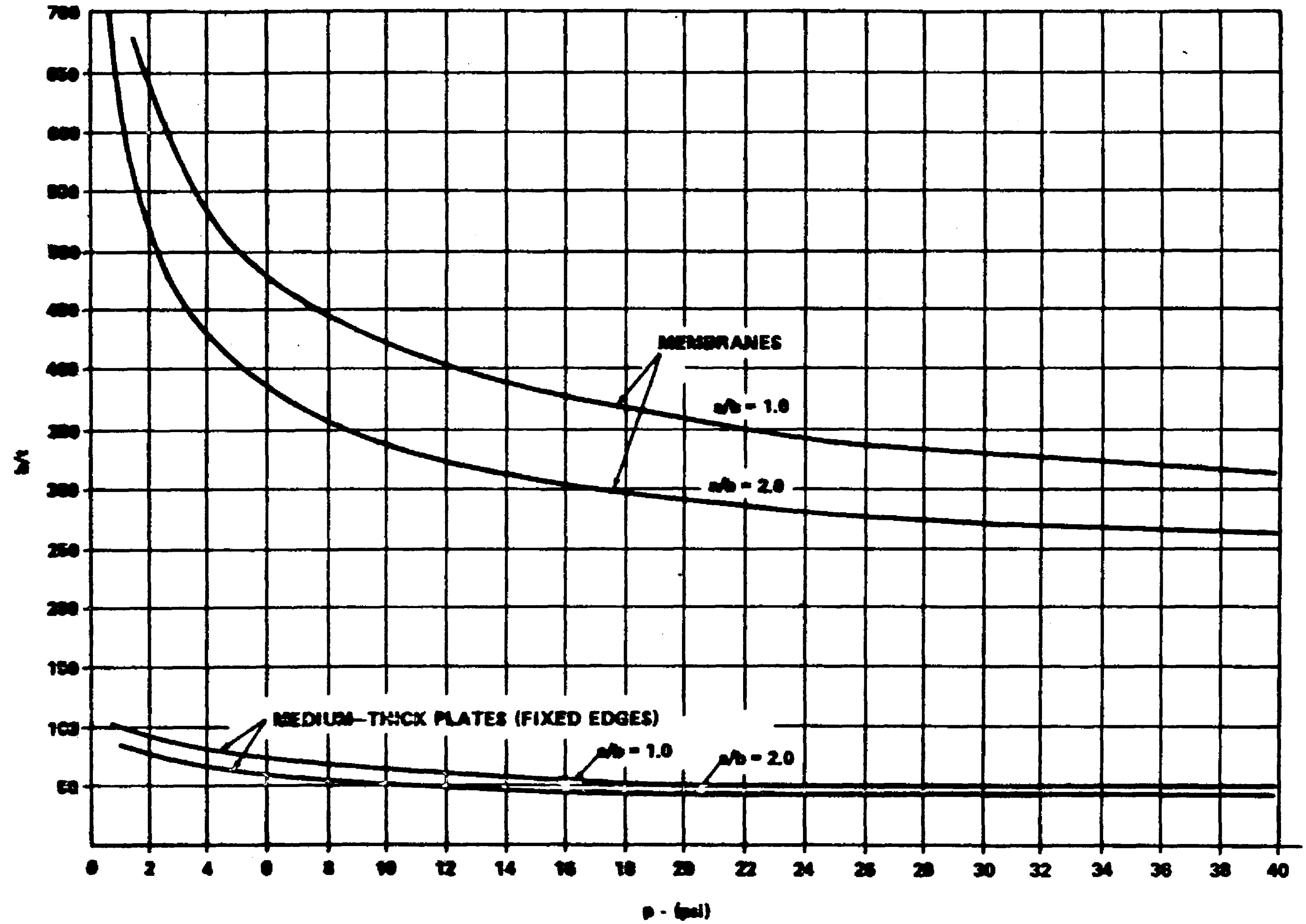


FIGURE B9-12. LIMITING REGIONS FOR RECTANGULAR MEMBRANE AND THICK PLATE THEORIES

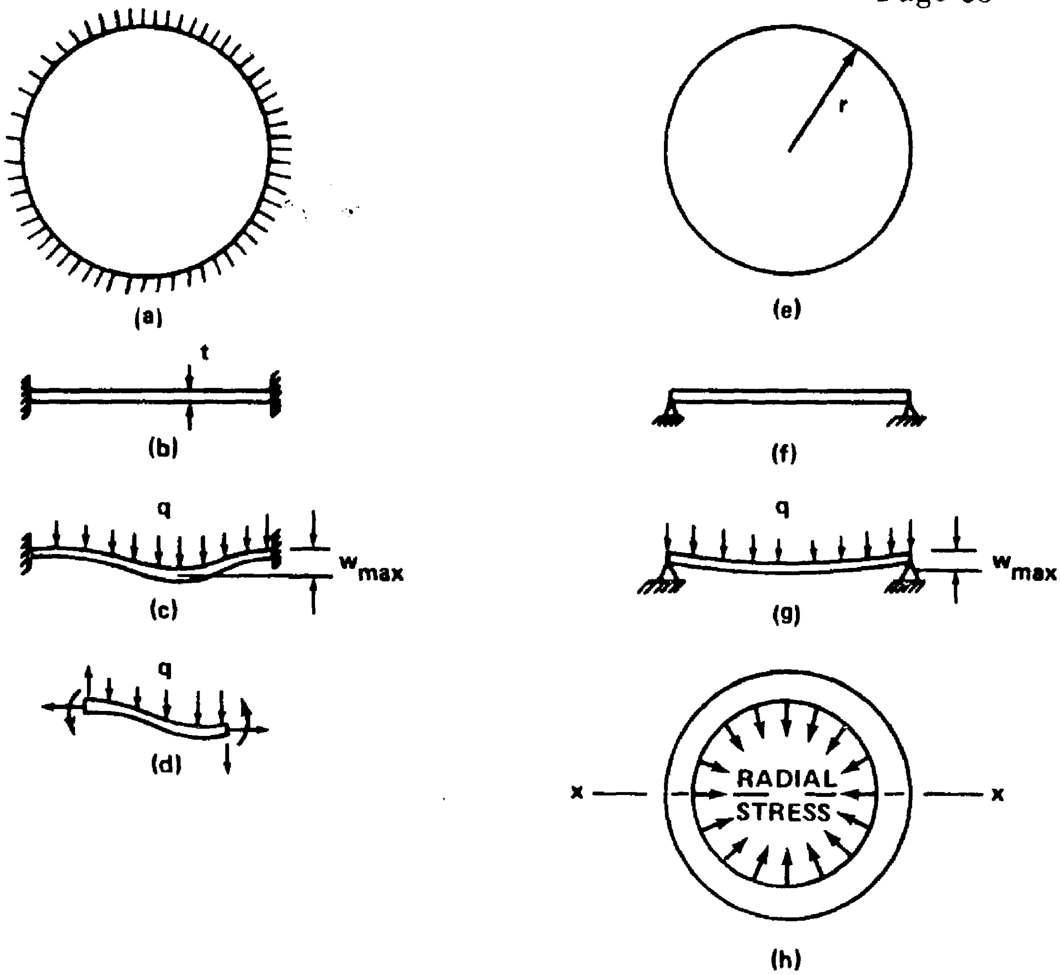


FIGURE B9-13. BEHAVIOR OF THIN CIRCULAR PLATE

as shown in Figs. B9-13e and f, radial stresses arise out of the tendency for outer concentric rings of the plate (such as shown in Fig. B9-13h) to retain their original diameter as the plate deflects. In Fig. B9-13h the concentric ring at the outer edge is shown cut from the plate. This ring tends to retain the original outside diameter of the unloaded plate; the radial tensile stresses acting on the inside of the ring, as shown in Fig. B9-13h, cause the ring diameter to decrease, and in doing so they introduce compressive stresses on every diametral section such as xx . These compressive stresses in the circumferential direction sometimes cause the plate to wrinkle or buckle near the edge, particularly if the plate is simply supported. The radial stresses are usually larger in the central portion of the plate than they are near the edge.

Stresses have been determined for a thin circular plate with clamped edges and the results are plotted in Fig. B9-14, where σ_{be} and σ_{bc} are the bending

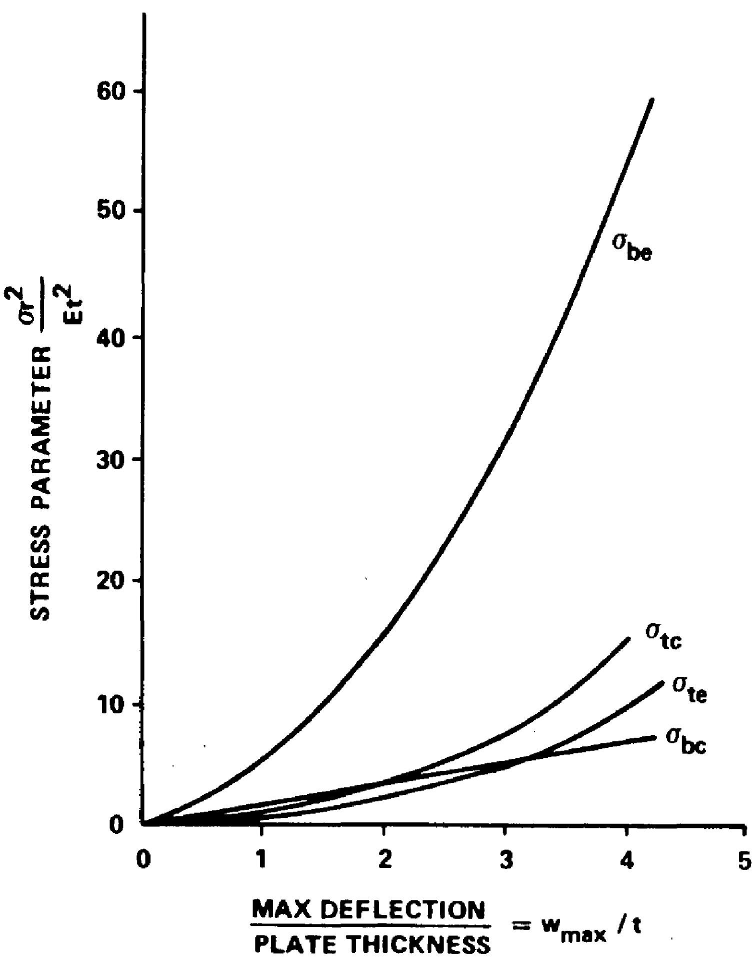


FIGURE B9-14. STRESSES IN THIN PLATES HAVING LARGE DEFLECTIONS, CIRCULAR PLATE WITH CLAMPED EDGES stresses in a radial plane at the edge and center of the plate, and σ_{te} and σ_{tc} are corresponding direct tensile stresses. It is noted that the bending stress σ_{be} at the fixed edge is the largest of these four stresses. The direct tensile stresses become relatively larger as the deflection increases.

Figure B9-15 presents a set of curves which show the relationship between load, deflection, and stress for a thin circular plate with clamped edges. For example, if the dimensions and the modulus of elasticity of the plate and the load q are given, the quantity qr^4/Et^4 can be computed. The value of w_{max}/t corresponding to this value of qr^4/Et^4 is found from the curve on the left. By projecting across to stress curves, corresponding stress parameters $\sigma_{max} r^2/Et^2$ are read at the center and at the edge of the plate.

Figure B9-16 presents curves similar to those of Fig. B9-15 for a plate whose edges are simply supported.

Also, Table B9-22 presents data for the calculation of approximate values of deflections and stresses in uniformly loaded circular plates, both clamped and simply supported. The deflection at the center w_0 is given by the equation,

$$\frac{w_0}{t} + A\left(\frac{w_0}{t}\right)^3 = B \; \frac{q}{E} \left(\frac{r}{t}\right)^4 \quad . \tag{33}$$

Also, the stresses in the middle plane are given by

$$\sigma_r = \alpha_r E\left(\frac{w_0}{r}\right)^2 \quad , \quad \sigma_t = \alpha_T E\left(\frac{w_0}{r}\right)^2 \quad , \tag{34}$$

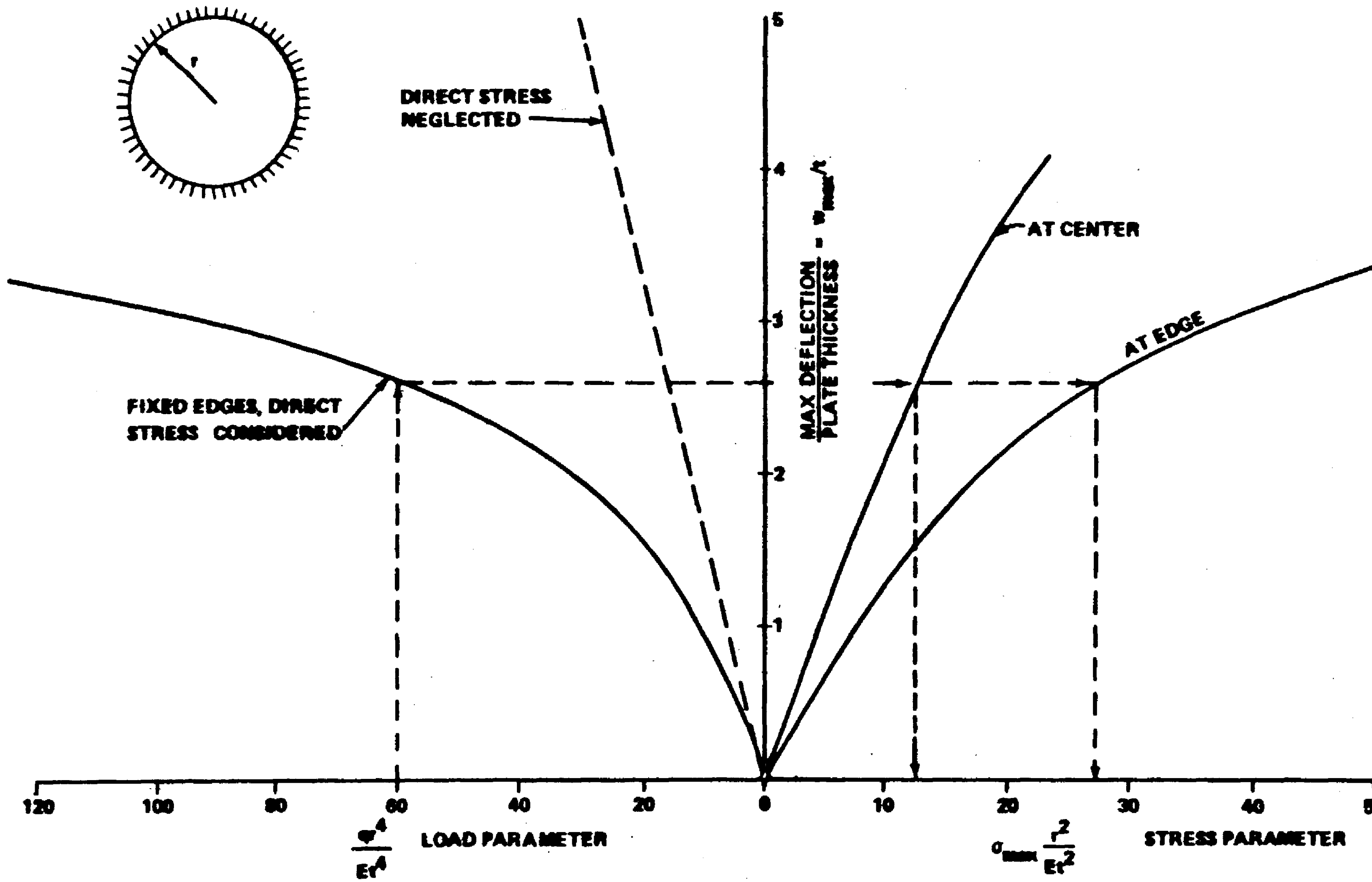


FIGURE B9-15. MAXIMUM STRESSES AND DEFLECTIONS IN THIN CIRCULAR PLATE WITH CLAMPED EDGES

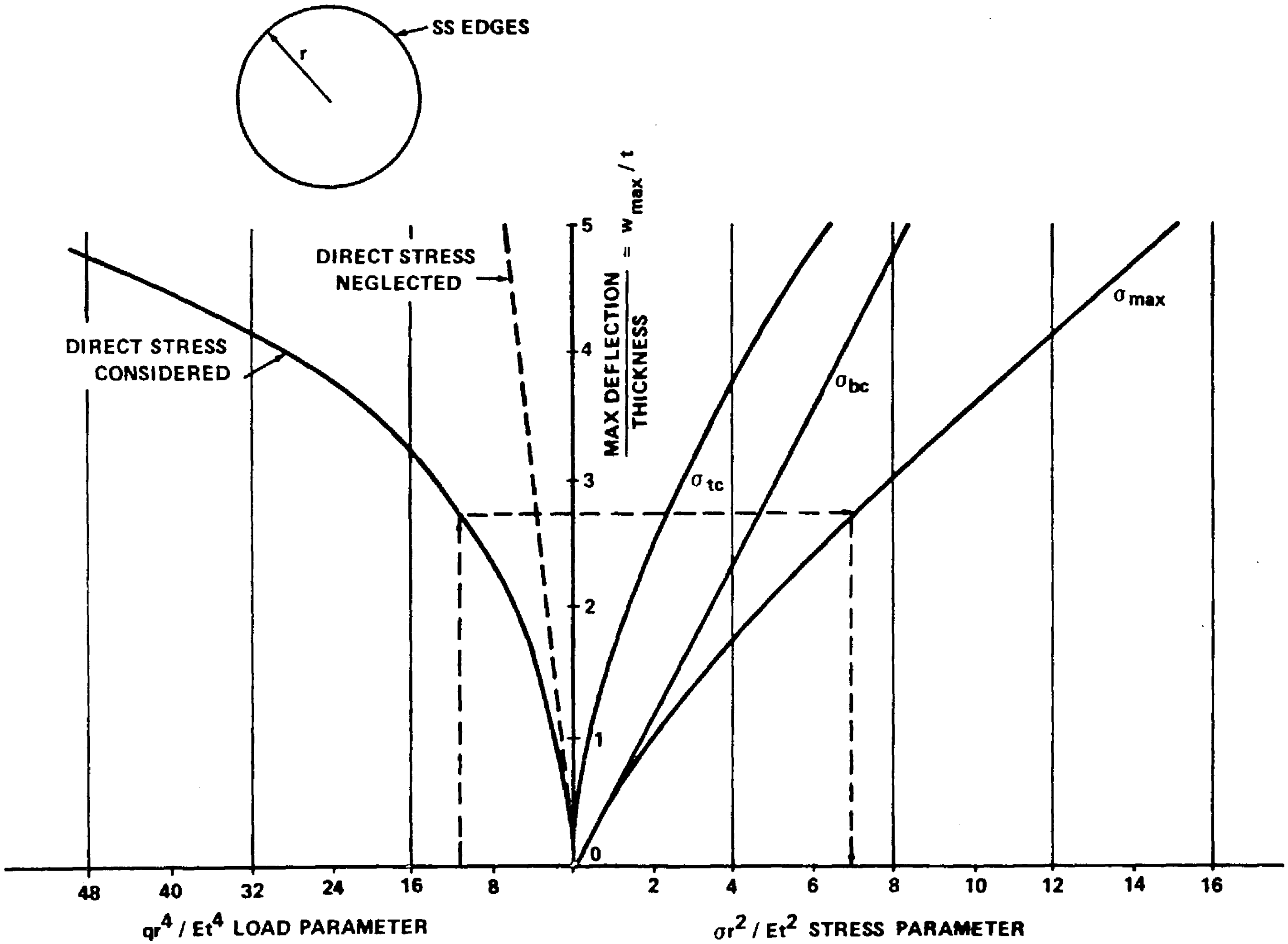


FIGURE B9-16. MAXIMUM STRESSES AND DEFLECTIONS IN THIN CIRCULAR PLATE WITH SIMPLY SUPPORTED EDGES

Table B9-22. Data for Calculation of Approximate Values of Deflections w_0 and Stresses in Uniformly Loaded Plates ($\mu = 0.3$)

Boundary Conditions		A	B	Center		Edge			
				$\alpha_r = \alpha_t$	$\beta_r = \beta_t$	α_r	α_t	β_r	β_t
Plate Clamped	Edge Immovable	0.471	0.171	0.976	2.86	0.476	0.143	-4.40	-1.32
	Edge Free To Move	0.146	0.171	0.500	2.86	0	-0.333	-4.40	-1.32
Plate Simply Supported	Edge Immovable	1.852	0.696	0.905	1.778	0.610	0.183	0	0.755
	Edge Free To Move	0.262	0.696	0.295	1.778	0	-0.427	0	0.755

and the extreme fiber bending stresses are given by

$$\sigma_r' = \beta_r \, E \, \frac{w_0 t}{r^2} \quad , \quad \sigma_t' = \beta_t \, E \, \frac{w_0 t}{r^2} \quad . \tag{35}$$

B9.4.2 Circular Plates — Loaded at the Center

An approximate solution of the problem of a circular plate loaded at the center with either clamped or simply supported edges has been obtained in Reference 1. Table B9-23 contains the coefficients necessary for solution of the center deflection w_0 from equations (33), (34), and (35).

B9.4.3 Rectangular Plates — Uniformly Loaded

For the case of a plate with clamped edges, an approximate solution has been obtained [1]. Numerical values of all the parameters have been computed for various intensities of the load q and for three different shapes of the plate $b/a = 1$, $b/a = 2/3$, and $b/a = 1/2$ for $\mu = 0.3$. The maximum deflections at the center of the plate are graphically represented in Fig. B9-17, in which

Table B9-23. Data for Calculation of Approximate Values of Deflections w_0 and Stresses in Centrally Loaded Plates (μ = 0.3)

Boundary Conditions		A	B	Center	Edge			
				$\alpha_r = \alpha_t$	α_r	α_t	β_r	β_t
Plate Clamped	Edge Immovable	0.443	0.217	1.232	0.357	0.107	-2.198	-0.659
	Edge Free To Move	0.200	0.217	0.875	0	-0.250	-2.198	-0.659
Plate Simply Supported	Edge Immovable	1.430	0.552	0.895	0.488	0.147	0	0.606
	Edge Free To Move	0.272	0.552	0.407	0	-0.341	0	0.606

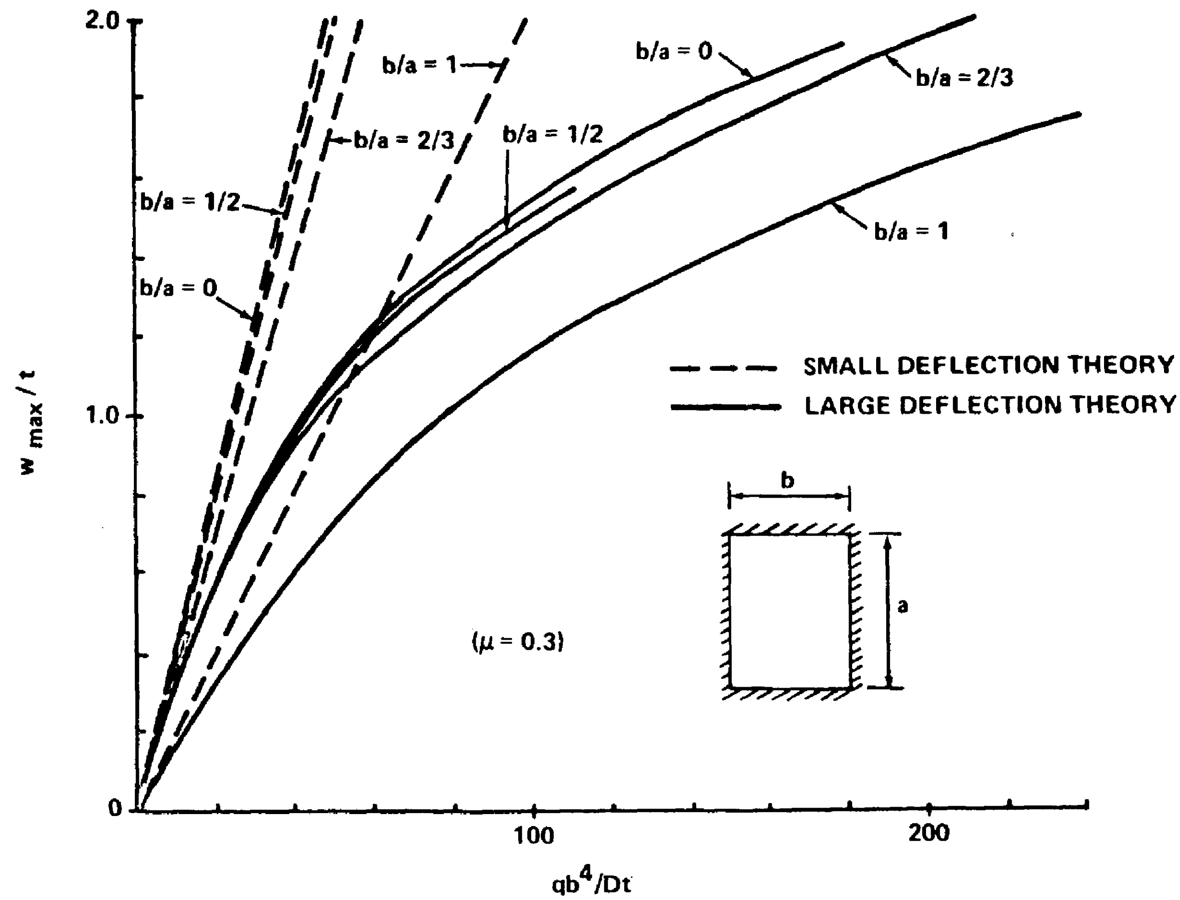


FIGURE B9-17. MAXIMUM DEFLECTIONS AT CENTER FOR RECTANGULAR PLATE WITH CLAMPED EDGES

w_{max}/t is plotted against qb^4/Dt. For comparison, the figure also includes the use of the theory of small deflections. Also included is the curve for $b/a = 0$, which represents deflections of an infinitely long plate. It can be seen that the deflections of finite plates with $b/a < 2/3$ are very close to those obtained for an infinitely long plate. The maximum values of the combined membrane and bending stress are at the middle of the long sides of the plate. They are given in graphical form in Fig. B9-18.

For the case of a rectangular plate, uniformly loaded with immovable, simply supported edges, a solution has been obtained [1]. Values for membrane stresses and extreme fiber bending stresses are given in Figs. B9-19 and B9-20, respectively. An approximate equation for maximum deflection, w_0, at the center of the plate in terms of the load q is given by:

$$q = \frac{w_0 E t^3}{a^4}\left[1.37 + 1.94\left(\frac{w_0}{t}\right)^2\right] . \tag{36}$$

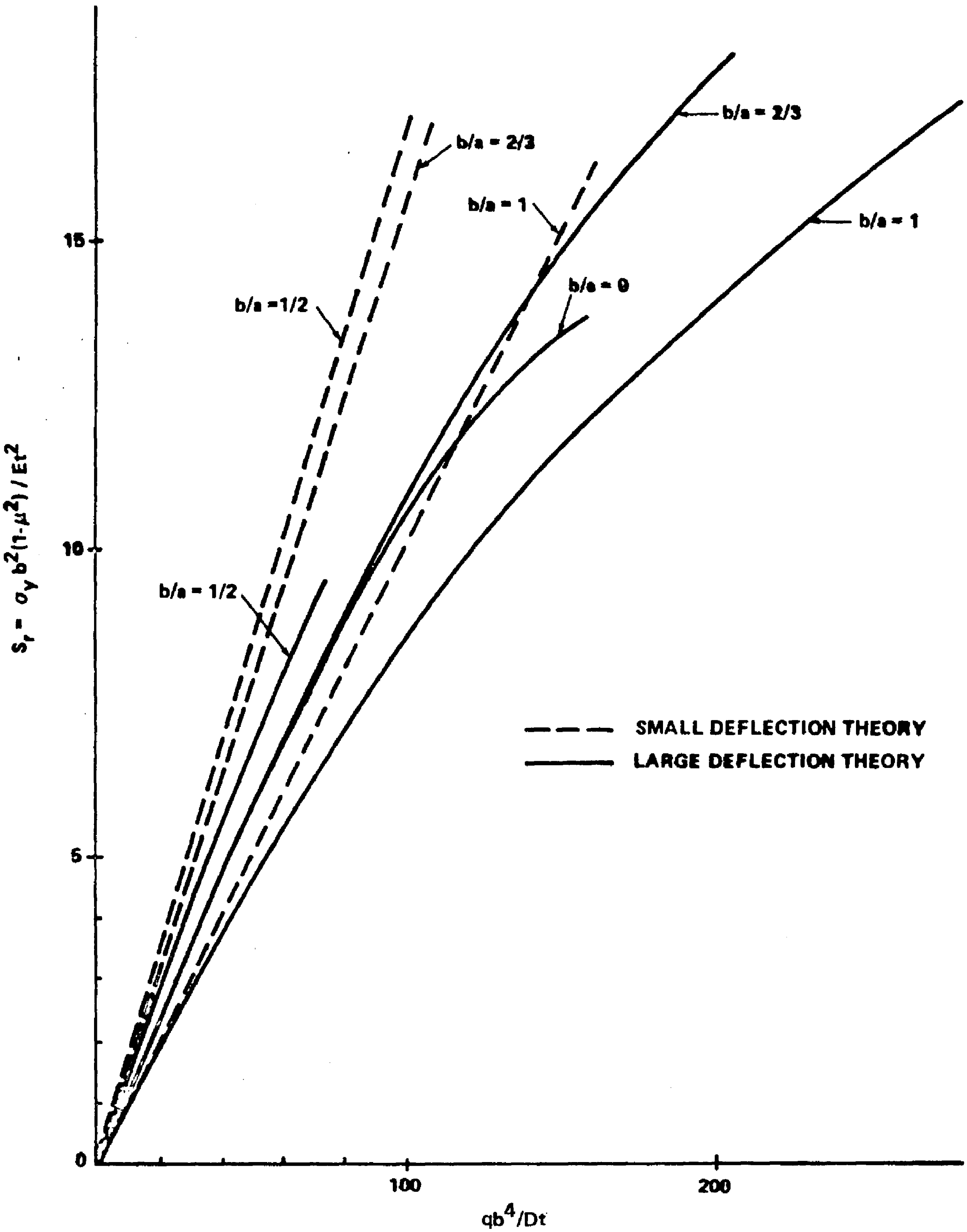


FIGURE B9-18. MAXIMUM STRESS AT CENTER OF LONG EDGE FOR RECTANGULAR PLATE WITH CLAMPED EDGES

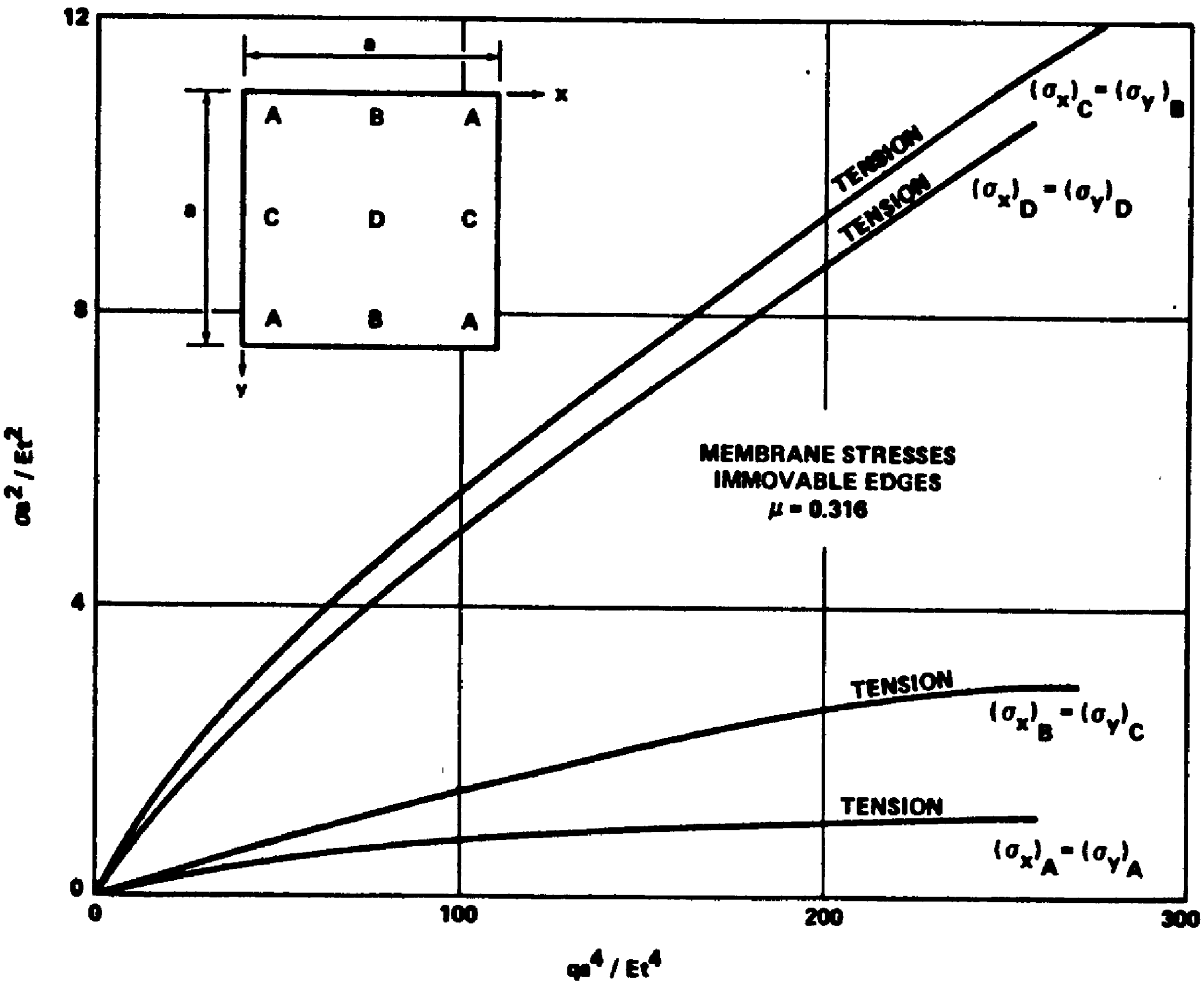


FIGURE B9-19. MEMBRANE STRESSES IN SQUARE PLATE, UNIFORMLY LOADED

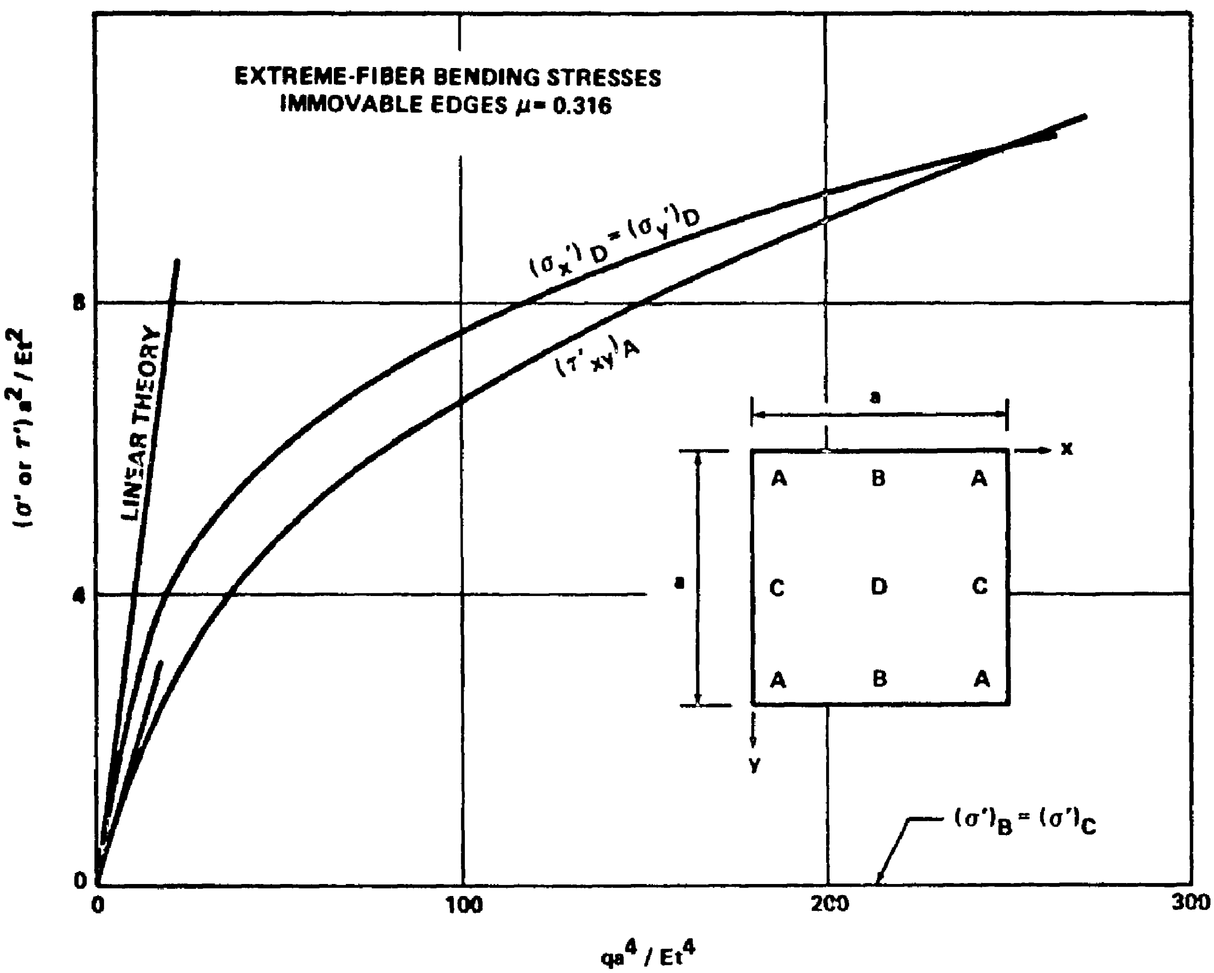


FIGURE B9-20. BENDING STRESSES IN SQUARE PLATE, UNIFORMLY LOADED

B9.5 ORTHOTROPIC PLATES

B9.5.1 Rectangular Plate

The deflection and the bending moments at the center of an orthotropic rectangular plate can be calculated from the following equations obtained from Reference 1.

$$w = \alpha \frac{q_0 b^4}{D_y} \quad , \tag{37}$$

$$M_x = \left(\beta_1 + \beta_2 \frac{E''}{E'_x} \sqrt{\frac{D_x}{D_y}} \right) \frac{q_0 a^2}{\epsilon} \quad , \tag{38}$$

$$M_y = \left(\beta_2 + \beta_1 \frac{E''}{E'_y} \sqrt{\frac{D_y}{D_x}} \right) q_0 b^2 \quad , \tag{39}$$

where α, β_1, and β_2 are numerical coefficients given in Table B9-24 and

$$\epsilon = \frac{a}{b} \sqrt[4]{\frac{D_y}{D_x}} \quad . \tag{40}$$

The four constants E'_x, E'_y, E'', and G in equations (37), (38), and (39) are needed to characterize the elastic properties of a material in the case of plane stress. These four constants are defined by equations (8) of Section B9.2.1.1. Equations (41) through (44) are expressions for rigidities and are subject to modifications according to the nature of the material and the geometry of the stiffening.

$$D_x = \frac{E'_x h^3}{12} \tag{41}$$

$$D_y = \frac{E'_y h^3}{12} \tag{42}$$

$$D_1 = \frac{E'' h^3}{12} \tag{43}$$

$$D_{xy} = \frac{Gh^3}{12} \tag{44}$$

Table B9-24. Constants α, β_1, and β_2 for A Simply Supported Rectangular Orthotropic Plate with $H = \sqrt{D_x D_y}$

ϵ	α	β_1	β_2
1.0	0.00407	0.0368	0.0368
1.1	0.00438	0.0359	0.0447
1.2	0.00565	0.0344	0.0524
1.3	0.00639	0.0324	0.0597
1.4	0.00709	0.0303	0.0665
1.5	0.00772	0.0280	0.0728
1.6	0.00831	0.0257	0.0785
1.7	0.00884	0.0235	0.0837
1.8	0.00932	0.0214	0.0884
1.9	0.00974	0.0191	0.0929
2.0	0.01013	0.0174	0.0964
2.5	0.01150	0.0099	0.1100
3.0	0.01223	0.0055	0.1172
4.0	0.01282	0.0015	0.1230
5.0	0.01297	0.0004	0.1245
∞	0.01302	0.0	0.1250

All values of rigidities based on purely theoretical considerations should be regarded as a first approximation and tests are recommended to obtain more reliable values. Usual values of the rigidities for three cases of practical interest are given below.

1. Plate Reinforced By Equidistant Stiffeners In One Direction: Consider a plate reinforced symmetrically with respect to its middle plane as shown in Fig. B9-21. The elastic constants of the material of the plating are E and ν, E' the Young's modulus, and I the moment of inertia of a stiffener, taken with respect to the middle axis of the cross section of the plate. The rigidity values are stated by equations (45) and (46):

$$D_x = \frac{Eh^3}{12(1-\nu^2)} = H \quad , \tag{45}$$

$$D_y = \frac{Eh^3}{12(1-\nu^2)} + \frac{E'I}{a_1} \quad , \tag{46}$$

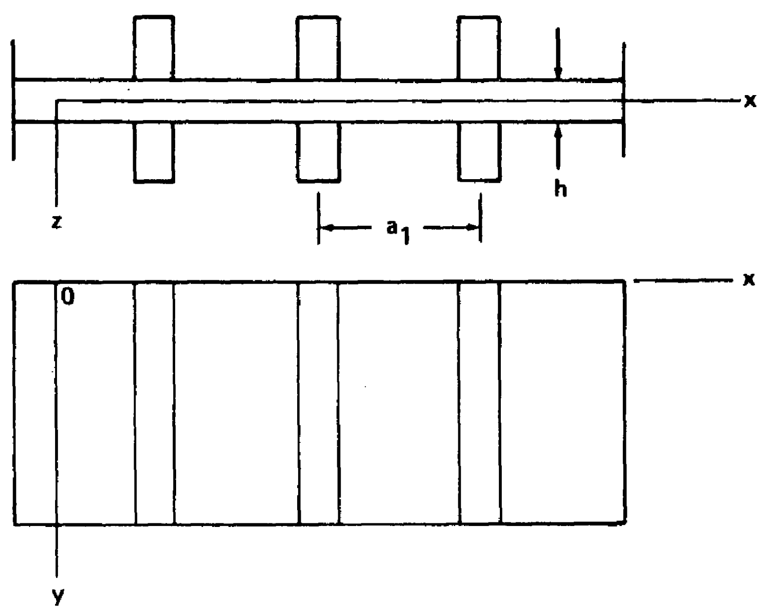


FIGURE B9-21. ORTHOTROPIC PLATE WITH EQUIDISTANT STIFFENERS

2. Plate Cross-Stiffened By Two Sets Of Equidistant Stiffeners: Assume the reinforcement to remain symmetrical about the plating. The moment of

inertia of one stiffener is I_1 , and b_1 is the spacing of the stiffeners in the x-direction. The corresponding values for stiffeners in the y-direction are I_2 and a_1 . The rigidity values for this case are stated by equations (47), (48), and (49):

$$D_x = \frac{Eh^3}{12(1-\nu^2)} + \frac{E' I_1}{b_1} \quad , \tag{47}$$

$$D_y = \frac{Eh^3}{12(1-\nu^2)} + \frac{E' I_2}{a_1} \quad , \tag{48}$$

$$H = \frac{Eh^3}{12(1-\nu^2)} \quad . \tag{49}$$

3. Plate Reinforced By A Set Of Equidistant Ribs: Refer to Fig. B9-22 and let E be the modulus of the material, I the moment of inertia of a T-section of width a_1 , and $\alpha = h/H$. Then, the rigidities are expressed by equations (50), (51), and (52):

$$D_x = \frac{Ea_1h^3}{12(a_1 - t + \alpha^3 t)} \quad , \tag{50}$$

$$D_y = \frac{EI}{a_1} \quad , \tag{51}$$

$$D_1 = 0 \quad . \tag{52}$$

The effect of the transverse contraction is neglected in the foregoing equations. The torsional rigidity may be calculated by means of equation (53):

$$D_{xy} = D'_{xy} + \frac{C}{2a_1} \quad , \tag{53}$$

in which D'_{xy} is the torsional rigidity of the plate without the ribs and C the torsional rigidity of one rib.

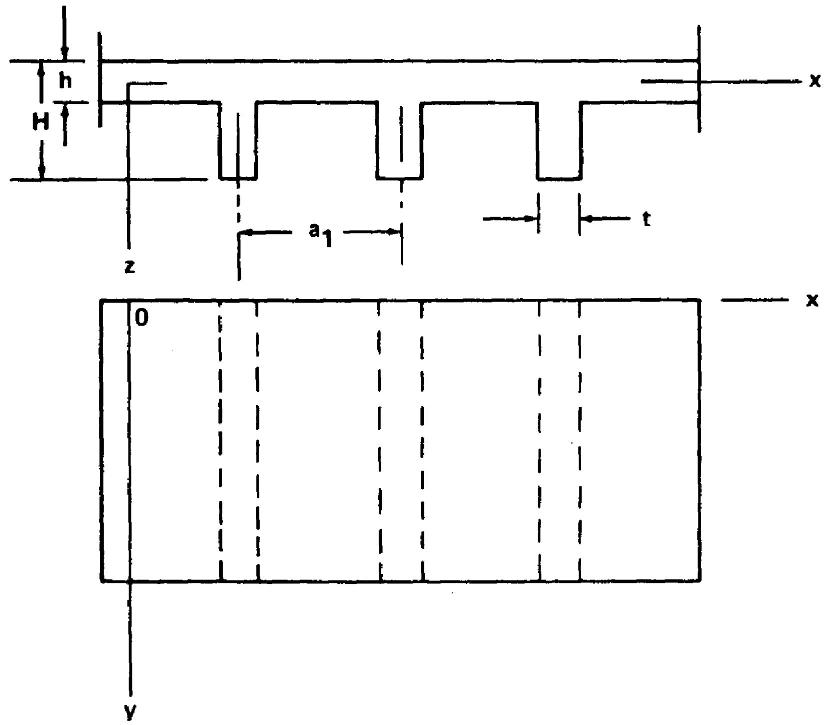


FIGURE B9-22. ORTHOTROPIC PLATE WITH STIFFENERS ON ONE SIDE

Formulas for the elastic constants of plates with integral waffle-like stiffening can be found in Reference 6.

B9.6 STRUCTURAL SANDWICH PLATES

B9.6.1 Small Deflection Theory

The information presented for the small deflection theory was obtained from Reference 7.

Structural sandwich is a layered composite formed by bonding two thin facings to a thick core. The main difference in design procedures for sandwich structural elements and those of homogeneous material is the inclusion of the effects of core shear properties on deflection, buckling, and stress for the sandwich. The basic design principles for a sandwich can be summarized into four conditions as follows:

1. Sandwich facings shall be at least thick enough to withstand chosen design stresses under design ultimate loads.
2. The core shall be thick enough and have sufficient shear rigidity and strength so that overall sandwich buckling, excessive deflection, and shear failure will not occur under design ultimate loads.
3. The core shall have high enough moduli of elasticity and the sandwich shall have great enough flatwise tensile and compressive strength so that wrinkling of either facing will not occur under design ultimate loads.
4. If the core is made of cellular or corrugated material and dimpling of the facings is not permissible, the cell size or corrugation spacing shall be small enough so that dimpling of either facing into the core spaces will not occur under design ultimate loads.

B9.6.1.1 Basic Principles for Design of Flat Sandwich Panels under Uniformly Distributed Normal Load

Assuming that a design begins with chosen design stresses and deflections and a given load to transmit, a flat rectangular or circular panel of sandwich construction under uniformly distributed normal load shall be designed to comply with the four basic design principles.

Detailed procedures giving theoretical formulas and graphs for determining dimensions of the facings and core, as well as necessary core properties, for simply supported panels are given in the following paragraphs. Double formulas are given, one formula for sandwich with isotropic facings of different materials and thicknesses and another formula for sandwich with each isotropic facing of the same material and thickness. Facing moduli of elasticity, $E_{1,2}$, and stress values, $F_{1,2}$, shall be compression or tension values at the condition of use; that is, if application is at elevated temperature, then facing properties at elevated temperature shall be used in design. For many combinations of facing materials it will be advantageous to choose thicknesses such that $E_1t_1 = E_2t_2$. The following procedures are restricted to linear elastic behavior.

B9.6.1.2 Determining Facing Thickness, Core Thickness , and Core Shear Modulus for Simply Supported Flat Rectangular Panels

This section gives procedures for determining sandwich facing and core thicknesses and core shear modulus so that chosen design facing stresses and allowable panel deflections will not be exceeded. The facing stresses, produced by bending moment, are maximum at the center of a simply supported panel under uniformly distributed normal load. If restraint exists at panel edges, a redistribution of stresses may cause higher stresses near panel edges. The procedures given apply only to panels with simply supported edges. Because facing stresses are caused by bending moment, they depend not only upon facing thickness but also upon the distance the facings are spaced, hence core thickness. Panel stiffness, hence deflection, is also dependent upon facing and core thickness.

If the panel is designed so that facing stresses are at chosen design levels, the panel deflection may be larger than allowable, in which case the core or facings must be thickened and the design facing stress lowered to meet deflection requirements. A solution is presented in the form of charts with which, by iterative process, the facing and core thicknesses and core shear modulus can be determined.

The average facing stress, F(stress at facing centroid), is given by the theoretical formulas:

$$F_{1,2} = K_2 \frac{pb^2}{ht_{1,2}} \qquad \text{(for unequal facings)} \quad , \tag{54}$$

and

$$F = K_2 \frac{pb^2}{ht} \qquad \text{(for equal facings)} \quad , \tag{55}$$

where p is the intensity of the distributed load; b is the panel width; h is the distance between facing centroids; t is facing thickness; 1 and 2 are subscripts denoting facings 1 and 2; and K_2 is a theoretical coefficient dependent on panel aspect ratio and sandwich bending and shear rigidities. If the core is isotropic (shear moduli alike in the two principal directions), K_2 values depend only upon panel aspect ratio. The values of K_2 for sandwich with orthotropic core are dependent not only on panel aspect ratio but also upon sandwich bending and shear rigidities as incorporated in the parameter $V = \frac{\pi^2 D}{b^2 U}$ which can be written as:

$$V = \frac{\pi^2 t_c E_1 t_1 E_2 t_2}{\lambda b^2 G_c (E_1 t_1 + E_2 t_2)} \tag{56}$$

$$V = \frac{\pi^2 t_c E t}{2 \lambda b^2 G_c} \qquad \text{(for equal facings)} \quad , \tag{57}$$

where U is sandwich shear stiffness; E is modulus of elasticity of facing; $\lambda = 1 - \mu^2$; μ is Poisson's ratio of facings [in formula (56) it is assumed that $\mu = \mu_1 = \mu_2$]; and G_c is the core shear modulus associated with axes parallel to panel side of length a and perpendicular to the plane of the panel. The core shear modulus associated with axes parallel to panel side of width b and perpendicular to the plane of the panel is denoted by (RG_c) .

Solving equations (54) and (55) for h/b gives

$$\frac{h}{b} = \sqrt{K_2} \; \frac{\sqrt{\frac{p}{F_{1,2}}}}{\sqrt{\frac{t_{1,2}}{h}}} \tag{58}$$

$$\frac{h}{b} = \sqrt{K_2}\,\frac{\sqrt{\frac{p}{F}}}{\sqrt{\frac{t}{h}}} \qquad \text{(for equal facings)} \quad . \tag{59}$$

A chart for solving formulas (58) and (59) graphically is given in Fig. B9-23. The formulas and charts include the ratio t/h, which is usually unknown, but by iteration satisfactory ratios of t/h and h/b can be found.

The deflection, δ, of the panel center is given by the theoretical formula:

$$\delta = \frac{K_1}{K_2} \cdot \frac{\lambda F_{1,2}}{E_{1,2}} \left(1 + \frac{E_{1,2} t_{1,2}}{E_{2,1} t_{2,1}}\right) \frac{b^2}{h} \tag{60}$$

$$\delta = 2 \frac{K_1}{K_2} \cdot \frac{\lambda F}{E} \cdot \frac{b^2}{h} \qquad \text{(for equal facings)} \quad , \tag{61}$$

where K_1 is a coefficient dependent upon panel aspect ratio and the value of V.

Solving equations (60) and (61) for h/b gives

$$\frac{h}{b} = \frac{\sqrt{\frac{K_1}{K_2}}\sqrt{\frac{\lambda F_{1,2}}{E_{1,2}}}\sqrt{1 + \frac{E_{1,2} t_{1,2}}{E_{2,1} t_{2,1}}}}{\sqrt{\frac{\delta}{h}}} \tag{62}$$

$$\frac{h}{b} = \frac{\sqrt{\frac{2K_1}{K_2}}\sqrt{\frac{\lambda F}{E}}}{\sqrt{\frac{\delta}{h}}} \qquad \text{(for equal facings)} \quad . \tag{63}$$

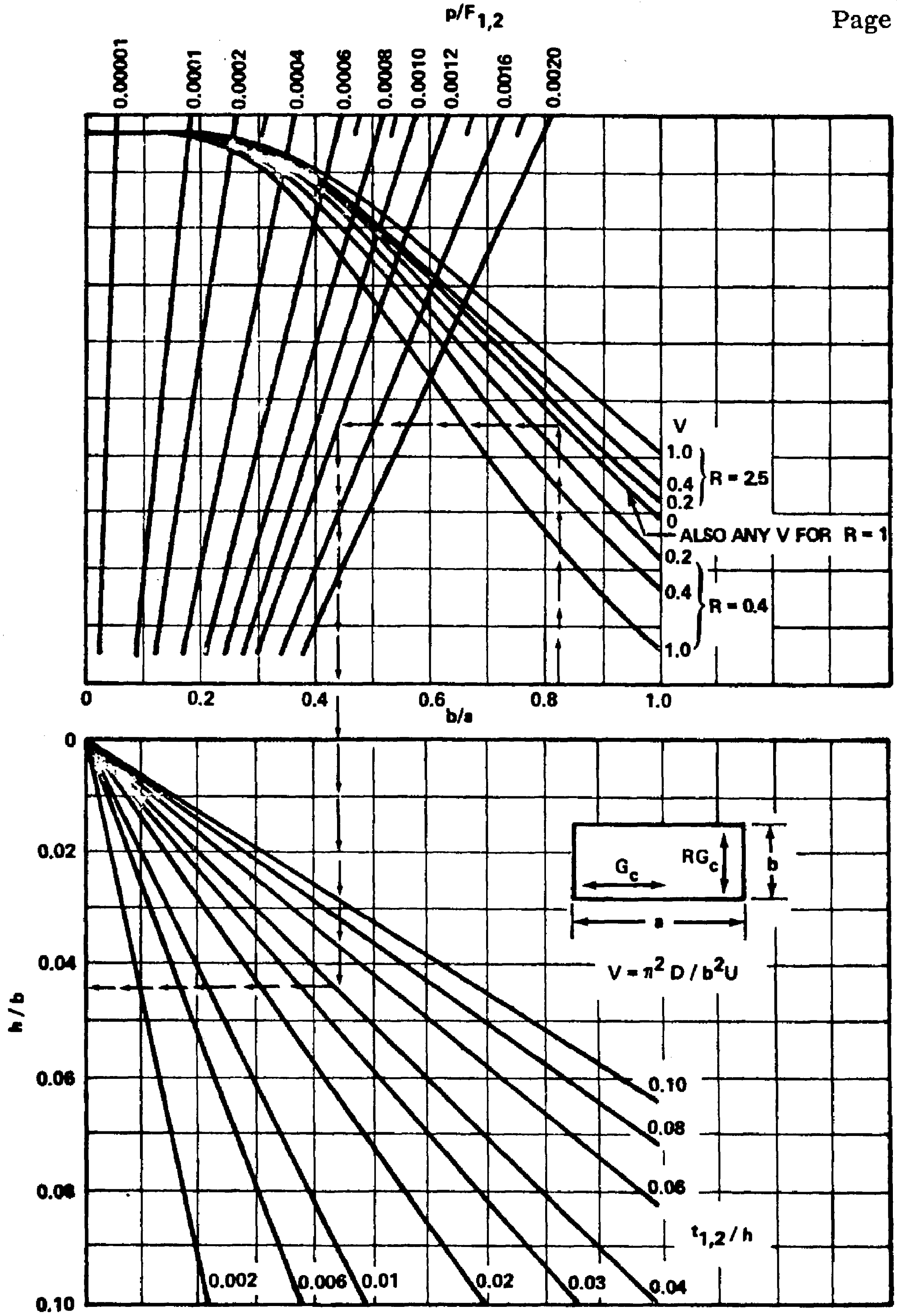


FIGURE B9-23. CHART FOR DETERMINING h/b RATIO FOR FLAT RECTANGULAR SANDWICH PANEL, WITH ISOTROPIC FACINGS, UNDER UNIFORMLY DISTRIBUTED NORMAL LOAD SO THAT FACING STRESS WILL BE $F_{1,2}$

Charts for solving formulas (62) and (63) are given in Figs. B9-24, B9-25, and B9-26. Use of the equations and charts beyond $\delta/h = 0.5$ is not recommended.

B9.6.1.3 Use of Design Charts

The sandwich must be designed by iterative procedures; these charts enable rapid determination of the various quantities sought. The charts were derived for a Poisson's ratio of the facings of 0.3 and can be used with small error for facings having other values of Poisson's ratio.

As a first approximation, it will be assumed that $V = 0$. If the design is controlled by facing stress criteria, as may be determined, this assumption will lead to an exact value of h if the core is isotropic, to a minimum value of h if the core is orthotropic with a greater core shear modulus across the panel width than across the length, and to too large a value of h if the core is orthotropic with a smaller core shear modulus across the panel width than across the length. If the design is controlled by deflection requirements, the assumption that $V = 0$ will produce a minimum value of h. The value of h is minimum because $V = 0$ if the core shear modulus is infinite. For any actual core, the shear modulus is not infinite; hence a thicker core must be used.

The following procedure is suggested:

1. Enter Fig. B9-23 with desired values for the parameters b/a and $p/F_{1,2}$, using the curve for $V = 0$. Assume a value for $t_{1,2}/h$ and determine

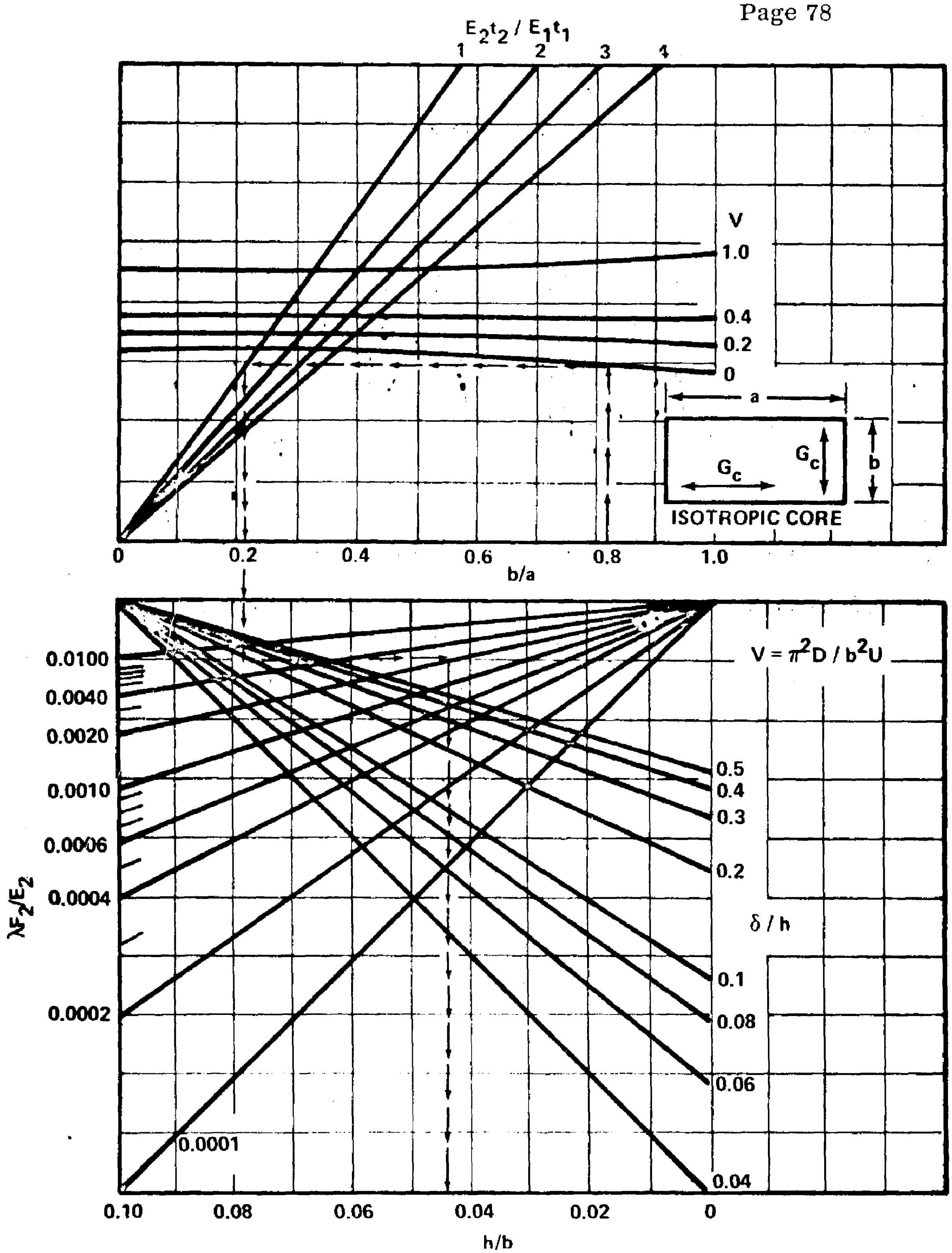


FIGURE B9-24. CHART FOR DETERMINING h/b RATIO FOR FLAT RECTANGULAR SANDWICH PANEL, WITH ISOTROPIC FACINGS AND ISOTROPIC CORE, UNDER UNIFORMLY DISTRIBUTED NORMAL LOAD PRODUCING DEFLECTION RATIO δ/h

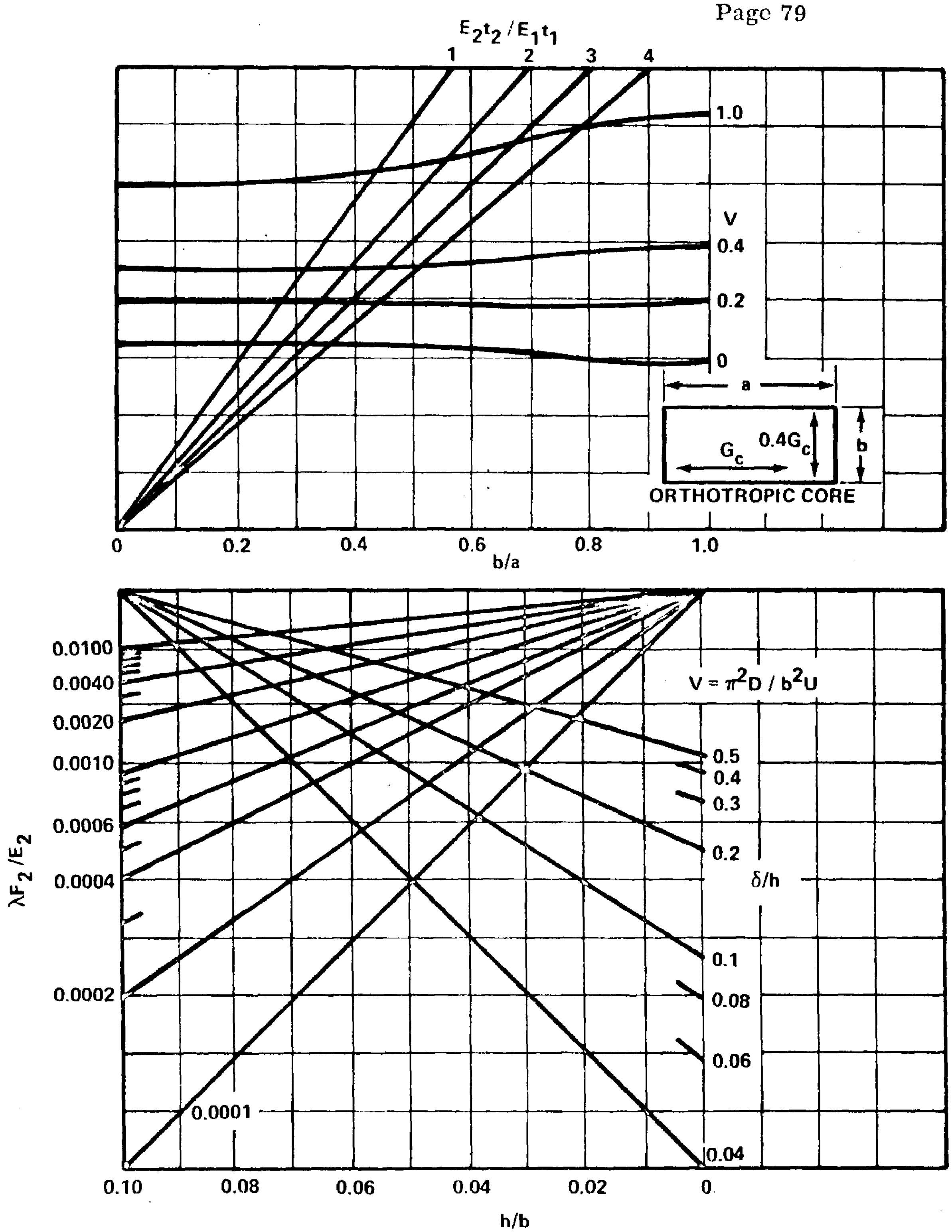


FIGURE B9-25. CHART FOR DETERMINING h/b RATIO FOR FLAT RECTANGULAR SANDWICH PANEL, WITH ISOTROPIC FACINGS AND ORTHOTROPIC (SEE SKETCH) CORE, UNDER UNIFORMLY DISTRIBUTED NORMAL LOAD PRODUCING DEFLECTION RATIO δ/h

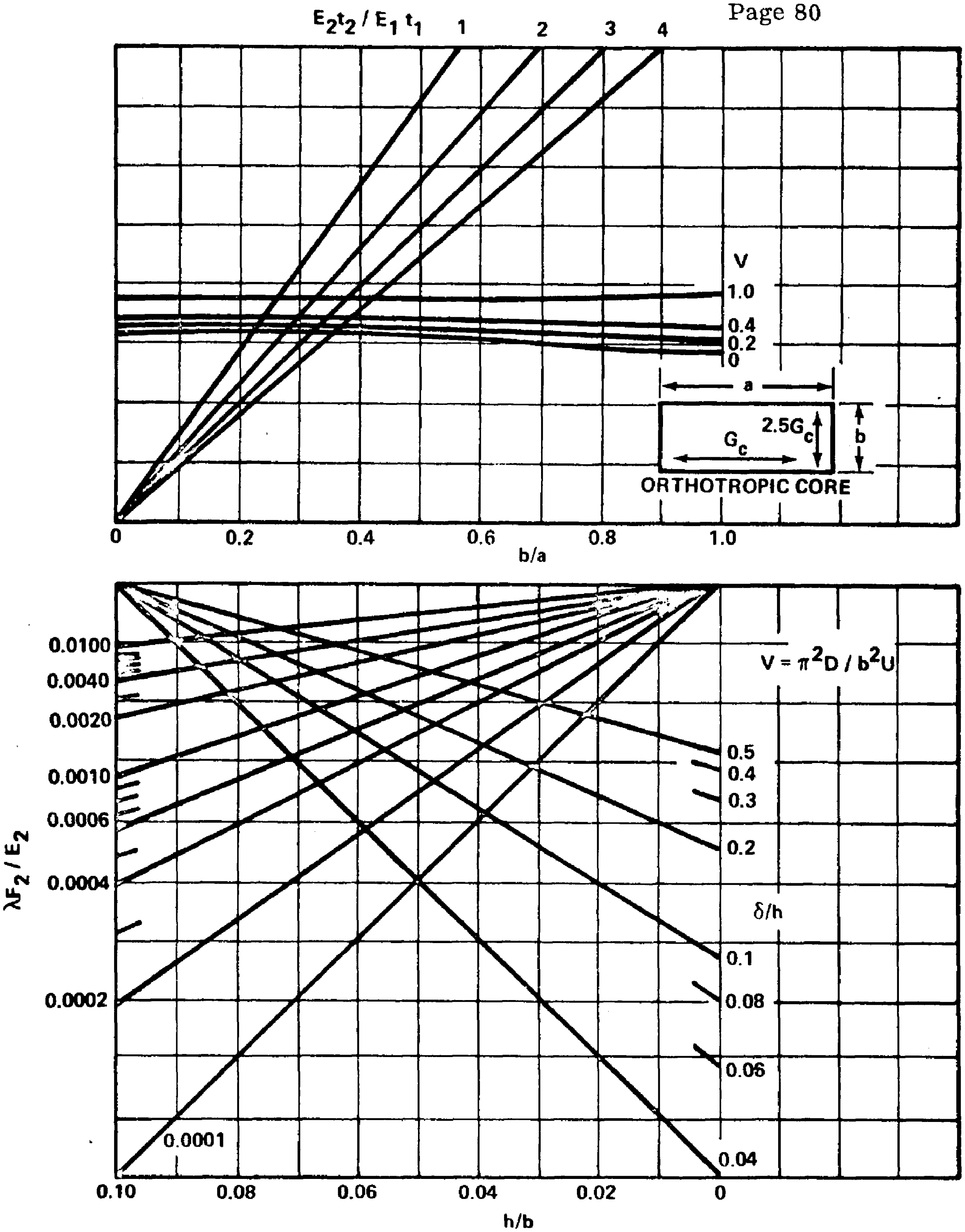


FIGURE B9-26. CHART FOR DETERMINING h/b RATIO FOR FLAT RECTANGULAR SANDWICH PANEL, WITH ISOTROPIC FACINGS AND ORTHOTROPIC (SEE SKETCH) CORE, UNDER UNIFORMLY DISTRIBUTED NORMAL LOAD PRODUCING DEFLECTION RATIO δ/h

h/b . Compute h and $t_{1,2}$. Modify ratio $t_{1,2}/h$ if necessary and determine more suitable values for h and $t_{1,2}$.

2. Enter Fig. B9-24 with desired values for the parameters b/a , $E_2 t_2/E_1 t_1$, and $\lambda F_2/E_2$, using the curve for $V = 0$. Assume a value of δ/h and determine h/b . Compute h and δ . Modify ratio δ/h if necessary and determine more suitable values for h and δ .

3. Repeat steps 1 and 2 using lower chosen design facing stresses until h determined by step 2 is equal to, or a bit less than, h determined by step 1.

4. Compute the core thickness, t_c , using the following formulas:

$$t_c = h - \frac{t_1 + t_2}{2} \tag{64}$$

$$t_c = h - t \qquad \text{(for equal facings)} \quad . \tag{65}$$

This first approximation was based on a core with an infinite shear modulus. Since actual core shear modulus values are not very large, a value of t_c somewhat larger must be used. Successive approximations can be made by entering Figs. B9-23 and B9-24, B9-25, or B9-26 with values of V as computed by equations (56) and (57). Figure B9-23 includes curves for sandwich with isotropic and certain orthotropic cores. Figure B9-24 applies to sandwich with isotropic core $(R = 1)$. Figure B9-25 applies to sandwich with orthotropic cores for which the shear modulus associated with the panel width is 0.4 of the shear modulus associated with the panel length $(R = 0.4)$. Figure B9-26 applies to sandwich with orthotropic cores for which the shear modulus associated with the panel width is 2.5 times the shear modulus associated with the panel length ($R = 2.5$).

NOTE: For honeycomb cores with core ribbon parallel to panel length a, $G_c = G_{TL}$ and the shear modulus parallel to panel width b is G_{TW}. For honeycomb cores with core ribbons parallel to panel width b, $G_c = G_{TW}$ and the shear modulus parallel to panel length a is G_{TL}.

In using Figs. B9-23 through B9-26 for $V \neq 0$, it is necessary to iterate because V is directly proportional to the core thickness t_c. As an aid to finally determine t_c and G_c, Fig. B9-27 presents a number of lines representing V for various values of G_c with V ranging from 0.01 to 2 and G_c ranging from 1000 to 1 000 000 psi. The following procedure is suggested:

1. Determine a core thickness using a value of 0.01 for V.

2. Compute the constant relating V to G_c:

$$\left[\frac{\pi^2 t_c E_1 t_1 E_2 t_2}{\lambda b^2 (E_1 t_1 + E_2 t_2)}\right] \text{ or } \left(\frac{\pi^2 t_c Et}{2\lambda b^2}\right) \quad \text{(for equal facings)} = VG_c \quad .$$

3. With this constant, enter Fig. B9-27 and determine necessary G_c.

4. If the shear modulus is outside the range of values for materials available, follow the appropriate line of Fig. B9-27 and pick a new value of V, for reasonable value of core shear modulus.

5. Reenter Figs. B9-23 through B9-26 with the new value of V and repeat all previous steps.

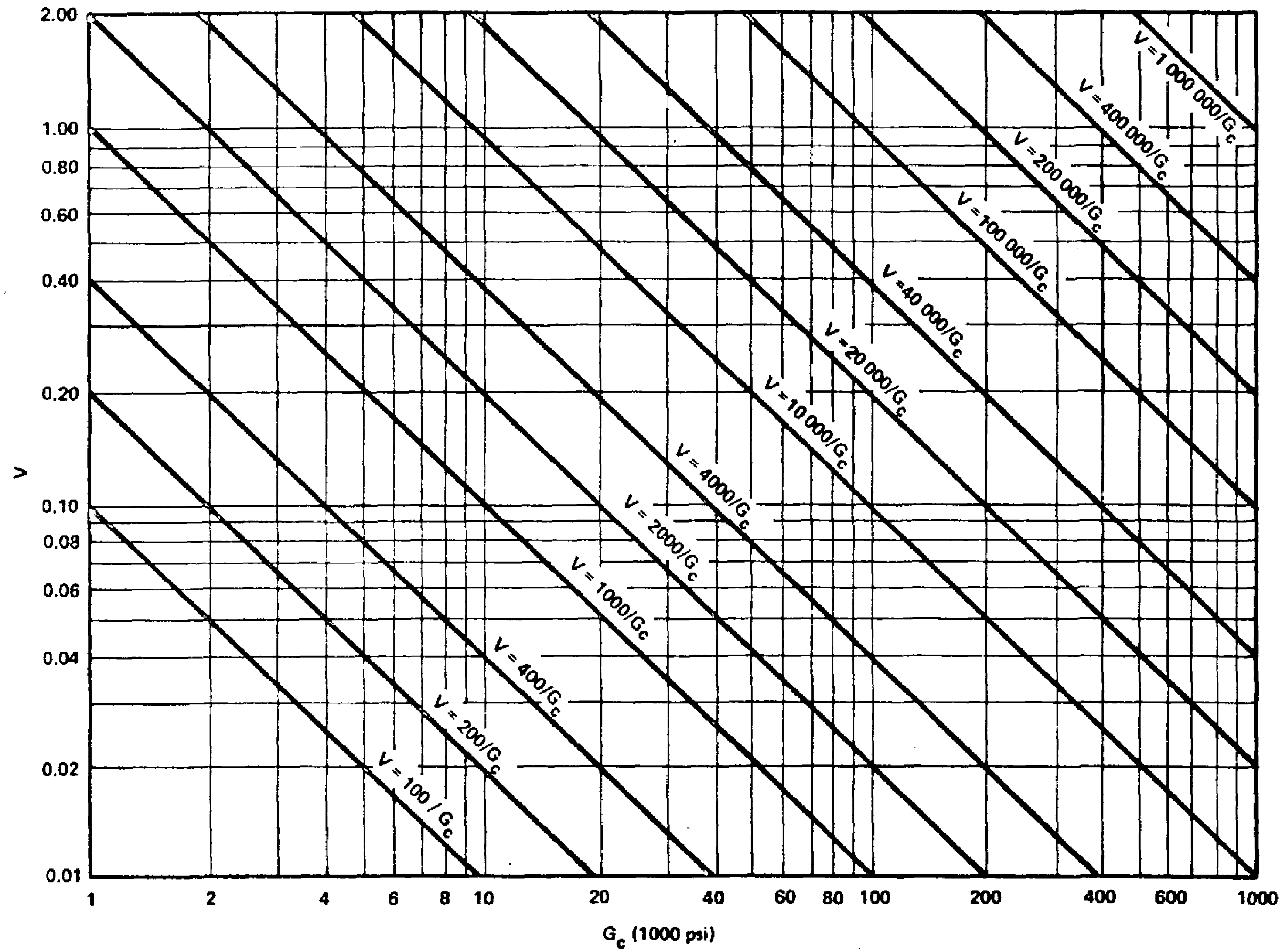


FIGURE B9-27. CHART FOR DETERMINING V AND G_c FOR SANDWICH PANELS UNDER UNIFORMLY DISTRIBUTED NORMAL LOAD

B9.6.1.4 Determining Core Shear Stress

This section gives the procedure for determining the maximum core shear stress of a flat rectangular sandwich panel under uniformly distributed normal load. The core shear stress is maximum at the panel edges, at mid-length of each edge. The maximum shear stress, F_{cs}, is given by the formula:

$$F_{cs} = K_3 p \frac{b}{h} \quad , \tag{66}$$

where K_3 is a theoretical coefficient dependent upon panel aspect ratio and the parameter V. If the core is isotropic, values of V do not affect the core shear stress.

The chart of Fig. B9-28 presents a graphical solution of formula (66). The chart should be entered with values of thicknesses and other parameters previously determined.

B9.6.1.5 Checking Procedure

The design shall be checked by using the graphs of Figs. B9-29, B9-30, and B9-31 to determine theoretical coefficients K_2, K_1, and K_3 to compute facing stresses, deflection, and core shear stresses.

B9.6.1.6 Determining Facing Thickness, Core Thickness, and Core Shear Modulus for Simply Supported Flat Circular Panels

This section gives procedures for determining sandwich facing and core thicknesses and core shear modulus so that chosen design facing stresses and allowable panel deflections will not be exceeded. The facing stresses, produced

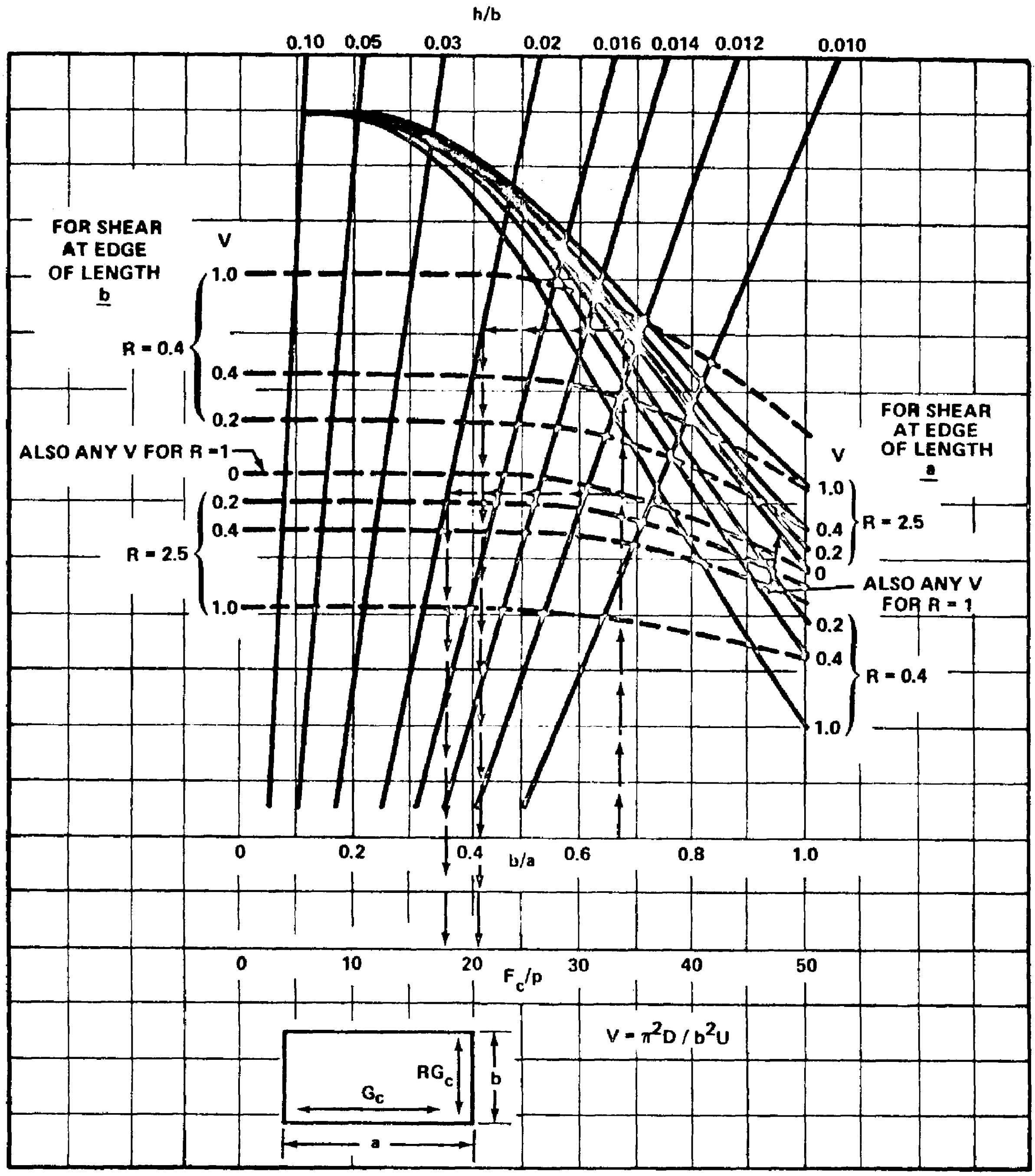


FIGURE B9-28. CHART FOR DETERMINING CORE SHEAR STRESS RATIO $\frac{F_{sc}}{p}$ FOR FLAT RECTANGULAR SANDWICH PANEL, WITH ISOTROPIC FACINGS, UNDER UNIFORMLY DISTRIBUTED NORMAL LOAD

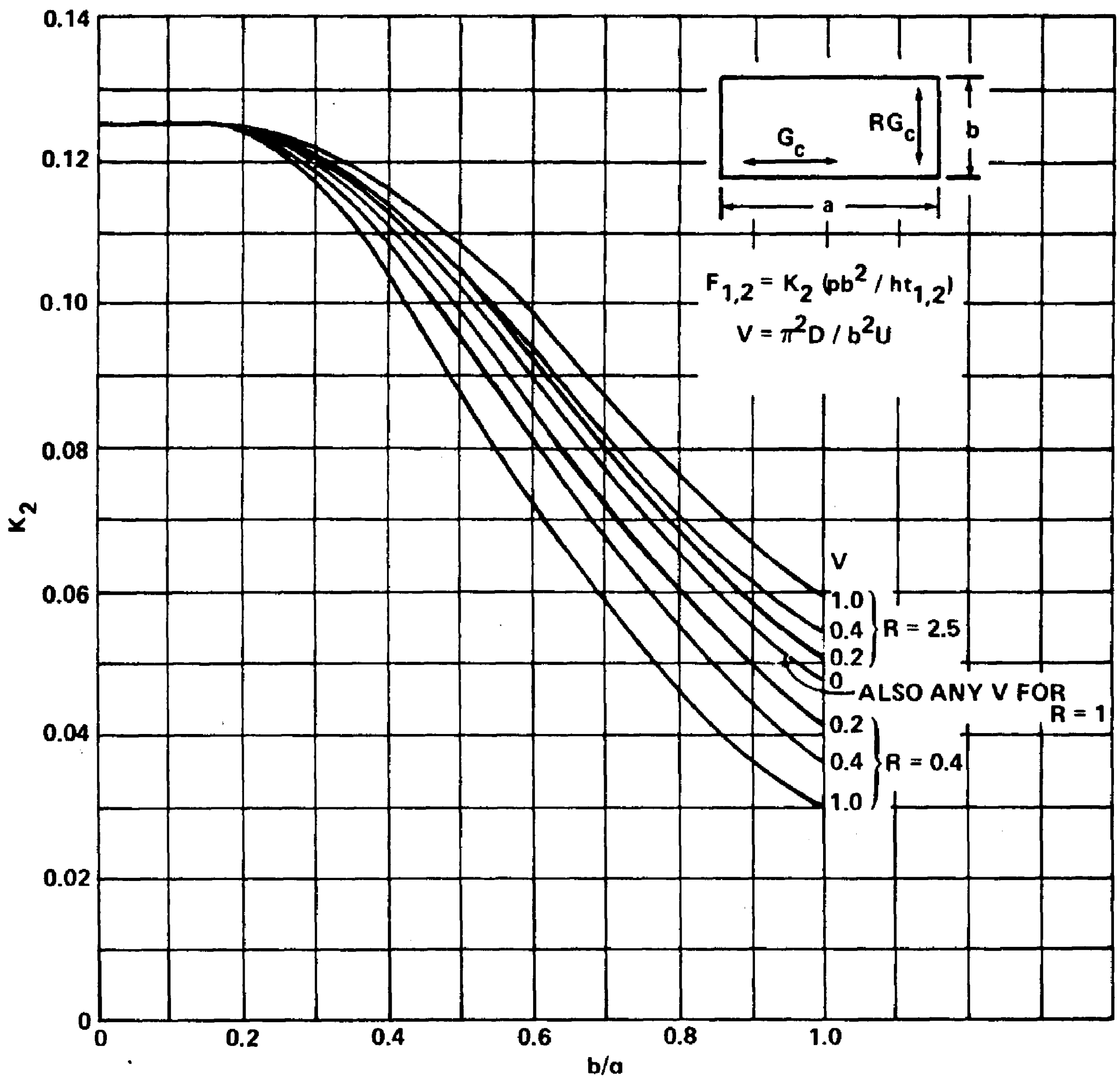


FIGURE B9-29. K_2 FOR DETERMINING FACING STRESS, F, OF FLAT RECTANGULAR SANDWICH PANELS WITH ISOTROPIC FACINGS AND ISOTROPIC OR ORTHOTROPIC CORE (SEE SKETCH) UNDER UNIFORMLY DISTRIBUTED NORMAL LOAD

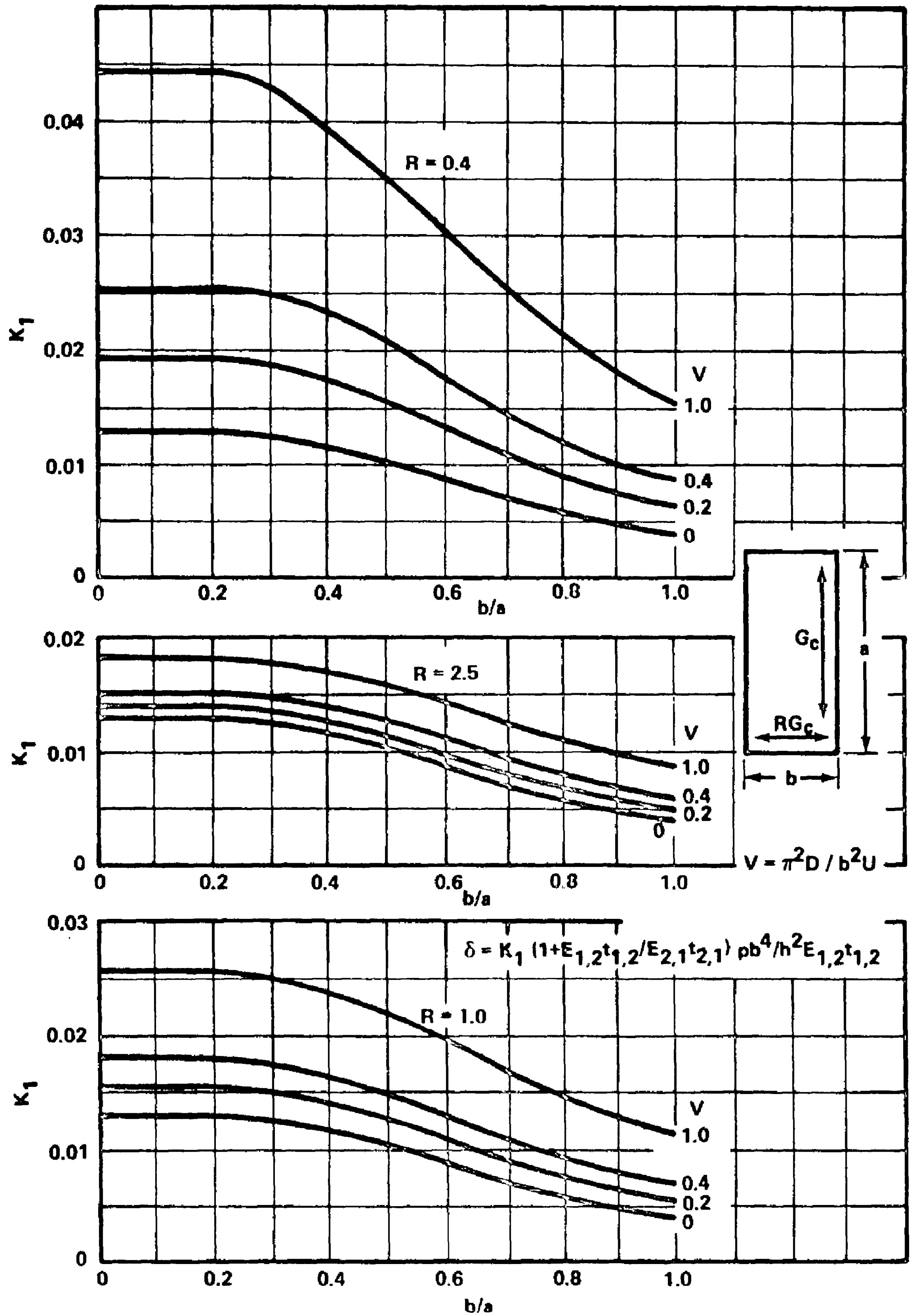
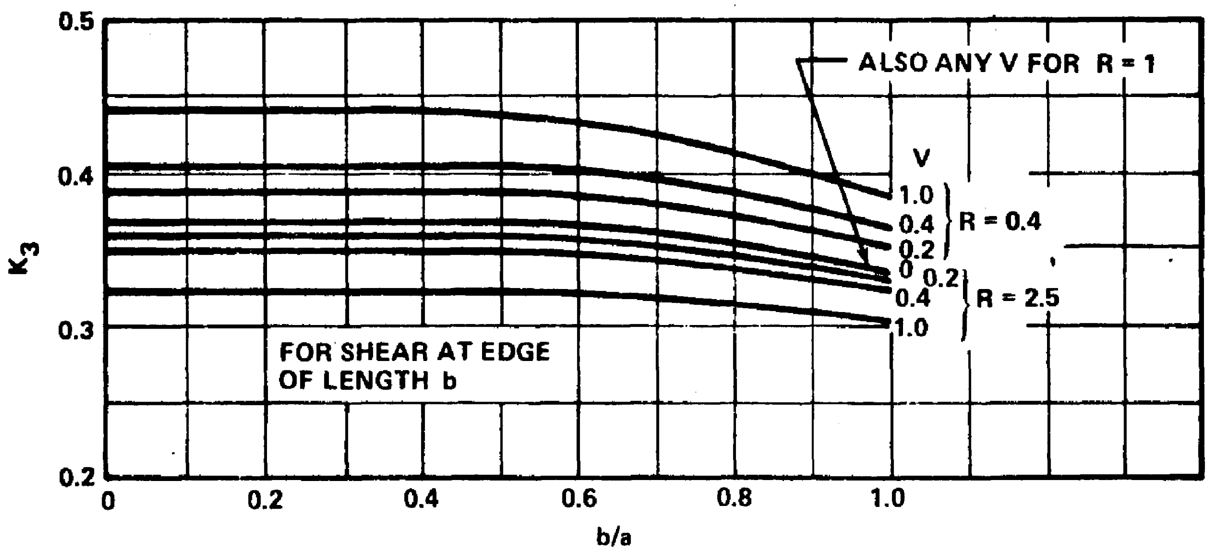
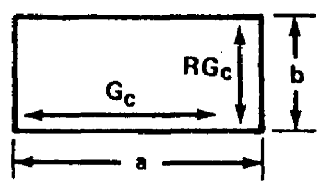


FIGURE B9-30. K_1 FOR DETERMINING MAXIMUM DEFLECTION, δ, OF FLAT RECTANGULAR SANDWICH PANELS WITH ISOTROPIC FACINGS AND ISOTROPIC OR ORTHOTROPIC CORE (SEE SKETCH) UNDER UNIFORMLY DISTRIBUTED NORMAL LOAD



$F_{sc} = K_3 p(b/h)$

$V = \pi^2 D / b^2 U$



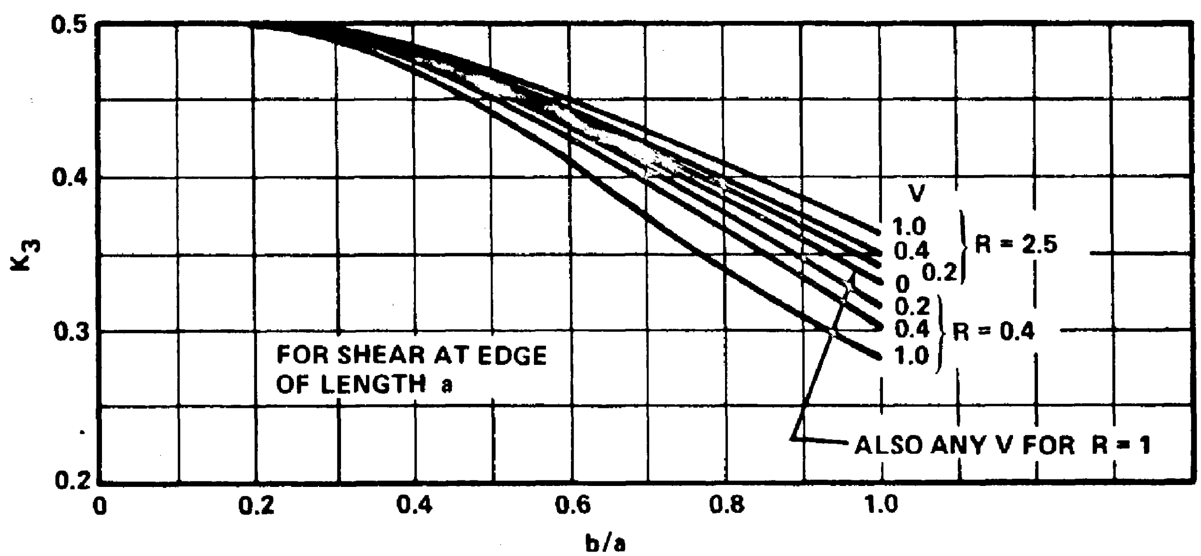


FIGURE B9-31. K_3 FOR DETERMINING MAXIMUM CORE SHEAR STRESS, F_{sc}, FOR FLAT RECTANGULAR SANDWICH PANELS WITH ISOTROPIC FACINGS AND ISOTROPIC OR ORTHOTROPIC CORE (SEE SKETCH) UNDER UNIFORMLY DISTRIBUTED NORMAL LOAD by bending moment, are maximum at the center of a simply supported circular panel under uniformly distributed normal load. If restraint exists at panel edges, a redistribution of stresses may cause higher stresses near panel edges. The procedures given apply only to panels with simply supported edges, isotropic facings, and isotropic cores. A solution is presented in the form of charts with which, by iterative process, the facing and core thicknesses and core shear modulus can be determined.

The average facing stress, F (stress at facing centroid), is given by the theoretical formula:

$$F_{1,2} = \frac{3+\mu}{16} \frac{pr^2}{t_{1,2}h} \tag{67}$$

$$F = \frac{3+\mu}{16} \frac{pr^2}{th} \quad \text{(for equal facings)} \quad , \tag{68}$$

where μ is Poisson's ratio of facings [in formula (67), it is assumed that $\mu = \mu_1 = \mu_2$] ; r is the radius of the circular panel; and other quantities are as previously defined (see Section B9.6.1.2).

Solving equations (67) and (68) for $\frac{h}{r}$ gives

$$\frac{h}{r} = \frac{\sqrt{3+\mu}\,\sqrt{\dfrac{p}{F_{1,2}}}}{4\sqrt{\dfrac{t_{1,2}}{h}}} \tag{69}$$

$$\frac{h}{r} = \frac{\sqrt{3+\mu}\,\sqrt{\dfrac{p}{F}}}{4\sqrt{\dfrac{t}{h}}} \quad \text{(for equal facings)} \quad . \tag{70}$$

A chart for solving formulas (69) and (70) graphically is given in Fig. B9-32. The formulas and chart include the ratio t/h, which is usually unknown, but by iteration satisfactory ratios of t/h and h/r can be found.

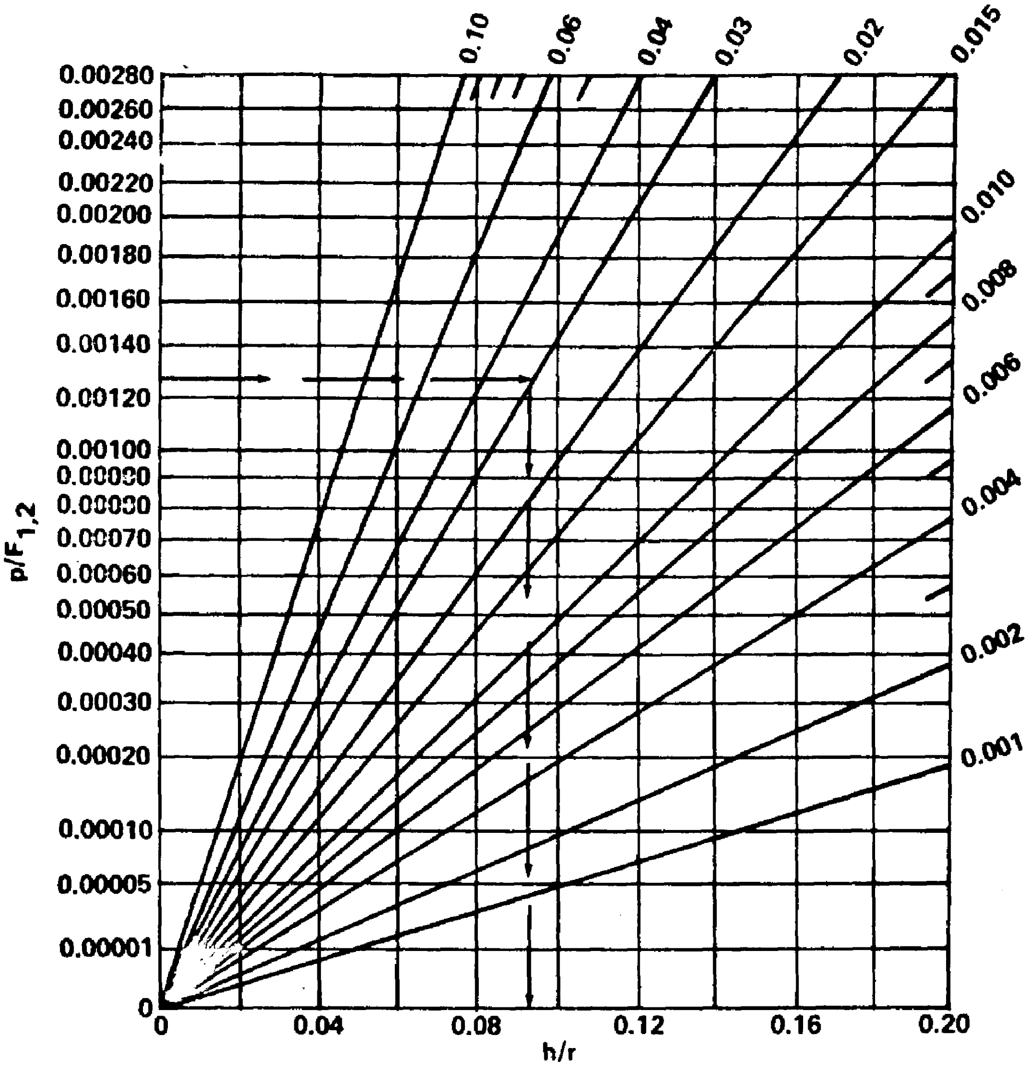


FIGURE B9-32. CHART FOR DETERMINING h/r RATIO FOR FLAT CIRCULAR SANDWICH PANEL, WITH ISOTROPIC FACINGS AND CORE, UNDER UNIFORMLY DISTRIBUTED NORMAL LOAD SO THAT FACING STRESS WILL BE $F_{1,2}$; $\mu = 0.3$

The deflection, δ, of the panel center is given by the theoretical formula:

$$\delta = K_4\left(1 + \frac{E_{1,2}t_{1,2}}{E_{2,1}t_{2,1}}\right)\frac{\lambda F_{1,2}r^2}{E_{1,2}h} \tag{71}$$

$$\delta = 2K_4\frac{\lambda F}{E}\frac{r^2}{h} \quad \text{(for equal facings)} \quad , \tag{72}$$

where K_4 depends on the sandwich bending and shear rigidities as incorporated

in the parameter $V = \frac{\pi^2 D}{(2r)^2 U}$ which can be written as

$$V = \frac{\pi^2 t_c E_1 t_1 E_2 t_2}{4\lambda r^2 G_c (E_1 t_1 + E_2 t_2)} \tag{73}$$

$$V = \frac{\pi^2 t_c E t}{8\lambda r^2 G_c} \quad \text{(for equal facings)} \quad , \tag{74}$$

where r is panel radius and all other terms are as previously defined in Section B9.6.1.2

Solving equations (71) and (72) for $\frac{h}{r}$, gives

$$\frac{h}{r} = \frac{\sqrt{K_4}\sqrt{\frac{\lambda F_{1,2}}{E_{1,2}}}\sqrt{1 + \frac{E_{1,2}t_{1,2}}{E_{2,1}t_{2,1}}}}{\sqrt{\frac{\delta}{h}}} \tag{75}$$

$$\frac{h}{r} = \frac{\sqrt{2K_4}\sqrt{\frac{\lambda F}{E}}}{\sqrt{\frac{\delta}{h}}} \quad \text{(for equal facings)} \quad . \tag{76}$$

A chart for solving formulas (75) and (76) is given in Fig. B9-33. Use of the equations and charts beyond $\delta/h = 0.5$ is not recommended.

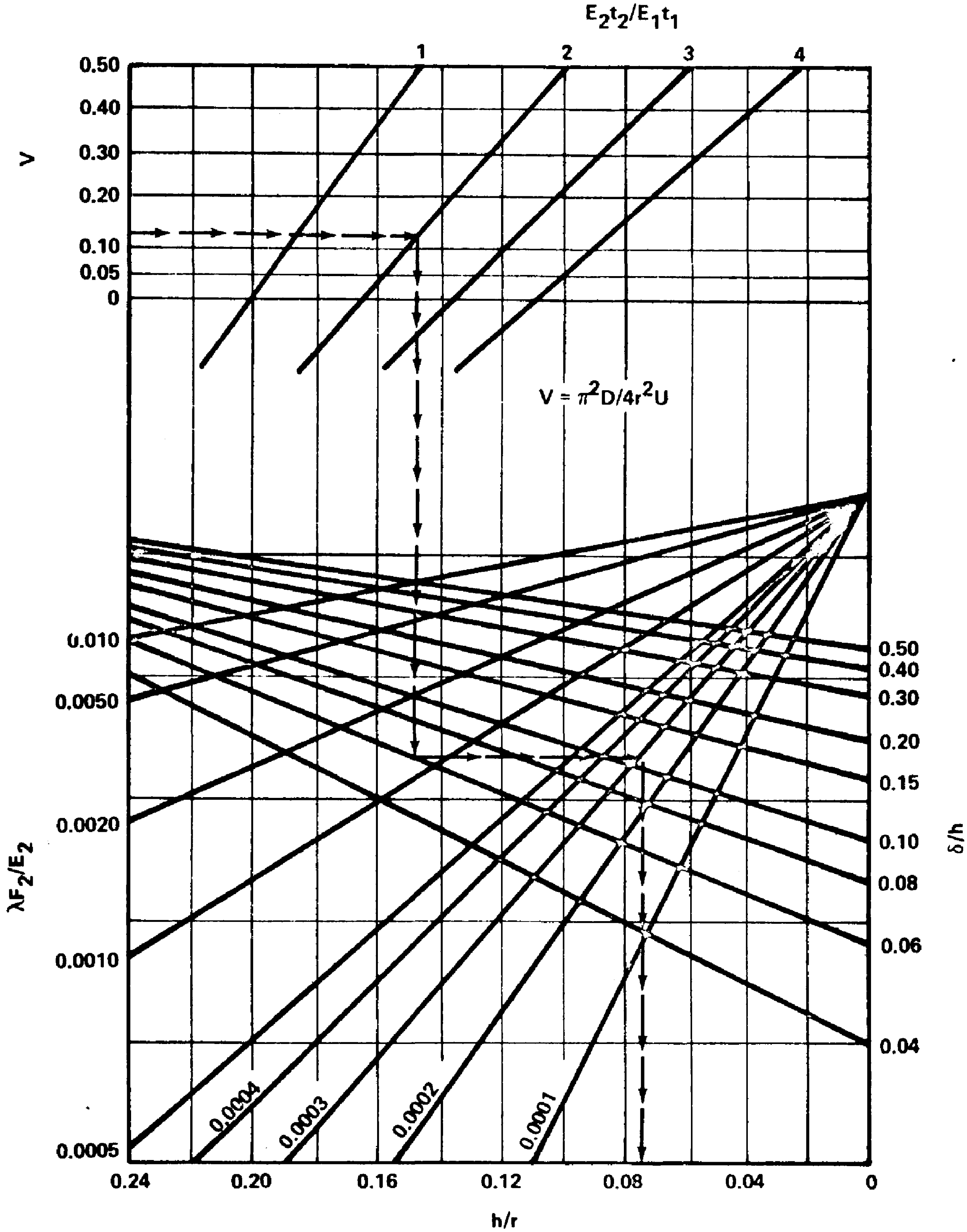


FIGURE B9-33. CHART FOR DETERMINING h/r RATIO FOR FLAT CIRCULAR SANDWICH, WITH ISOTROPIC FACINGS AND CORE, UNDER UNIFORMLY DISTRIBUTED NORMAL LOAD PRODUCING CENTER DEFLECTION RATIO δ/h

B9.6.1.7 Use of Design Charts

The sandwich must be designed by iterative procedures and the charts enable rapid determination of the various quantities sought. The charts were derived for Poisson's ratio of the facings of 0.3 and can be used with small error for facings having other values of Poisson's ratio.

As a first approximation, it will be assumed that $V = 0$. If the design is controlled by facing stress criteria, as may be determined, this assumption will lead to an exact value of h. If the design is controlled by deflection requirements, the assumption that $V = 0$ will produce a minimum value of h. The value of h is minimum because $V = 0$ if the core shear modulus is infinite. For any actual core, the shear modulus is not infinite; hence a thicker core must be used.

The following procedure is suggested:

1. Enter Fig. B9-28 with the desired value for the parameter $\frac{p}{F_{1,2}}$. Assume a value for $\frac{t_{1,2}}{h}$ and determine h/r. Compute h and $t_{1,2}$. Modify ratio $\frac{t_{1,2}}{h}$ if necessary and determine more suitable values for h and $t_{1,2}$.

2. Enter Fig. B9-33 with desired values of the parameters $\frac{E_2 t_2}{E_1 t_1}$ and $\frac{\lambda F_2}{E_2}$ and assume $V = 0$. Assume a value for δ/h and determine h/r. Compute h and δ. Modify ratio δ/h if necessary and determine more suitable values for h and δ.

3. Repeat steps 1 and 2, using lower chosen design facing stresses, until h determined by step 2 is equal to, or a bit less than, h determined by step 1.

4. Compute the core thickness t_c, using the following formulas:

$$t_c = h - \frac{t_1 + t_2}{2}$$

$$t_c = h - t \qquad \text{(for equal facings)} \quad .$$

This first approximation was based on a core with an infinite shear modulus. Since actual core shear modulus values are not very large, a value of t_c somewhat larger must be used. Successive approximations can be made by entering Fig. B9-33 with values of V as computed by equations (73) and (74).

In using Fig. B9-33 for $V \neq 0$ it is necessary to iterate because V is directly proportional to the core thickness t_c. As an aid to finally determine t_c and G_c, Fig. B9-27 can again be used. The constant relating V to G_c may be computed from the formula

$$VG_c = \left[\frac{\pi^2 t_c E_1 t_1 E_2 t_2}{4\lambda r^2 (E_1 t_1 + E_2 t_2)}\right] \text{ or } \left(\frac{\pi^2 t_c E t}{8\lambda r^2}\right) \quad \text{(for equal facings)} \quad .$$

With this constant, Fig. B9-27 may be entered. Use of the figure is as described in Section B9.6.1.3.

B9.6.1.8 Determining Core Shear Stress

This section gives the procedure for determining the maximum core shear stress of a flat circular sandwich panel under uniformly distributed normal load. The core shear stress is maximum at the panel edge. The maximum shear stress, F_{cs}, is given by the formula

$$F_{cs} = \frac{pr}{2h} \quad . \tag{77}$$

B9.6.1.9 Checking Procedure

The design shall be checked by computing the facing stresses using equation (67) and the deflection using equation (71). The value of K_4 to be used in equation (71) is given by

$$K_4 = \frac{16}{\pi^2(3 + \mu)} \left[\frac{(5 + \mu)\pi^2}{64(1 + \mu)} + V\right] \quad , \tag{78}$$

which reduces $K_4 = 0.309 + 0.491V$ when $\mu = 0.3$. Values of V may be computed using equation (73).

An alternate method for computing the deflection at the panel center is given by the formula

$$\delta = K_5\left(1 + \frac{E_{1,2}t_{1,2}}{E_{2,1}t_{2,1}}\right)\frac{\lambda pr^4}{\pi^2 E_{1,2}t_{1,2}h^2} \tag{79}$$

$$\delta = 2K_5\frac{\lambda pr^4}{\pi^2 Eth^2} \tag{80}$$

where

$$K_5 = \frac{(5 + \mu)\pi^2}{64(1 + \mu)} + V$$

which reduces to $K_5 = 0.629 + V$ when $\mu = 0.3$.

The core selected for the panel should be checked to be sure that it has a core shear modulus value, G_c, at least as high as that assumed in computing the deflection in equation (71) and that the core shear strength is sufficient to withstand the maximum core shear stress calculated from equation (77).

B9.6.2 Large Deflection Theory

Most of the literature classifies large deflection theory as having a deflection-to-plate thickness ratio greater than 0.50. In Figs. B9-34, B9-35, B9-36, and B9-37, a small difference is noted between the linear and nonlinear theory for deflection-to-plate ratios less than 0.50.

B9.6.2.1 Rectangular Sandwich Plate with Fixed Edge Conditions

The curves of Fig. B9-34 were obtained from Reference 8 with the following corresponding nomenclature for a rectangular sandwich plate with fixed edge conditions (shear deformations are not included):

w_o	Center deflection of plate
h	Thickness of the core layer
a, b	Half length of panel in x and y directions
λ	a/b
E	Elastic constant of the face layers
ν	Poisson's ratio of the core layer
p	External load per unit area
t	Thickness of the face layer
Q	$12a^3(1 - \nu^2)\ p/th^2E$

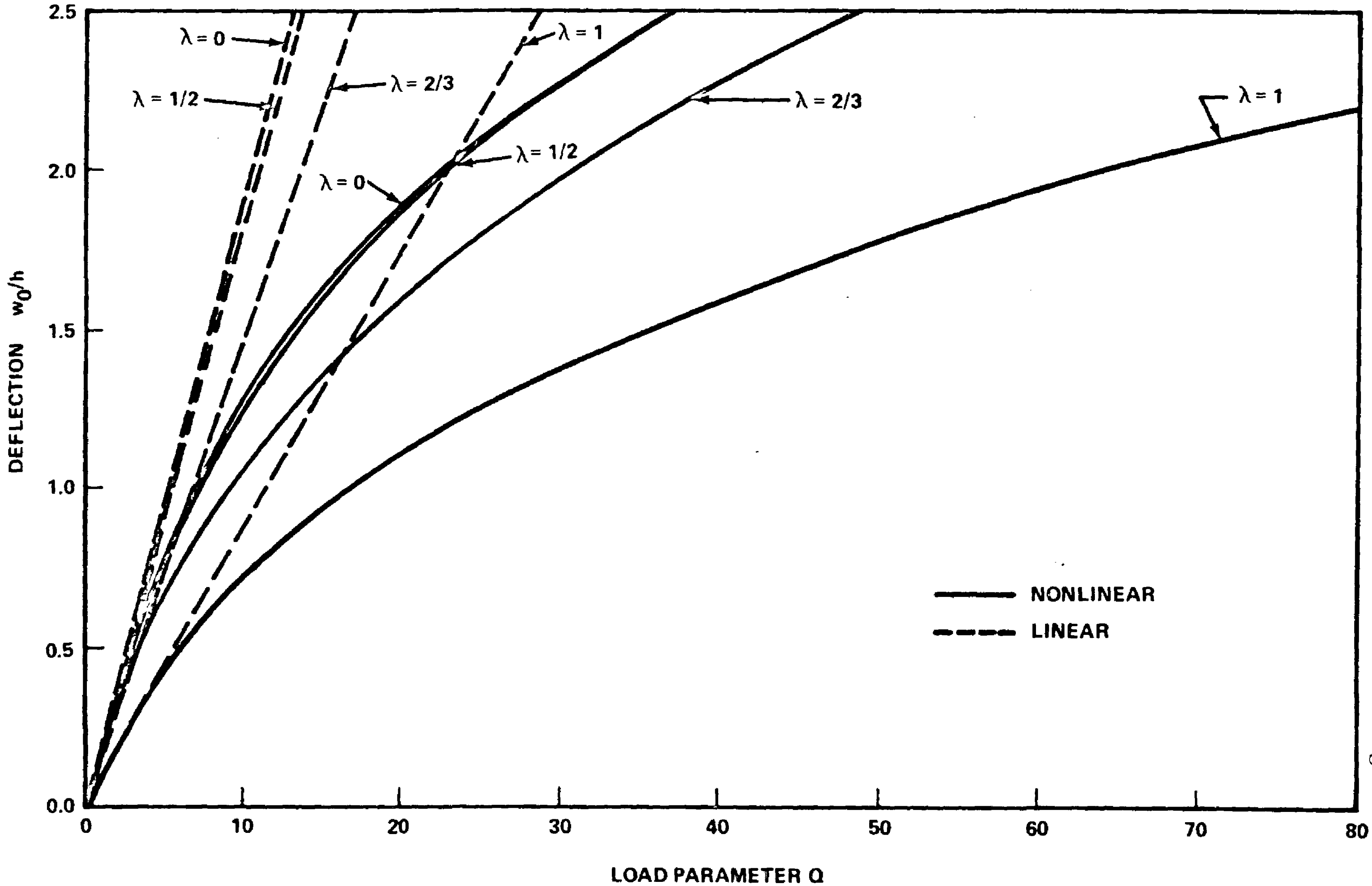


FIGURE B9-34. DEFLECTION OF RECTANGULAR SANDWICH PLATES WITH THE EDGES CLAMPED

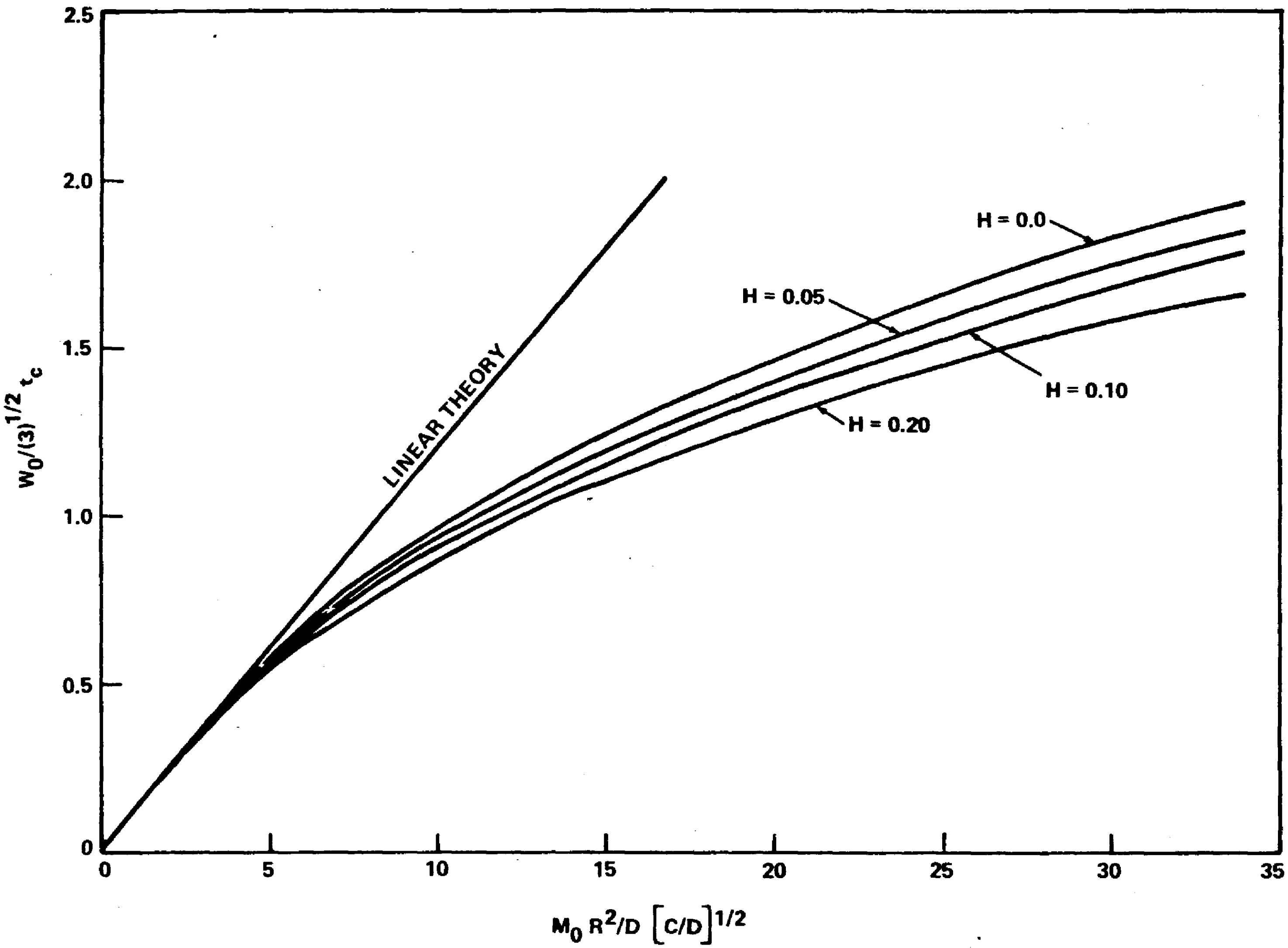


FIGURE B9-35. CENTER DEFLECTION w_o VERSUS APPLIED EDGE MOMENT M_0 FOR ν_f = 0.25

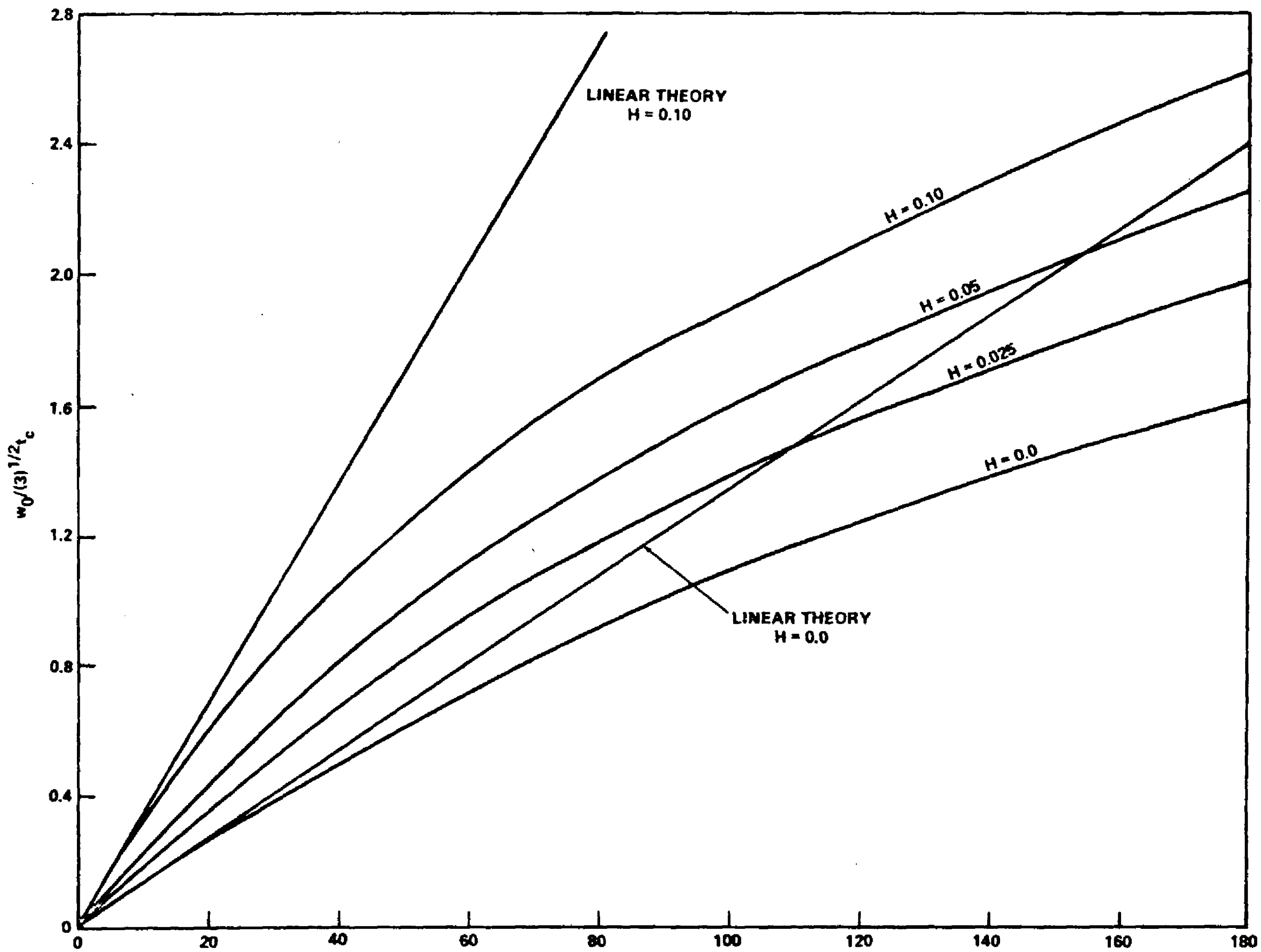


FIGURE B9-36. CENTER DEFLECTION w_o VERSUS APPLIED LOAD q FOR A CIRCULAR PLATE WITH CLAMPED, MOVABLE EDGE

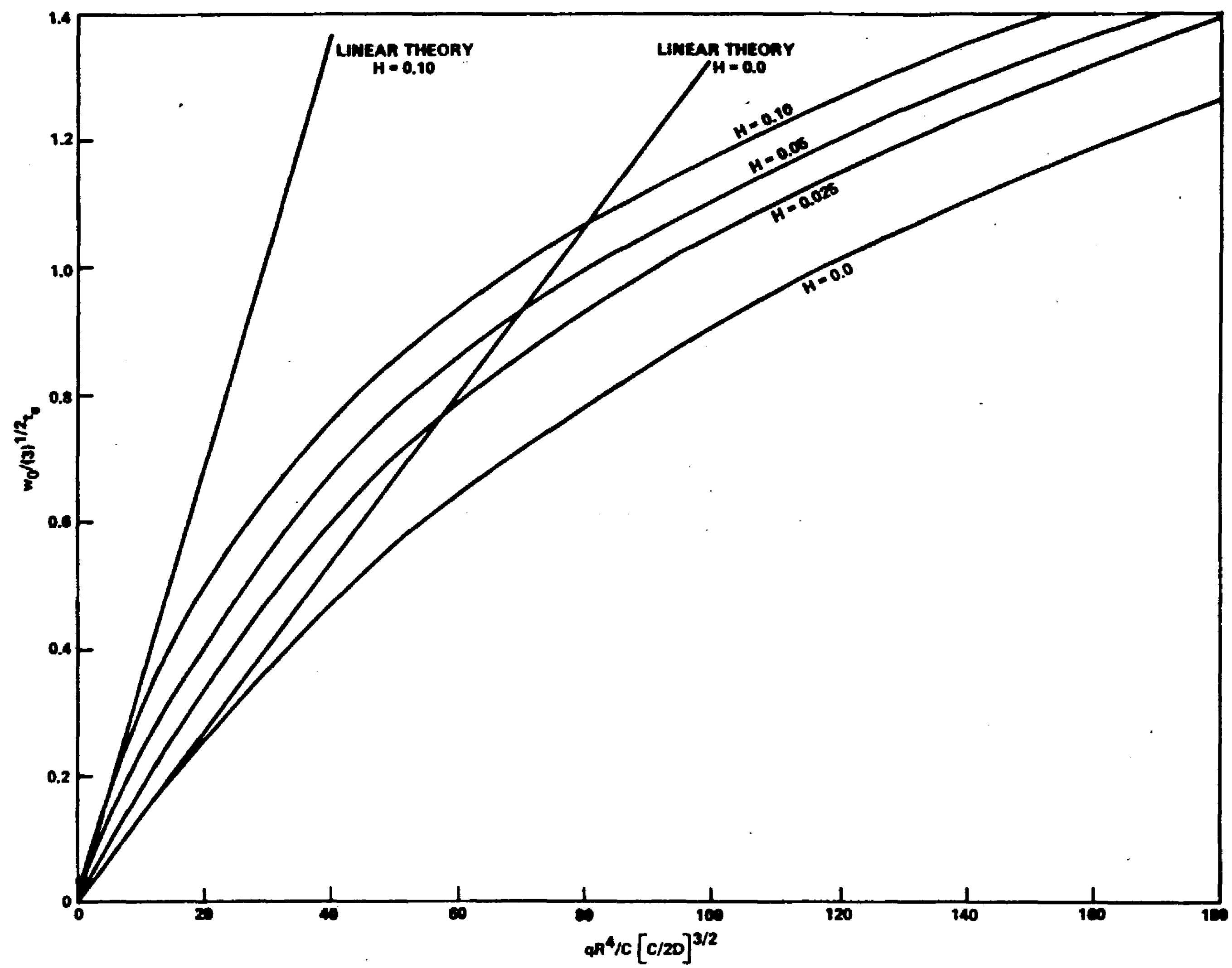
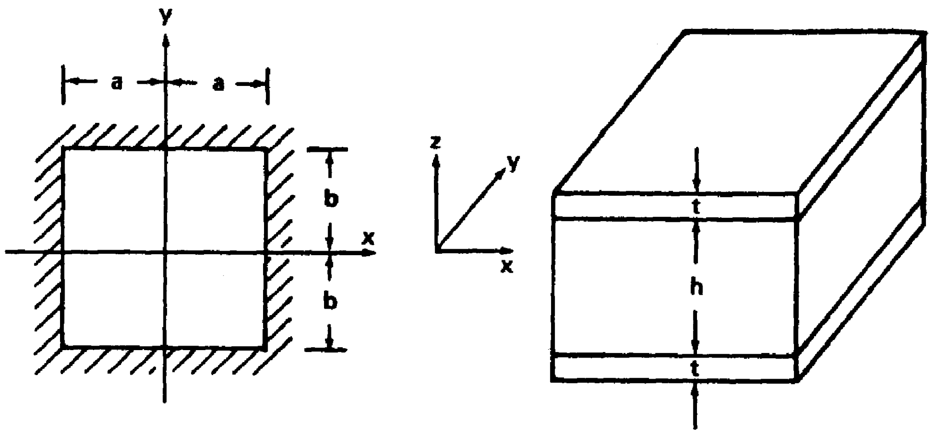


FIGURE B9-37. CENTER DEFLECTION w_o VERSUS APPLIED LOAD q FOR A CIRCULAR PLATE WITH CLAMPED IMMOVABLE EDGE (ν_f = 0.25)



B9.6.2.2 Circular Sandwich Plate with Simply Supported Movable, Clamped Movable, and Clamped Immovable Boundary Conditions

The curves of Figs. B9-35, B9-36, and B9-37 were obtained from Reference 9 for a circular sandwich plate for the following states of loading and boundary conditions:

1. Moments uniformly distributed around a simply supported, radially movable boundary,

2. Uniformly loaded plate with a clamped, radially movable boundary, and

3. Uniformly loaded plate with a clamped, radially immovable boundary.

The equations are nondimensionalized for each state of loading. The effect of shear deformation is characterized by the nondimensional parameter H. If H = 0, then shear deformation is neglected; a nonzero value of H signifies shear distortion in the core. Nomenclature of the symbols is as follows:

w_o	Normal deflection at the plate center
t_c	Thickness of the core
H	Measure of effect of core shear deformation $= \frac{D}{BR^2}$
M_o	Applied edge moment
R	Radius of a circular plate
C	In-plane rigidity $= 2E_f t_{f_2}$
D	Bending rigidity $= E_f t_f t_c / 2(1 - \nu_f^2)$
q	Applied, transverse load
ν_f	Poisson's ratio of face sheet
B	Transverse shear rigidity $= G_c t_c$
G_c	Shear modulus of the core
E_f	Modulus of elasticity of the face sheet
t_f	Thickness of the face sheet

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