SECTION B7.2

LOCAL LOADS ON

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B7.2.0.0 LOCAL LOADS ON THIN SHELLS

The method contained in this section for determining stresses and displacements in thin shells is based on analyses performed by P. P. Bijlaard. [1] These analyses represent the local loads and radial displacement in the form of a double Fourier series. The equations developed using these series and the necessary equilibrium considerations are readily solved by numerical techniques for stresses and displacements.

The stresses and displacements calculated by the methods of this section can be superimposed upon the stresses caused by other loadings if the specified limitations are observed.

The equations for determining stresses in spherical shells caused by local loads have been evaluated within the parametric ranges of space vehicle interest for radial load and overturning moment. The results of this evaluation have been plotted and are contained in this section for use in determining the stresses. A direct method is presented for determining the stresses in spherical shells caused by locally applied shear load or twisting moment. No method is provided to calculate displacements of spherical shells caused by local loads.

The equations for determining stresses and displacements in cylindrical shells caused by local loads have been programmed in Fortran IV for radial load and overturning moment. A direct method is presented for determining the stresses in cylindrical shells caused by a locally applied shear load or twisting moment. No method is provided to calculate displacements of cylindrical shells caused by locally applied shear loads or twisting moments.

B7.2.1.0 LOCAL LOADS ON SPHERICAL SHELLS*

This section presents a method of obtaining spherical shell membrane and bending stresses resulting from loads induced through rigid attachments at the attachment-to-shell juncture. Shell and attachment parameters are used to obtain nondimensional stress resultants from curves for the radial load and overturning moment load condition. The values of the stress resultants are then used to calculate stress components. Shear stresses caused by shearing loads and twisting moment can be calculated directly.

Local load stresses reduce rapidly at points removed from the attachment-to-shell juncture. The shaded areas in Figure B7.2.1.0-1 locate the region where stresses caused by local loads are considered.

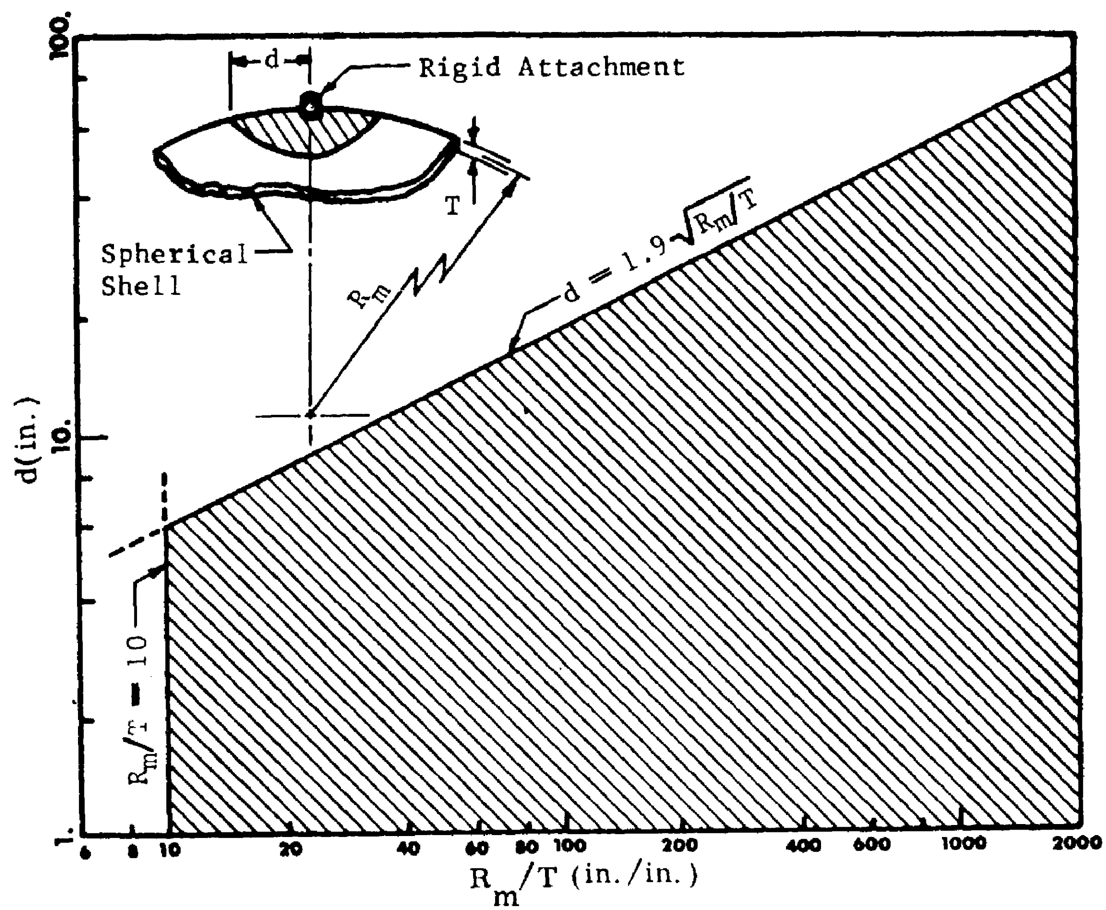

Fig. B7.2.1.0-1 Local Loads Area of Influence

* This section is adapted from the Welding Research Council Bulletin. No. 107, "Local Stresses in Spherical and Cylindrical Shells Due to External Loadings" [5].

B7.2.1.1 GENERAL

I NOTATION

- a - fillet radius at attachment-to-shell juncture, in.
- c - half width of square attachment, in.
- d - distance defined by Figure B7.2.1.0-1
- E - modulus of elasticity, psi
- f_x - normal meridional stress, psi
- f_y - normal circumferential stress, psi
- f_{xy} - shear stress, psi
- K_n, K_b - stress concentration parameters for normal stresses and bending stresses, respectively
- M_a - Applied overturning moment, in.-lb.
- M_T - applied twisting moment, in.-lb.
- M_j - internal bending moment stress resultant per unit length of shell, in.-lb/in.
- N_j - internal normal force stress resultant per unit length of shell, lb/in.
- P - applied concentrated radial load, lb.
- R - radius of the shell, in.
- r - radius of the attachment-to-shell, in.
- T - thickness of the shell, in.
- t - thickness of hollow attachment-to-shell, in.
- U - shell parameter, in./in.
- V_a - applied concentrated shear load, lb.
- ρ - hollow attachment-to-shell parameter, in./in.
- θ - circumferential angular coordinate, rad.
- Υ - hollow attachment-to-shell parameter, in./in.
- ϕ - meridional angular coordinate, rad.

B7.2.1.1 GENERAL (Cont'd)

I NOTATION (Cont'd)

Subscripts

a - applied (a = 1 or a = 2)
b - bending
i - inside
j - internal (j = x or j = y)
m - mean (average of outside and inside)
n - normal
o - outside
x - meridional coordinate
y - circumferential coordinate
z - radial coordinate

1 - applied load coordinate
2 - applied load coordinate

B7.2.1.1 GENERAL

II SIGN CONVENTION

Local loads applied at an attachment-to-shell induce a biaxial state of stress on the inside and outside surfaces of the shell. The meridional stress (f_x), circumferential stress (f_y), shear stress (f_{xy}), the positive directions of the applied loads (M_a, M_T, V_a, and P), and the stress resultants (M_j and N_j) are indicated in Figure B7.2.1.1-1.

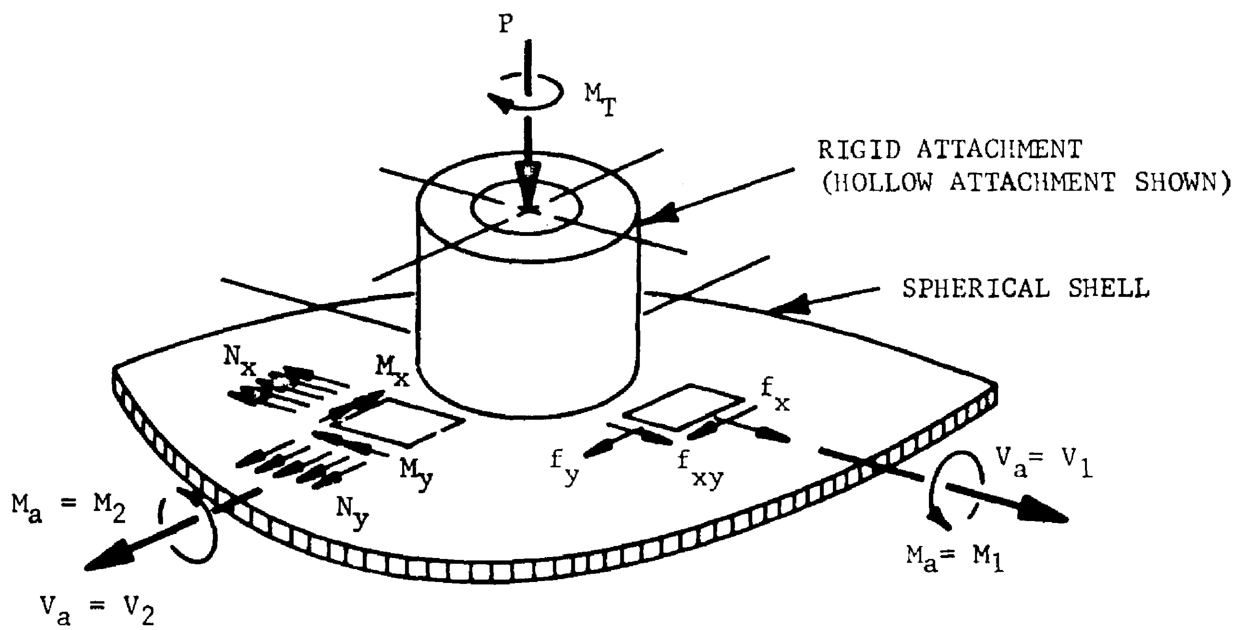

Fig. B7.2.1.1-1 Stresses, Stress Resultants, and Loads

The geometry of the shell and attachment, and the local coordinate system (1-2-3) are indicated in Figure B7.2.1.1-2. It is possible to predict the sign of the induced stresses, tensile (+) or compressive (-), by considering the deflection of the shell resulting from various modes of loading.

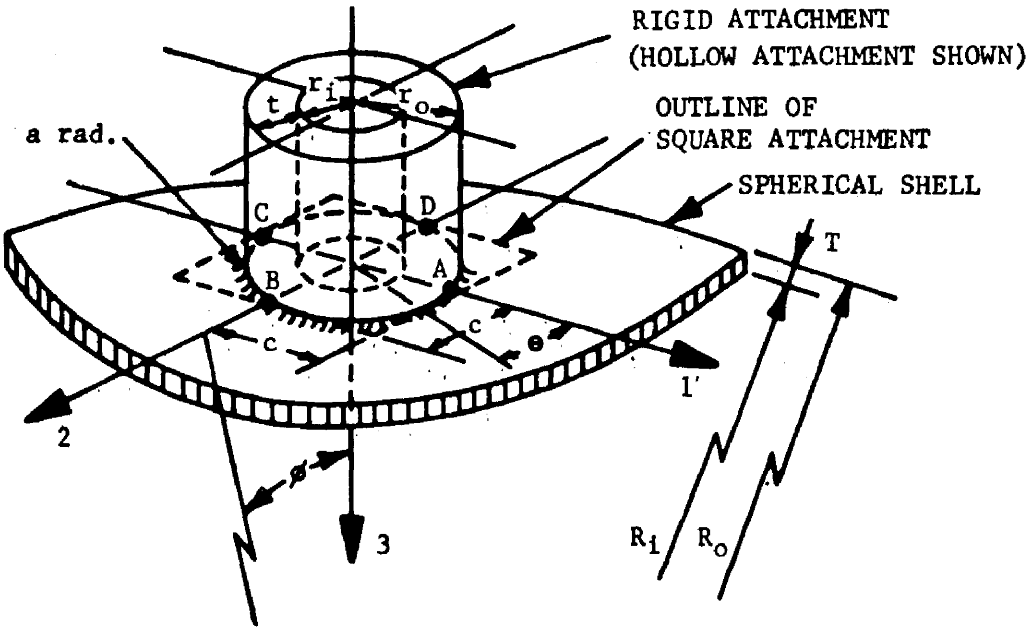

Fig. B7.2.1.1-2 Shell and Attachment Geometry

Mode I, Figure B7.2.1.1-3 shows a positive radial load (P) transmitted to the shell by a rigid attachment. The load (P) causes compressive membrane stresses and local bending stresses adjacent to the attachment. The compressive membrane stresses are similar to the stresses induced by an external pressure. The local bending stresses result in tensile bending stresses on the inside of the shell and compressive bending stresses on the outside of the shell at points C and A.

Mode II, Figure B7.2.1.1-3 shows a negative overturning moment (M_a) transmitted to the shell by a rigid attachment. The overturning moment (M_a) causes compressive and tensile membrane stresses and local bending stresses adjacent to the attachment. Tensile membrane stresses induced in the shell at C are similar to the stresses caused by an internal pressure. Compressive membrane stresses induced in the shell at A are similar to the stresses caused by an external pressure. The local bending stresses cause tensile bending stresses in the shell at C on the outside and A on the inside, and cause compressive bending stresses in the shell at A on the outside and at C on the inside.

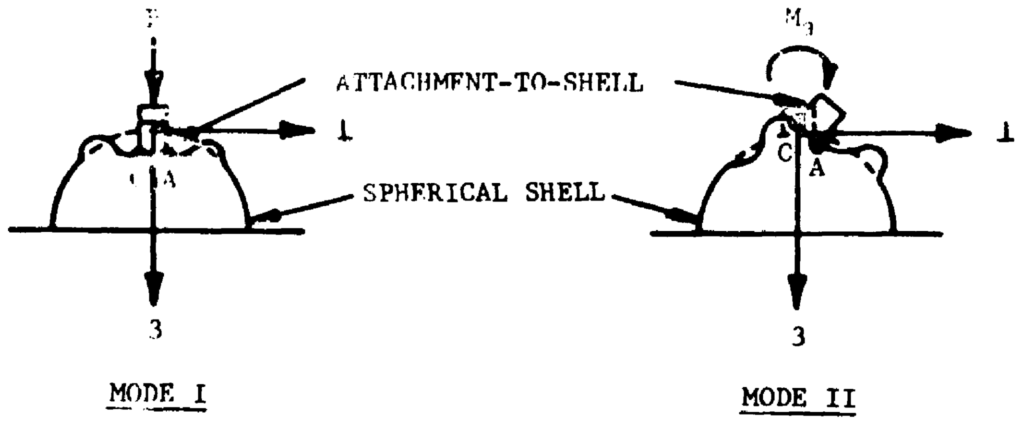

Fig. B7.2.1.1-3 Loading Modes

B7.2.1.1 GENERAL

III LIMITATIONS OF ANALYSIS

Four general areas must be considered for limitations: attachment size and shell thickness, attachment location, shift in maximum stress location, and stresses caused by shear loads.

A Size of Attachment with Respect to Shell Size

The analysis is applicable to small attachments relative to the shell size and to thin shells. The limitations on these conditions are shown by the shaded area of Figure B7.2.1.1-4.

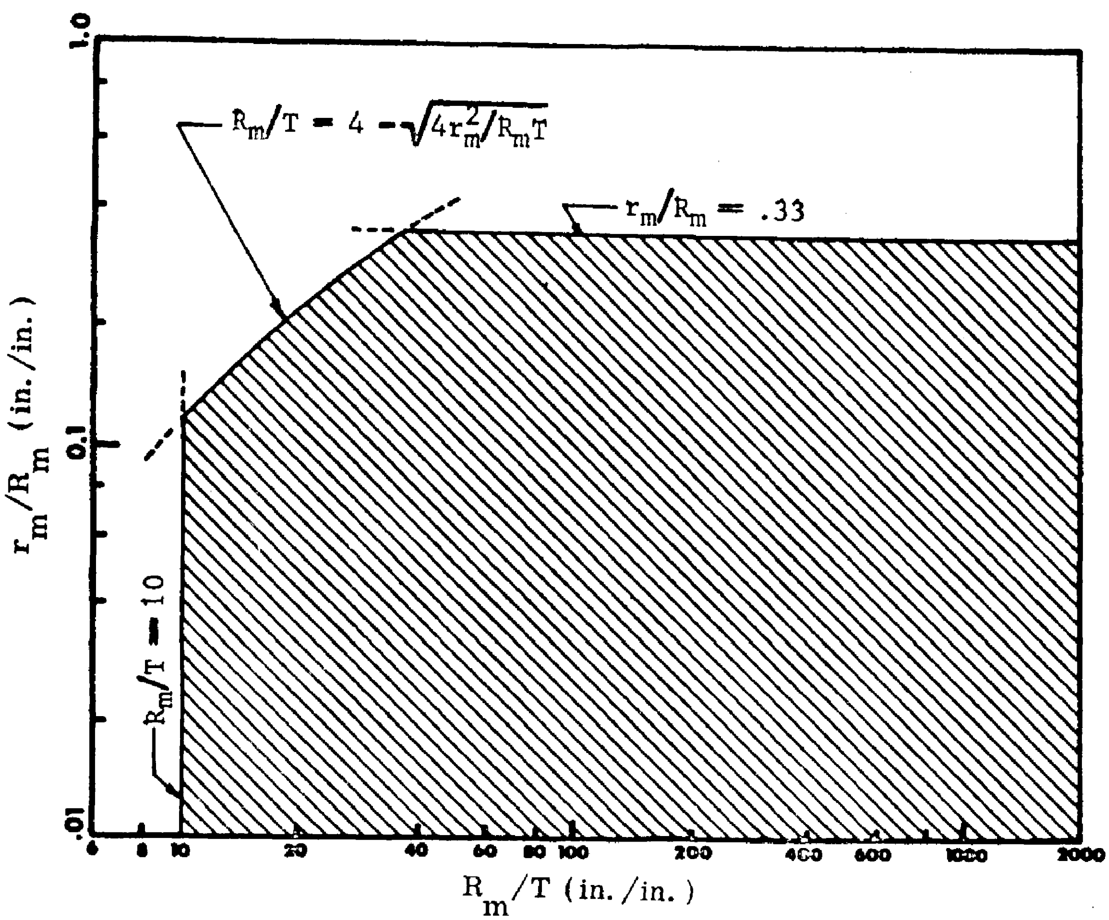

Fig. B7.2.1.1-4

B <u>Location of Attachment with Respect to Boundary Conditions of Shell</u>

The analysis is applicable when any part of the area of influence shown in Figure B7.2.1.0-1 does not contain any stress perturbations. These perturbations may be caused by discontinuity, thermal loading, liquid-level loading, change in section and material change.

C <u>Shift in Maximum Stress Location</u>

Under certain conditions the stresses in the shell may be higher at points removed from the attachment-to-shell juncture than at the juncture. The following conditions should be carefully considered:

1. In some instances, stresses will be higher in the hollow attachment wall than they are in the shell. This is most likely when the attachment opening is not reinforced, when reinforcement is placed on the shell and not on the attachment, and when very thin attachments are used.

2. For some load conditions certain stress resultants peak at points slightly removed from the attachment-to-shell juncture. The maximum value of these stress resultants is determined from the curves in Section B7.2.1.5 and is indicated by dashed lines.

When conditions are encountered that deviate from the limitations of the analysis, Appendix A of Reference 2 should be consulted.

D <u>Stresses Caused by Shear Loads</u>

An accurate stress distribution caused by a shear load (V_a) applied to a spherical shell is not available. The actual stress distribution consists of varying shear and membrane stresses around the rigid attachment. The method [2] presented here assumes that the shell resists the shear load by shear only. If this assumption appears unreasonable, it can be assumed that the shear load is resisted totally by membrane stresses or by some combination of membrane and shear stresses.

B7.2.1.2 STRESSES +

I GENERAL

Stress resultants at attachment-to-shell junctures are obtained from the nondimensional stress-resultant curves in section B7.2.1.5. These curves are plots of the shell parameter (U) versus a nondimensional form of the stress resultants (M_j and N_j). Figures B7.2.1.5-1 and B7.2.1.5-2 are used for solid attachments and Figures B7.2.1.5-3 through B7.2.1.5-22 are used for hollow attachments. Additional attachment parameters (Υ and ρ) are required to use Figures B7.2.1.5-3 through B7.2.1.5-22.

The general equation for stresses in a shell at a rigid attachment juncture in terms of the stress resultants is:

$$f_j = K_n(N_j/T) \pm K_b(6M_j/T^2) .$$

The stress concentration parameters (K_n and K_b) are functions of the ratio of fillet radius to shell thickness (a/T). The value of the stress concentration parameters for $R \gg r$ is equal to unity except in the following cases:

(a) Attachment-to-shell juncture is brittle material;

(b) Fatigue analysis is necessary at attachment-to-shell juncture.

When stress concentration parameters are used they can be determined from Figure B7.2.1.2-1.

The value of the stress resultant at the juncture is indicated by a solid line on the nondimensional stress resultant curves. When the maximum value for a stress-resultant does not occur at the attachment-to-shell juncture, it is indicated on the nondimensional stress-resultant curves by dashed lines. An incorrect but conservative analysis would assume this maximum stress to be at the juncture.

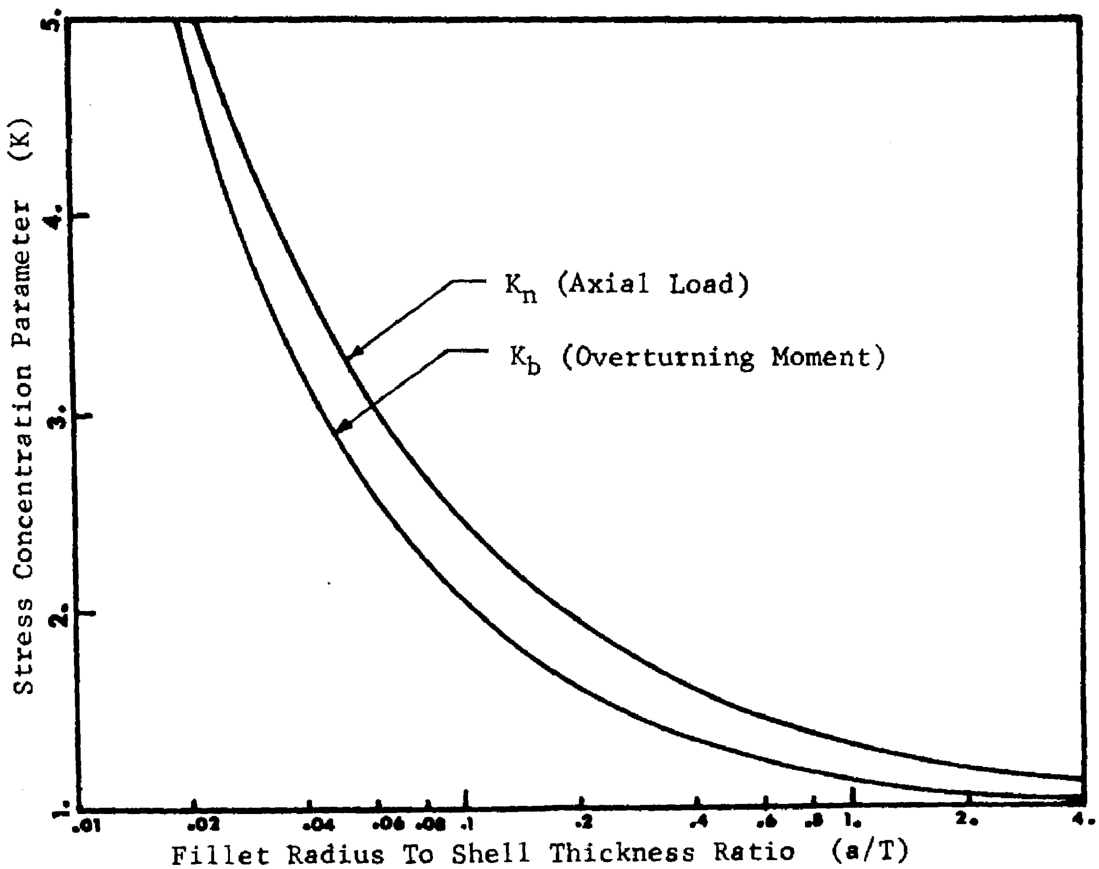

Fig. B7.2.1.2-1 Stress Concentration Parameters for R≫r

The stress calculation sheets (Figs. B7.2.1.2-2 and B7.2.1.2-3) can be used to calculate inside and outside stresses at four points (A, B, C, D on Figure B7.2.1.1-2) around the attachment. The stress calculation sheets also determine the proper sign of the stresses when the applied loads follow the sign convention used in Figure B7.2.1.1-1. The stress calculation sheets provide a place to record applied loads, geometry, parameters and all values calculated or obtained from the step-by-step procedures in paragraphs III-VI below.

STRESS CALCULATION SHEET FOR STRESSES IN SPHERICAL SHELLS CAUSED BY LOCAL LOADS (HOLLOW ATTACHMENT)

APPLIED LOADS	SHELL GEOMETRY	PARAMETERS
P = ______	T = ______	U = ______
V_1 = ______	t = ______	T = ______
V_2 = ______	R_m = ______	ρ = ______
M_T = ______	r_m = ______	K_n = ______
M_1 = ______	r_o = ______	K_b = ______
M_2 = ______	a = ______	

STRESS	LOAD	NON-DIMENSIONAL STRESS RESULTANT	ADJUSTING FACTOR	STRESS COMPONENT	STRESSES* A_i	A_o	B_i	B_o	C_i	C_o	D_i	D_o
MERIDIONAL STRESS (f_x)	P	$\frac{N_x T}{P}$ =	$\frac{K_n P}{T^2}$ =	$\frac{K_n N_x}{T}$ =	–	–	–	–	–	–	–	–
		$\frac{M_x}{P}$ =	$\frac{6K_b P}{T^2}$ =	$\frac{6K_b M_x}{T^2}$ =	+	–	+	–	+	–	+	–
	M_1	$\frac{N_x T\sqrt{R_m T}}{M_1}$ =	$\frac{K_n M_1}{T^2\sqrt{R_m T}}$ =	$\frac{K_n N_x}{T}$ =			–	–			+	+
		$\frac{M_x \sqrt{R_m T}}{M_1}$ =	$\frac{6K_b M_1}{T^2\sqrt{R_m T}}$ =	$\frac{6K_b M_x}{T^2}$ =			+	–			–	+
	M_2	$\frac{N_x T\sqrt{R_m T}}{M_2}$ =	$\frac{K_n M_2}{T^2\sqrt{R_m T}}$ =	$\frac{K_n N_x}{T}$ =	+	+			–	–		
		$\frac{M_x\sqrt{R_m T}}{M_2}$ =	$\frac{6K_b M_2}{T^2\sqrt{R_m T}}$ =	$\frac{6K_b M_x}{T^2}$ =	–	+			+	–		
	TOTAL MERIDIONAL STRESSES (f_x)											
CIRCUMFERENTIAL STRESS (f_y)	P	$\frac{N_y T}{P}$ =	$\frac{K_n P}{T^2}$ =	$\frac{K_n N_y}{T}$ =	–	–	–	–	–	–	–	–
		$\frac{M_y}{P}$ =	$\frac{6K_b P}{T^2}$ =	$\frac{6K_b M_y}{T^2}$ =	+	–	+	–	+	–	+	–
	M_1	$\frac{N_y T\sqrt{R_m T}}{M_1}$ =	$\frac{K_n M_1}{T^2\sqrt{R_m T}}$ =	$\frac{K_n N_y}{T}$ =			–	–			+	+
		$\frac{M_y \sqrt{R_m T}}{M_1}$ =	$\frac{6K_b M_1}{T^2\sqrt{R_m T}}$ =	$\frac{6K_b M_y}{T^2}$ =			+	–			–	+
	M_2	$\frac{N_y T\sqrt{R_m T}}{M_2}$ =	$\frac{K_n M_2}{T^2\sqrt{R_m T}}$ =	$\frac{K_n N_y}{T}$ =	+	+			–	–		
		$\frac{M_y \sqrt{R_m T}}{M_2}$ =	$\frac{6K_b M_2}{T^2\sqrt{R_m T}}$ =	$\frac{6K_b M_y}{T^2}$ =	–	+			+	–		
	TOTAL CIRCUMFERENTIAL STRESS (f_y)											
SHEAR STRESS (f_{xy})	V_1		$\pi r_o T$ =	$\frac{V_1}{\pi r_o T}$ =			–	–			+	+
	V_2		$\pi r_o T$ =	$\frac{V_2}{\pi r_o T}$ =	–	–			+	+		
	M_T		$2\pi r_o^2 T$ =	$\frac{M_T}{2\pi r_o^2 T}$ =	–	–	+	+	–	–	+	+
	TOTAL SHEAR STRESS (f_{xy})											
PRINCIPAL STRESSES**	f_{max}	$\frac{f_x + f_y}{2} + \sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2}$ ***										
	f_{min}	$\frac{f_x + f_y}{2} - \sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2}$ ***										
	$f_{xy}max$	$\pm\sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2}$										

* IF LOAD IS OPPOSITE TO THAT SHOWN IN FIGURE B7.2.1.1-1 THEN REVERSE THE SIGN SHOWN.
** SEE SECTION A3.1.0.
*** CHANGE SIGN OF THE RADICAL IF $(f_x + f_y)$ IS NEGATIVE.

Fig. B7.2.1.2-2 Stress Calculation Sheet (Hollow Attachment)

STRESS CALCULATION SHEET FOR STRESSES IN SPHERICAL SHELLS CAUSED BY LOCAL LOADS (SOLID ATTACHMENT)

APPLIED LOADS	SHELL GEOMETRY	PARAMETERS
P = ______	T = ______	U = ______
V_1 = ______	R_m = ______	K_n = ______
V_2 = ______	r_o = ______	K_b = ______
M_T = ______	a = ______	
M_1 = ______		
M_2 = ______		

STRESS	LOAD	NON-DIMENSIONAL STRESS RESULTANT	ADJUSTING FACTOR	STRESS COMPONENT	STRESSES* A_i	A_o	B_i	B_o	C_i	C_o	D_i	D_o
MERIDIONAL STRESS (f_x)	P	$\frac{N_x T}{P}$ =	$\frac{K_n P}{T^2}$ =	$\frac{K_n N_x}{T}$ =	−	−	−	−	−	−	−	−
		$\frac{M_x}{P}$ =	$\frac{6K_b P}{T^2}$ =	$\frac{6K_b M_x}{T^2}$ =	+	−	+	−	+	−	+	−
	M_1	$\frac{N_x T\sqrt{R_m T}}{M_1}$ =	$\frac{K_n M_1}{T^2\sqrt{R_m T}}$ =	$\frac{K_n N_x}{T}$ =	▨	▨	−	−	▨	▨	+	+
		$\frac{M_x\sqrt{R_m T}}{M_1}$ =	$\frac{6K_b M_1}{T^2\sqrt{R_m T}}$ =	$\frac{6K_b M_x}{T^2}$ =	▨	▨	+	−	▨	▨	−	+
	M_2	$\frac{N_x T\sqrt{R_m T}}{M_2}$ =	$\frac{K_n M_2}{T^2\sqrt{R_m T}}$ =	$\frac{K_n N_x}{T}$ =	+	+	▨	▨	−	−	▨	▨
		$\frac{M_x\sqrt{R_m T}}{M_2}$ =	$\frac{6K_b M_2}{T^2\sqrt{R_m T}}$ =	$\frac{6K_b M_x}{T^2}$ =	−	+	▨	▨	+	−	▨	▨
	TOTAL MERIDIONAL STRESSES (f_x)											
CIRCUMFERENTIAL STRESS (f_y)	P	$\frac{N_y T}{P}$ =	$\frac{K_n P}{T^2}$ =	$\frac{K_n N_y}{T}$ =	−	−	−	−	−	−	−	−
		$\frac{M_y}{P}$ =	$\frac{6K_b P}{T^2}$ =	$\frac{6K_b M_y}{T^2}$ =	+	−	+	−	+	−	+	−
	M_1	$\frac{N_y T\sqrt{R_m T}}{M_1}$ =	$\frac{K_n M_1}{T^2\sqrt{R_m T}}$ =	$\frac{K_n N_y}{T}$ =	▨	▨	−	−	▨	▨	+	+
		$\frac{M_y\sqrt{R_m T}}{M_1}$ =	$\frac{6K_b M_1}{T^2\sqrt{R_m T}}$ =	$\frac{6K_b M_y}{T^2}$ =	▨	▨	+	−	▨	▨	−	+
	M_2	$\frac{N_y T\sqrt{R_m T}}{M_2}$ =	$\frac{K_n M_2}{T^2\sqrt{R_m T}}$ =	$\frac{K_n N_y}{T}$ =	+	+	▨	▨	−	−	▨	▨
		$\frac{M_y\sqrt{R_m T}}{M_2}$ =	$\frac{6K_b M_2}{T^2\sqrt{R_m T}}$ =	$\frac{6K_b M_y}{T^2}$ =	−	+	▨	▨	+	−	▨	▨
	TOTAL CIRCUMFERENTIAL STRESS (f_y)											
SHEAR STRESS (f_{xy})	V_1		$\pi r_o T$ =	$\frac{V_1}{\pi r_o T}$ =	▨	▨	−	−	▨	▨	+	+
	V_2		$\pi r_o T$ =	$\frac{V_2}{\pi r_o T}$ =	−		▨	▨	+	+	▨	▨
	M_T		$2\pi r_o^2 T$ =	$\frac{M_T}{2\pi r_o^2 T}$ =	−	−	+	+		−	+	+
	TOTAL SHEAR STRESS (f_{xy})											
PRINCIPAL STRESSES**	f_{max}	$\frac{f_x + f_y}{2} + \sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2}$ ***										
	f_{min}	$\frac{f_x + f_y}{2} - \sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2}$ ***										
	$f_{xy\,max}$	$\pm\sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2}$										

* IF LOAD IS OPPOSITE TO THAT SHOWN IN FIGURE B7.2.1.1-1 THEN REVERSE THE SIGN SHOWN.
** SEE SECTION A3.1.0.
*** CHANGE SIGN OF THE RADICAL IF ($f_x + f_y$) IS NEGATIVE.

Fig. B7.2.1.2-3 Stress Calculation Sheet (Solid Attachment)

B7.2.1.2 STRESSES

II PARAMETERS

The following applicable parameters must be evaluated:

A Geometric Parameters

1. Shell Parameters (U)

a. round attachment

$$U = r_0/(R_m T)^{\frac{1}{2}}$$

b. square attachment

$$U = 1.413c/(R_m T)^{\frac{1}{2}}$$

2. Attachment Parameters (Υ and ρ)

a. hollow round attachment

$$\Upsilon = r_m/t$$

$$\rho = T/t$$

b. hollow square attachment

$$\Upsilon = 1.143c/t$$

$$\rho = T/t$$

B Stress Concentration Parameters

1. Membrane stress-stress concentration parameter $(K_n)^*$

$$K_n = 1 + (T/5.6a)^{0.65}$$

2. Bending stress-stress concentration parameters $(K_b)^*$

$$K_b = 1 + (T/9.4a)^{0.80}$$

* K_n and K_b values can be determined from Figure B7.2.1.2-1 with a/T values.

B7.2.1.2 STRESSES

III STRESSES RESULTING FROM RADIAL LOAD

A radial load will cause membrane and bending stress components in both the meridional and circumferential directions.

A Meridional Stresses (f_x)

Step 1. Calculate the applicable geometric parameters as defined in paragraph II above.

Step 2. Using the geometric parameters calculated in step 1, obtain the membrane nondimensional stress resultant ($N_x T/P$) for a solid attachment from Figure B7.2.1.5-1 or for a hollow attachment from Figures B7.2.1.1.5-3 through B7.2.1.5-12.

Step 3. Using P and T values and the membrane nondimensional stress resultant ($N_x T/P$), calculate the membrane stress component N_x/T from:

$N_x/T = (N_x T/P) \cdot (P/T^2)$.

Step 4. Using the geometric parameters calculated in step 1 and the same figures as step 2, obtain the bending nondimensional stress resultant (M_x/P).

Step 5. Using P and T values and the bending nondimensional stress resultant (M_x/P), calculate the bending stress component $6M_x/T^2$ from :

$6M_x/T^2 = (M_x/P) \cdot (6P/T^2)$.

Step 6. Using the criteria in paragraph I, obtain values for the stress concentration parameters (K_n and K_b).

Step 7. Using the stress components calculated in steps 3 and 5 and the the stress concentration parameters calculated in step 6, determine the meridional stress (f_x) from :

$f_x = K_n (N_x/T) \pm K_b (6M_x/T^2)$.

Proper consideration of the sign will give values for the meridional stress on the inside and outside surfaces of the shell.

B <u>Circumferential Stresses (f_y)</u>

The circumferential stress can be determined by following the seven steps outlined above in paragraph A and by using the same curves to obtain the nondimensional stress resultants ($N_y T/P$ and M_y/P) and the following equations to calculate the stress components and circumferential stress:

$$N_y/T = (N_y T/P) \cdot (P/T^2)$$

$$6M_y/ T^2 \ (M_y/P) \cdot (6P/T^2)$$

$$f_y = K_n(N_y/T) \pm K_b (6 M_y/T^2).$$

B7.2.1.2 STRESSES

IV STRESSES RESULTING FROM OVERTURNING MOMENT

An overturning moment will cause membrane and bending stress components in both the meridional and circumferential directions.

A Meridional Stresses (f_x)

Step 1. Calculate the applicable geometric parameters as defined in paragraph II above.

Step 2. Using the geometric parameters calculated in step 1, obtain the membrane nondimensional stress resultant $[N_x T(R_m T)^{\frac{1}{2}}/M_a]$ for a solid attachment from Figure B7.2.1.5-2, or for a hollow attachment from Figures B7.2.1.5-13 through B7.2.1.5-22.

Step 3. Using M_a, R_m and T values and the membrane nondimensional stress resultant $[N_x T(R_m T)^{\frac{1}{2}}/M_a]$, calculate the membrane stress component N_x/T from:

$$N_x/T = [N_x T(R_m T)^{\frac{1}{2}}/M_a]\ [M_a/T^2(R_m T)^{\frac{1}{2}}].$$

Step 4. Using the geometric parameters calculated in step 1 and the same figures as step 2, obtain the bending nondimensional stress resultant $[M_x(R_m T)^{\frac{1}{2}}/M_a]$.

Step 5. Using M_a, R_m and T values and the bending nondimensional stress resultant $[M_x (R_m T)^{\frac{1}{2}}/M_a]$, calculate the bending stress component $6M_x/T^2$ from:

$$6M_x/T^2 = [M_x(R_m T)^{\frac{1}{2}}/M_a]\ [6M_a/T^2(R_m T)^{\frac{1}{2}}].$$

Step 6. Using the criteria in paragraph I, obtain values for the stress concentration parameters (K_n and K_b).

Step 7. Using the stress components calculated in steps 3 and 5 and the stress concentration parameters calculated in step 6, determine the meridional stress (f_x) from:

$f_x = K_n(N_x/T) \pm K_b(6M_x/T^2)$

Proper consideration of the sign will give values for the meridional stress on the inside and outside surfaces of the shell.

B <u>Circumferential Stress (f_y)</u>

The circumferential stress can be determined by following the seven steps outlined above in paragraph A and by using the same figures to obtain the nondimensional stress resultants $[N_y T(R_m T)^{\frac{1}{2}}/M_a]$ and the following equations to calculate the stress components and circumferential stress:

$$N_y/T = [N_y T(M_m T)^{\frac{1}{2}}/M_a][M_a/T^2(R_m T)^{\frac{1}{2}}]$$

$$6M_y/T^2 = [M_x(R_m T)^{\frac{1}{2}}/M_a][6M_a/T^2(R_m T)^{\frac{1}{2}}]$$

$$f_y = K_n(N_x/T) \pm K_b(6M_x/T^2).$$

B7.2.1.2 STRESSES

V STRESSES RESULTING FROM SHEAR LOAD

A shear load (V_a) will cause a membrane shear stress (f_{xy}) in the shell at the attachment-to-shell juncture. The shear stress is determined as follows:

A Round Attachment

$$f_{xy} = \frac{V_a}{r_0 T} \sin \Theta \qquad \text{for } V_a = V_1$$

$$\text{or } f_{xy} = \frac{V_a}{r_0 T} \cos \Theta \qquad \text{for } V_a = V_2$$

B Square Attachment

$$\left.\begin{array}{l} f_{xy} = V_a / 4cT \ (\text{at } \Theta = 90° \text{ and } 270°) \\ f_{xy} = 0 \quad (\text{at } \Theta = 0° \text{ and } 180°) \end{array}\right\} \text{for } V_a = V_1$$

or

$$\left.\begin{array}{l} f_{xy} = V_a / 4cT \ (\text{at } \Theta = 0° \text{ and } 180°) \\ f_{xy} = 0 \quad (\text{at } \Theta = 90° \text{ and } 270°) \end{array}\right\} \text{for } V_a = V_2$$

B7.2.1.2 STRESSES

VI STRESSES RESULTING FROM TWISTING MOMENT

A Round Attachment

A twisting moment (M_T) applied to a round attachment will cause a shear stress (f_{xy}) in the shell at the attachment-to-shell juncture. The shear stress is pure shear and is constant around the juncture. The shear stress is determined as follows:

$$f_{xy} = M_T / 2\pi \ r_0^2 \ T.$$

B Square Attachment

A twisting moment applied to a square attachment will cause a complex stress field in the shell. No acceptable methods for analyzing this loading are available.

B7.2.1.3 STRESSES RESULTING FROM ARBITRARY LOADING

I CALCULATION OF STRESSES

Most loadings that induce local loads on spherical shells are of an arbitrary nature. Stresses are determined by the following procedure:

Step 1. Resolve the applied arbitrary load (forces and/or moments into axial forces, shear forces, overturning moments and twisting moment components. (See paragraph B7.2.1.6, Example Problem.) The positive directions of the components and the point of application of the force components (intersection of centerline of attachment with attachment-shell interface) are indicated in Figure B7.2.1.1-1.

Step 2. Evaluate inside and outside stresses at points A, B, C and D for each component of the applied arbitrary load by the methods in paragraph B7.2.1.2.

Step 3. Obtain the stresses for the arbitrary loading by combining the meridional, circumferential and shear stresses evaluated by step 2 for each of the points A, B, C and D on the inside and outside of the shell. Proper consideration of signs is necessary.

B7.2.1.3 STRESSES RESULTING FROM ARBITRARY LOADING

II LOCATION AND MAGNITUDE OF MAXIMUM STRESSES

The location and magnitude of the maximum stresses caused by an arbitrary load require a consideration of the following:

A The determination of principal stresses (f_{max}, f_{min} and $f_{xy} = 0$ or $f_{xy} = max$) for the calculated stresses (f_x, f_y and f_{xy}) at a specific point.

B The orientation of the coordinate system (1, 2, 3) in Figures B7.2.1.1-1 and B7.2.1.1-2 with respect to an applied arbitrary load may give different values for principal stresses. These different values are caused by a different set of components.

C Whether or not the value for a stress resultant is obtained from the dashed lines or solid lines in Figures B7.2.1.5-3 through B7.2.1.5-22.

B7.2.1.4 ELLIPSOIDAL SHELLS

The analysis presented in this section (B7.2.1.0) can be applied to ellipsoidal shells with attachment at the apex because the radii of curvature are equal. For attachments not located at the apex (points of unequal radii), the analysis is incorrect, and the error increases for attachments at greater distances from the apex.

B7.2.1.5 NONDIMENSIONAL STRESS RESULTANT CURVES

I LIST OF CURVES

A Solid Attachments

1. Nondimensional Stress Resultants for Radial Load (P) B7.2.1.5-1
2. Nondimensional Stress Resultants for Overturning Moment (M_a) B7.2.1.5-2

B Hollow Attachments

1. Nondimensional Stress Resultants for Radial Load (P)

Υ = 5	ρ = 0.25	B7.2.1.5-3
Υ = 5	ρ = 1.0	B7.2.1.5-4
Υ = 5	ρ = 2.0	B7.2.1.5-5
Υ = 5	ρ = 4.0	B7.2.1.5-6
Υ = 15	ρ = 1.0	B7.2.1.5-7
Υ = 15	ρ = 2.0	B7.2.1.5-8
Υ = 15	ρ = 4.0	B7.2.1.5-9
Υ = 15	ρ = 10.0	B7.2.1.5-10
Υ = 50	ρ = 4.0	B7.2.1.5-11
Υ = 50	ρ = 10.0	B7.2.1.5-12

2. Nondimensional Stress Resultants for Overturning Moment (M_a)

Υ = 5	ρ = 0.25	B7.2.1.5-13
Υ = 5	ρ = 1.0	B7.2.1.5-14
Υ = 5	ρ = 2.0	B7.2.1.5-15
Υ = 5	ρ = 4.0	B7.2.1.5-16
Υ = 15	ρ = 1.0	B7.2.1.5-17
Υ = 15	ρ = 2.0	B7.2.1.5-18
Υ = 15	ρ = 4.0	B7.2.1.5-19
Υ = 15	ρ = 10.0	B7.2.1.5-20
Υ = 50	ρ = 4.0	B7.2.1.5-21
Υ = 50	ρ = 10.0	B7.2.1.5-22

B7.2.1.5 NONDIMENSIONAL STRESS RESULTANT CURVES

II CURVES

The following curves (Figs. B7.2.1.5-1 — B7.2.1.5-22) are plots of nondimensional stress resultants versus a shell parameter for the axial load and overturning moment loadings and for various attachment parameters.

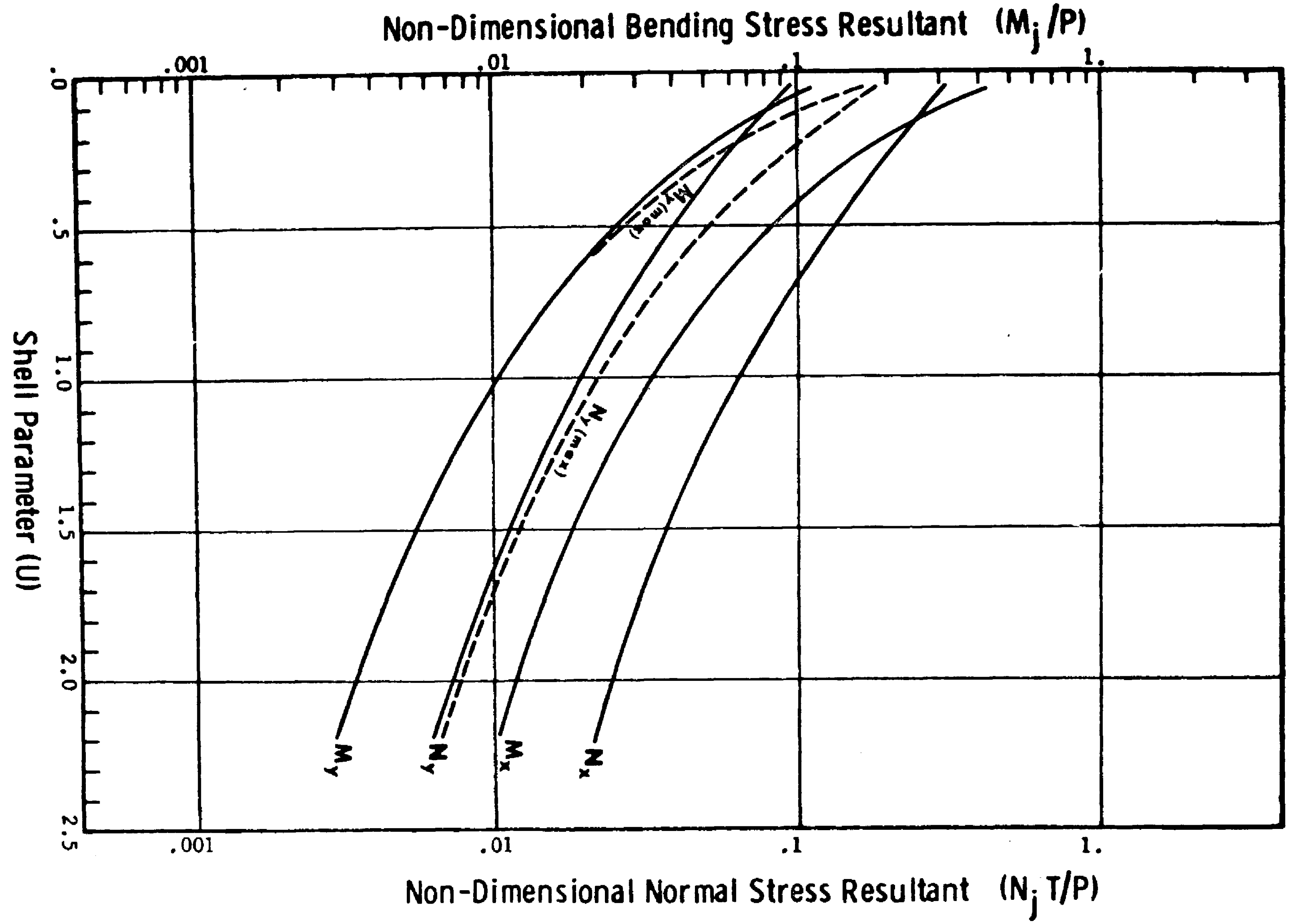

Figure B7.2.1.5-1 Non-Dimensional Stress Resultants for Radial Load (P) Solid Attachment

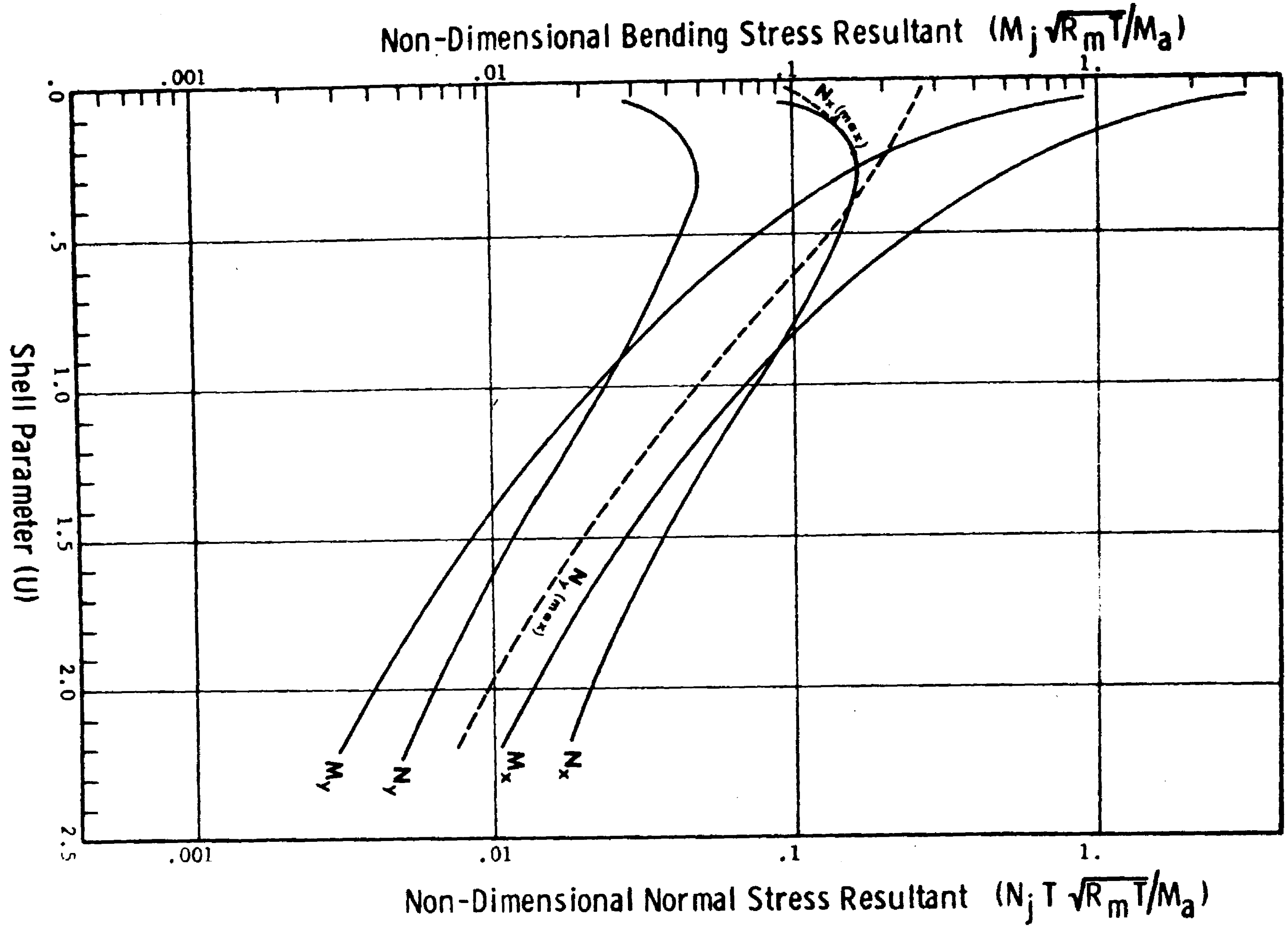

Figure B7.2.1.5-2 Non-Dimensional Stress Resultants for Overturning Moment (M_a) Solid Attachment

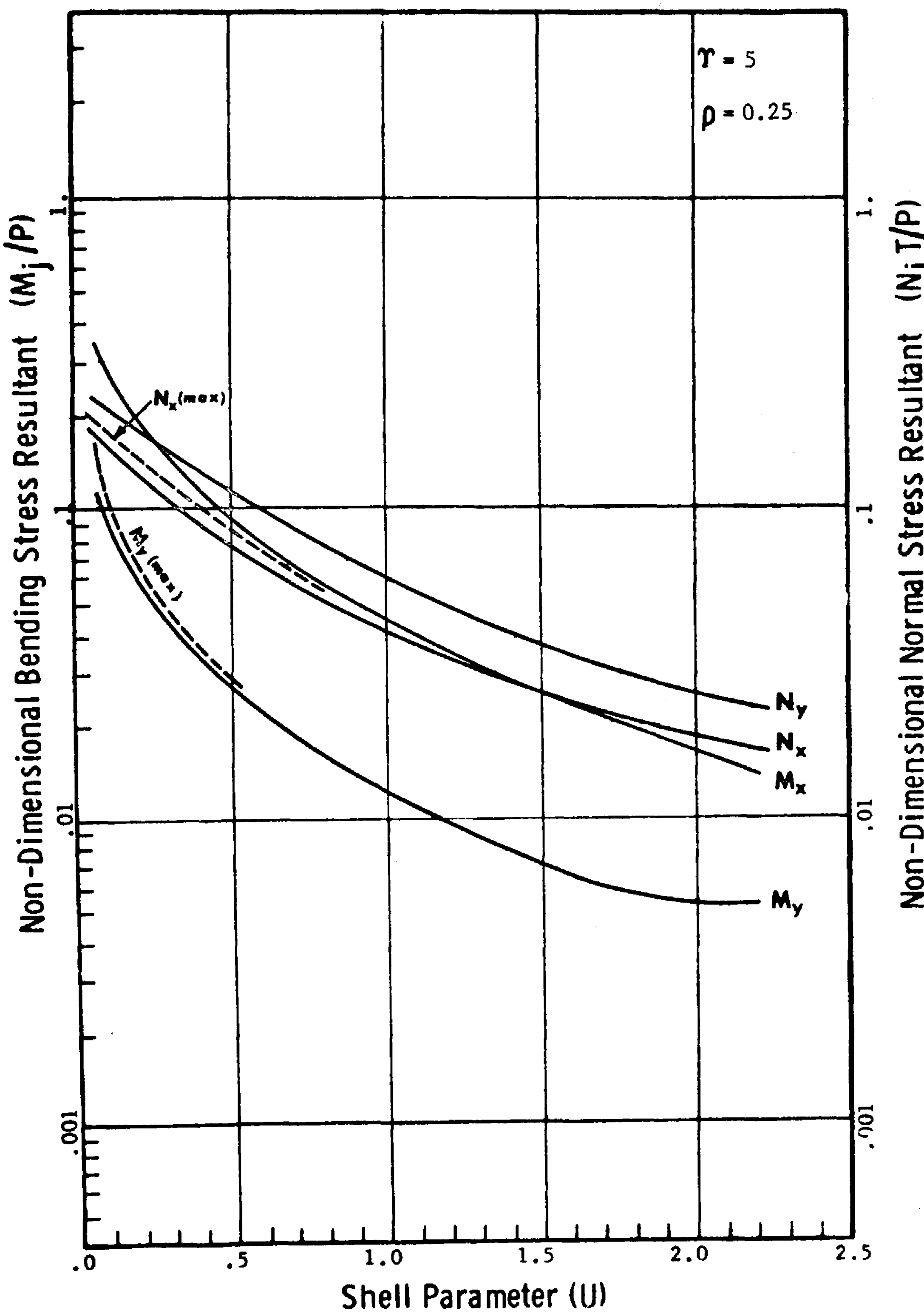

Figure B7.2.1.5-3 Non-Dimensional Stress Resultants for Radial Load (P) Hollow Attachment $\Upsilon = 5$ and $\rho = 0.25$

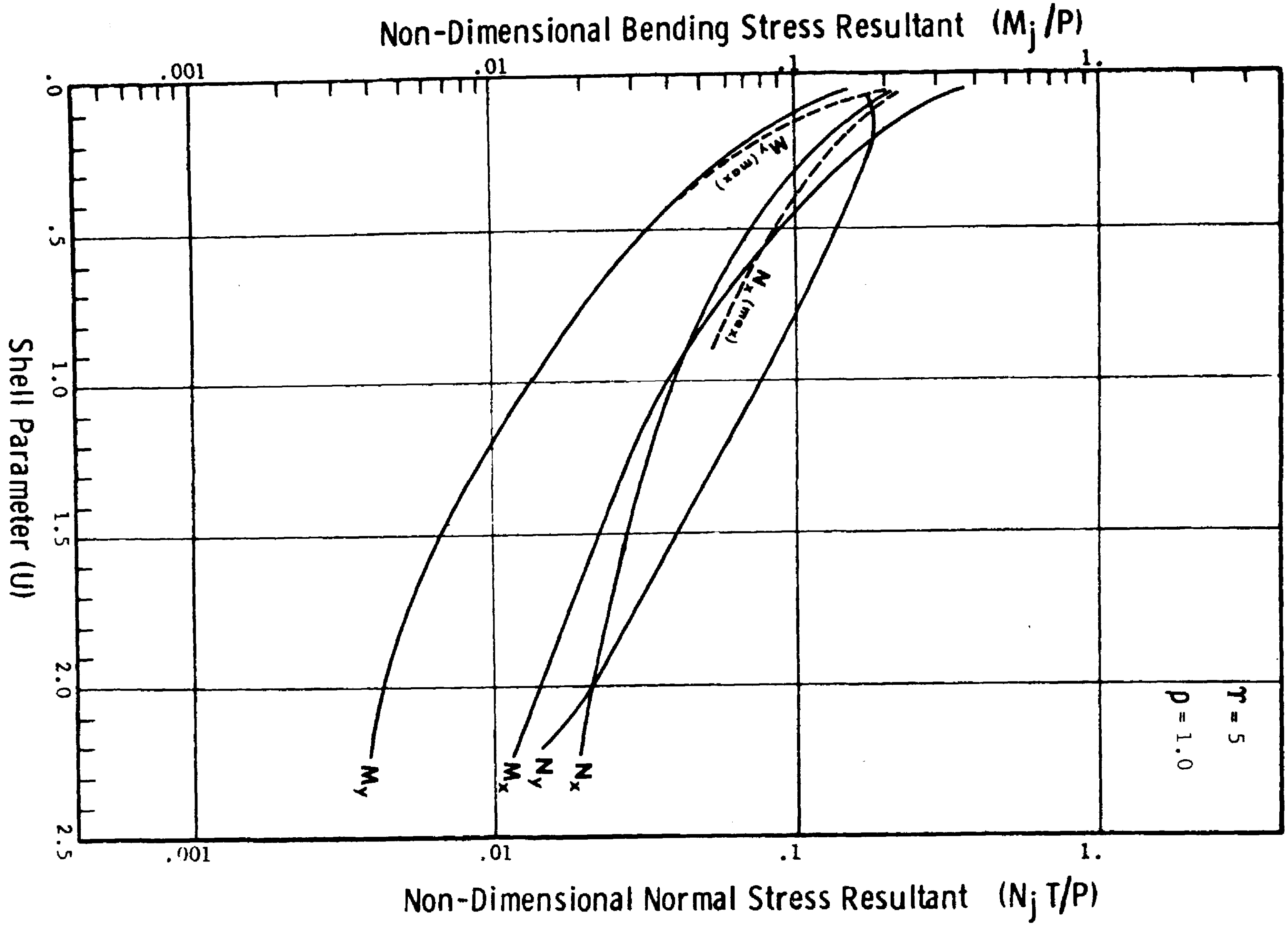

Figure B7.2.1.5-4 Non-Dimensional Stress Resultants for Radial Load (P)
Hollow Attachment $\tau = 5$ and $\rho = 1.0$

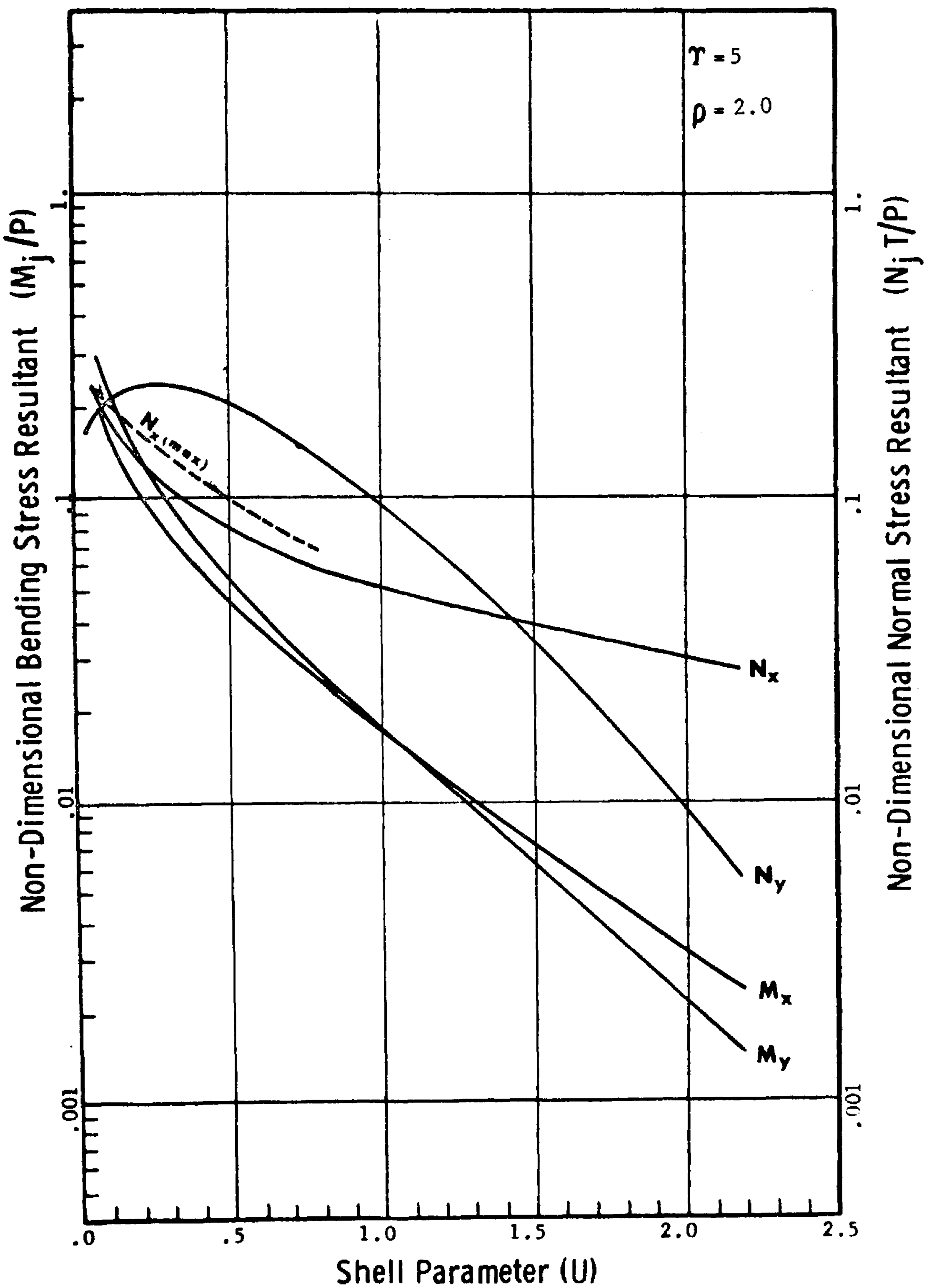

Figure B7.2.1.5-5 Non-Dimensional Stress Resultants for Radial Load (P) Hollow Attachment $\Upsilon = 5$ and $\rho = 2.0$

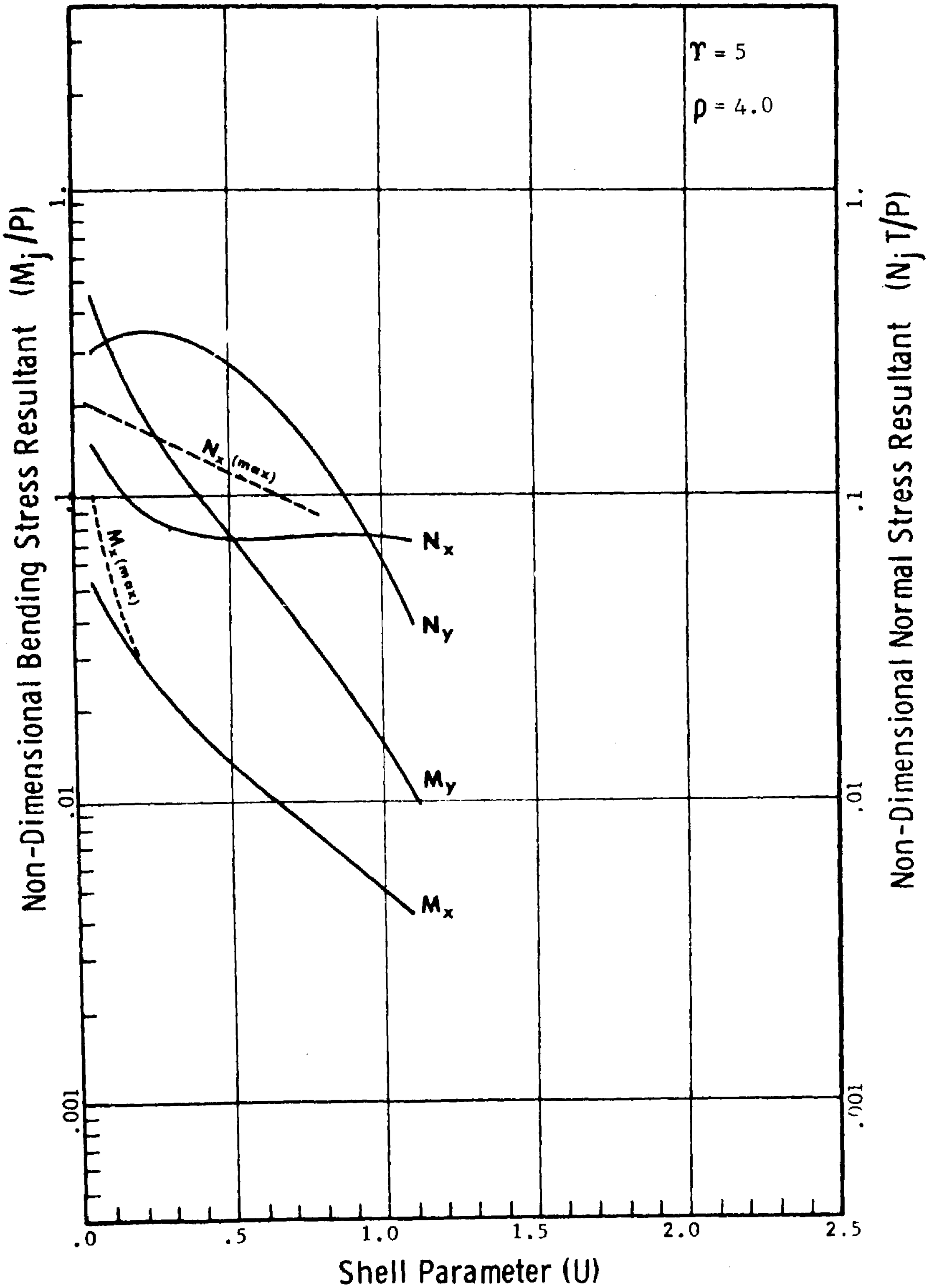

Figure B7.2.1.5-6 Non-Dimensional Stress Resultants for Radial Load (P) Hollow Attachment $\Upsilon = 5$ and $\rho = 4.0$

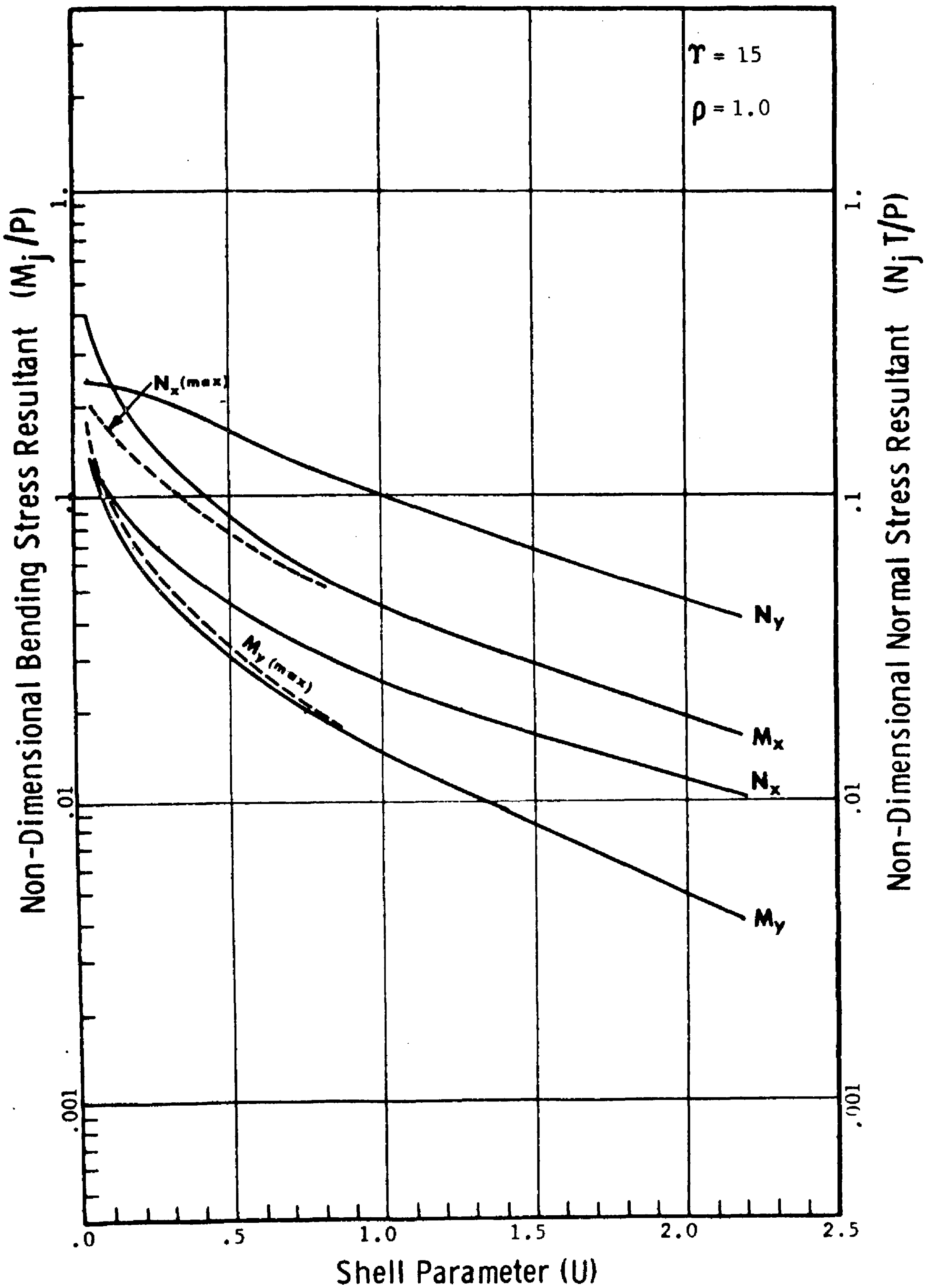

Figure B7.2.1.5-7 Non-Dimensional Stress Resultants for Radial Load (P) Hollow Attachment $\Upsilon = 15$ and $\rho = 1.0$

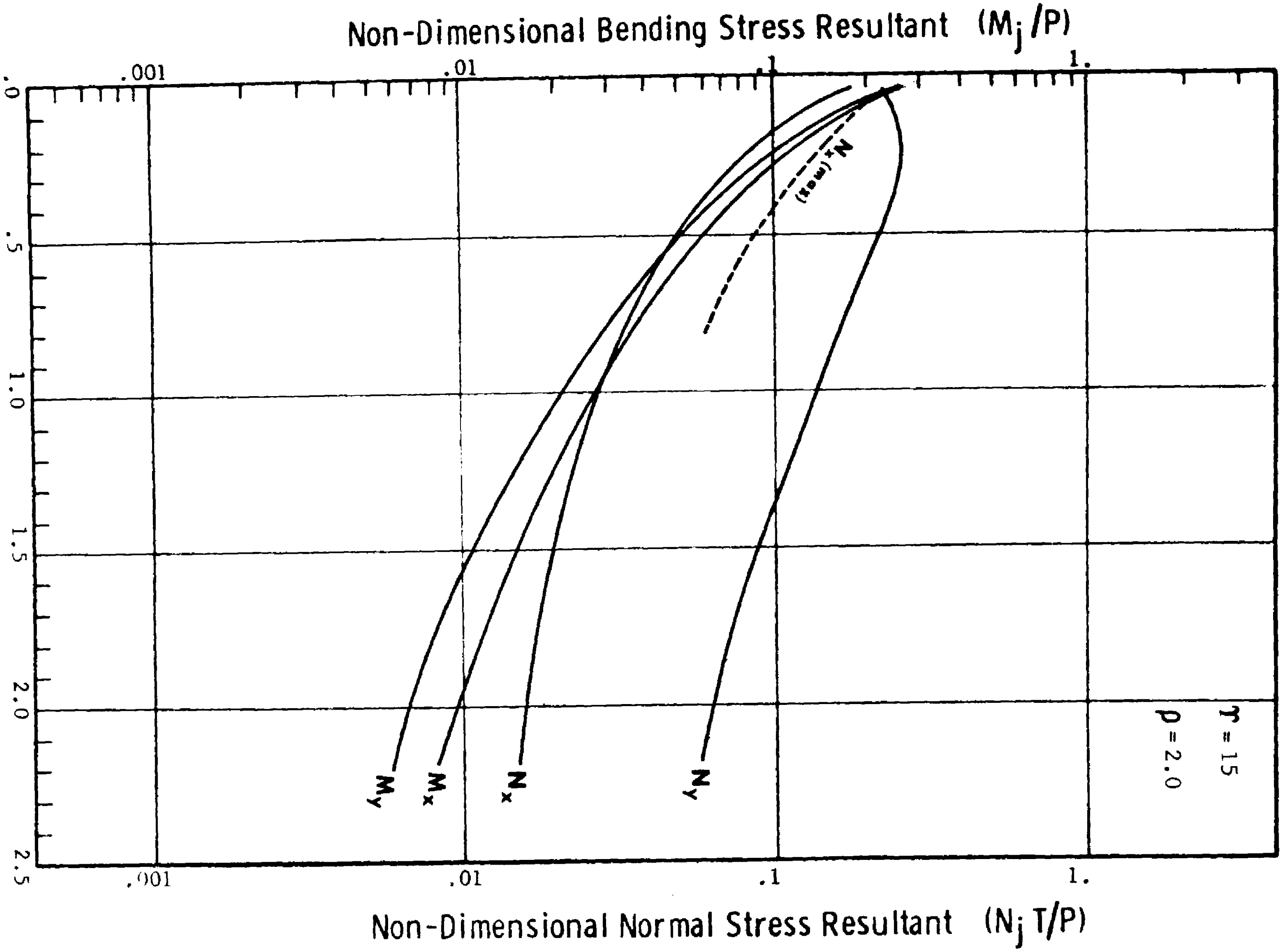

Figure B7.2.1.5-8 Non-Dimensional Stress Resultants for Radial Load (P) Hollow Attachment Υ = 15 and ρ = 2.0

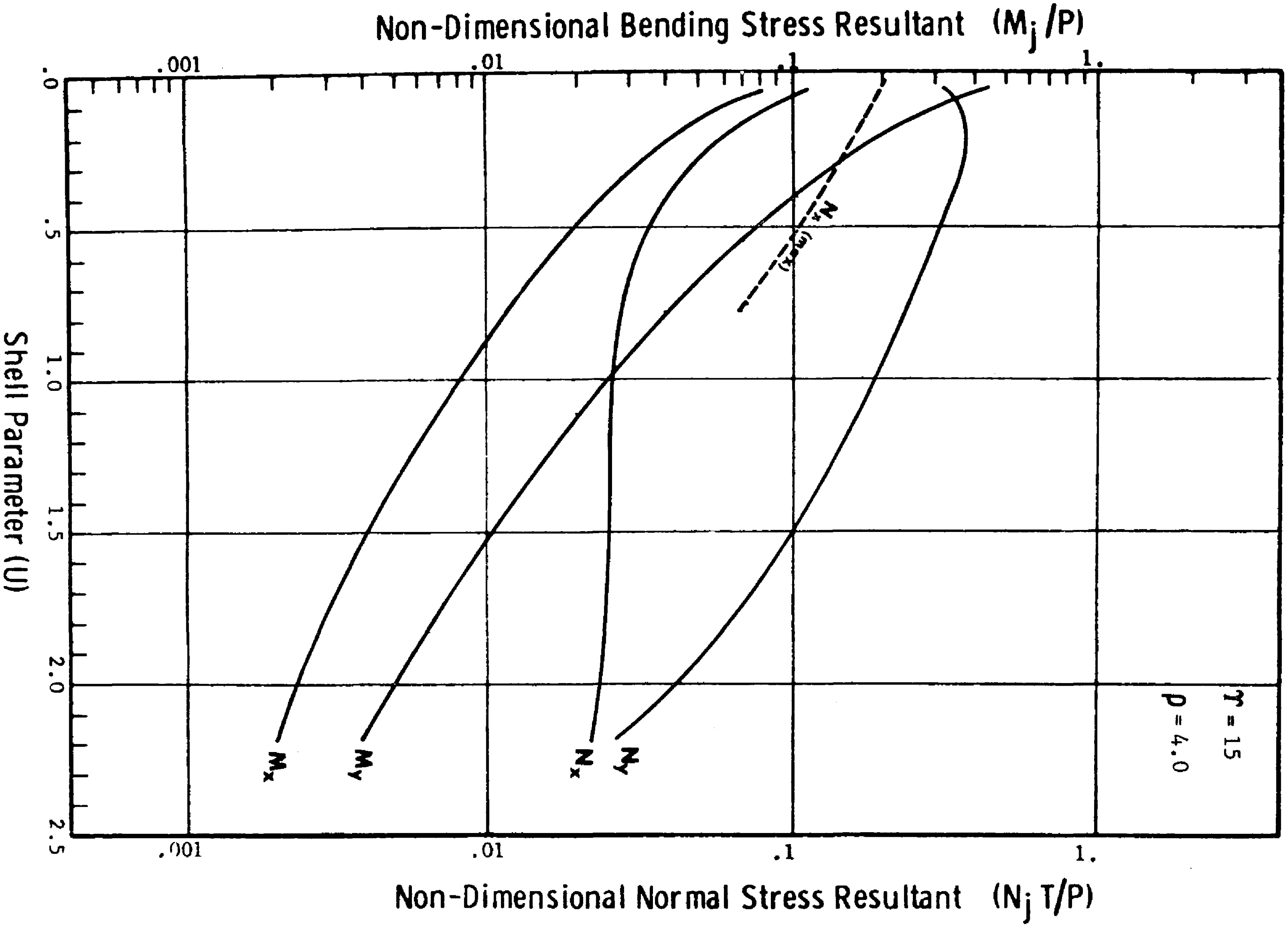

Figure B7.2.1.5-9 Non-Dimensional Stress Resultants for Radial Load (P) Hollow Attachment ϒ = 15 and ρ = 4.0

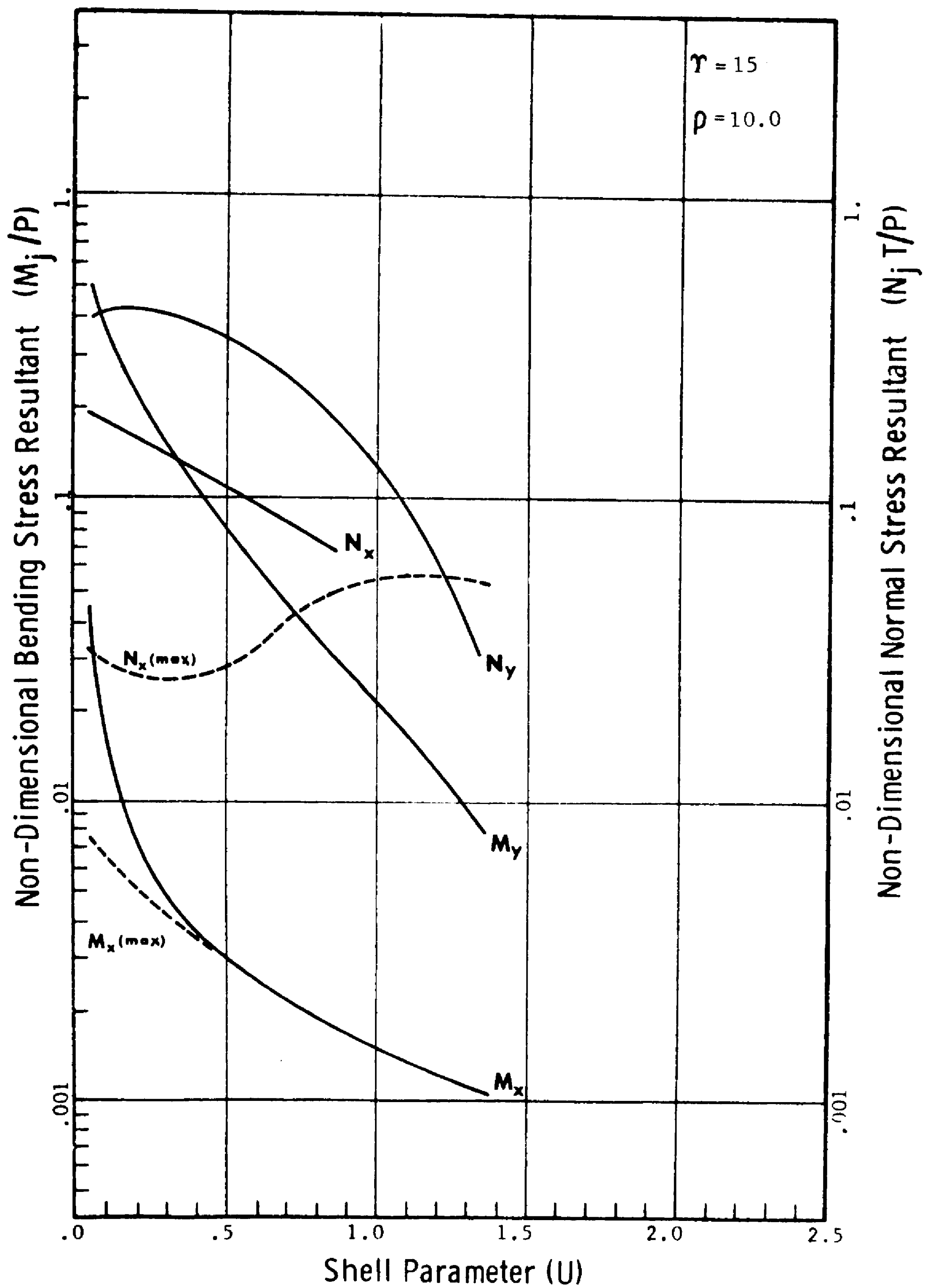

Figure B7.2.1.5-10 Non-Dimensional Stress Resultants for Radial Load (P)
Hollow Attachment ϒ = 15 and ρ = 10.0

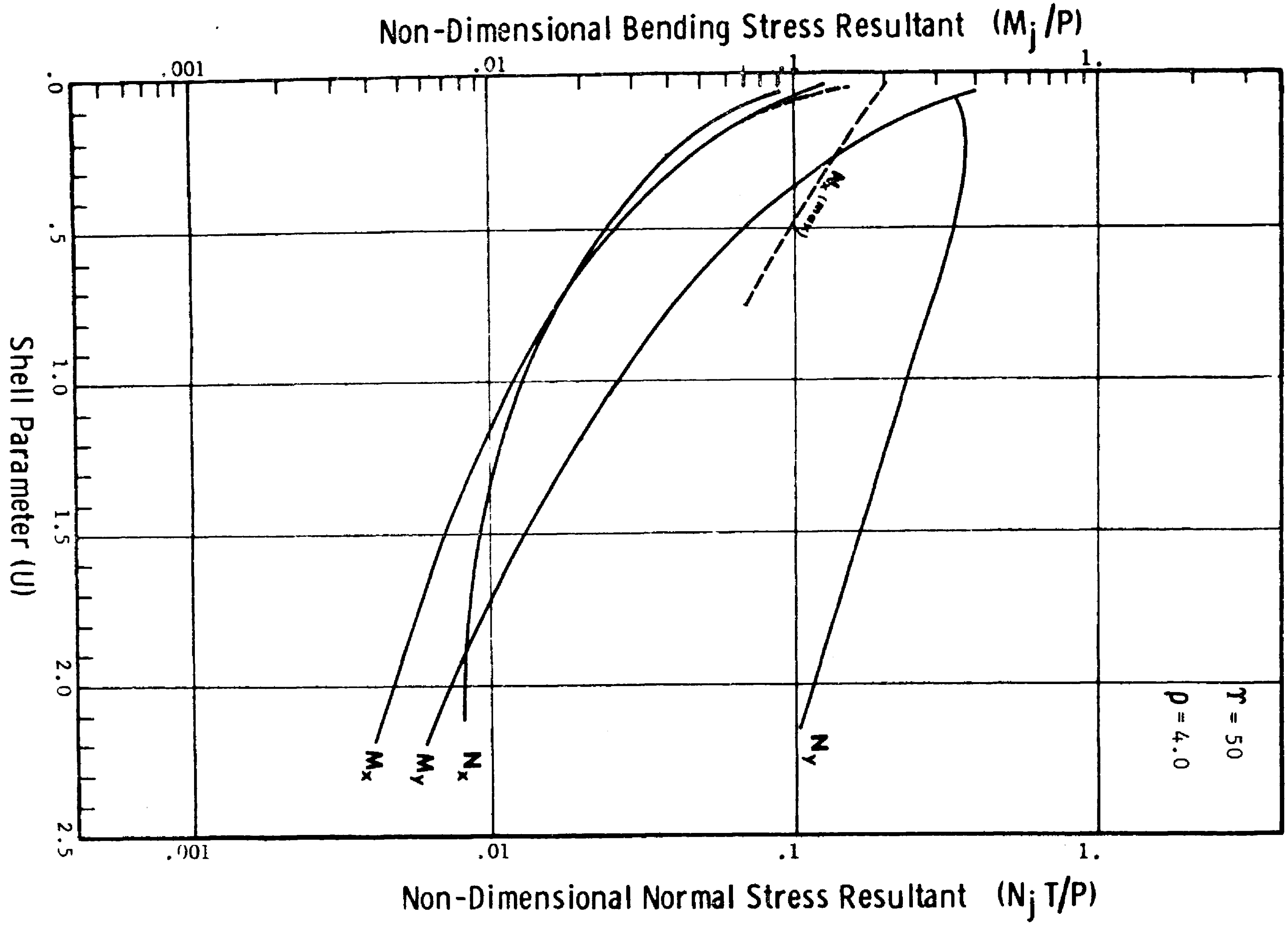

Figure B7.2.1.5-11 Non-Dimensional Stress Resultants for Radial Load (P)
Hollow Attachment $\Upsilon = 50$ and $\rho = 4.0$

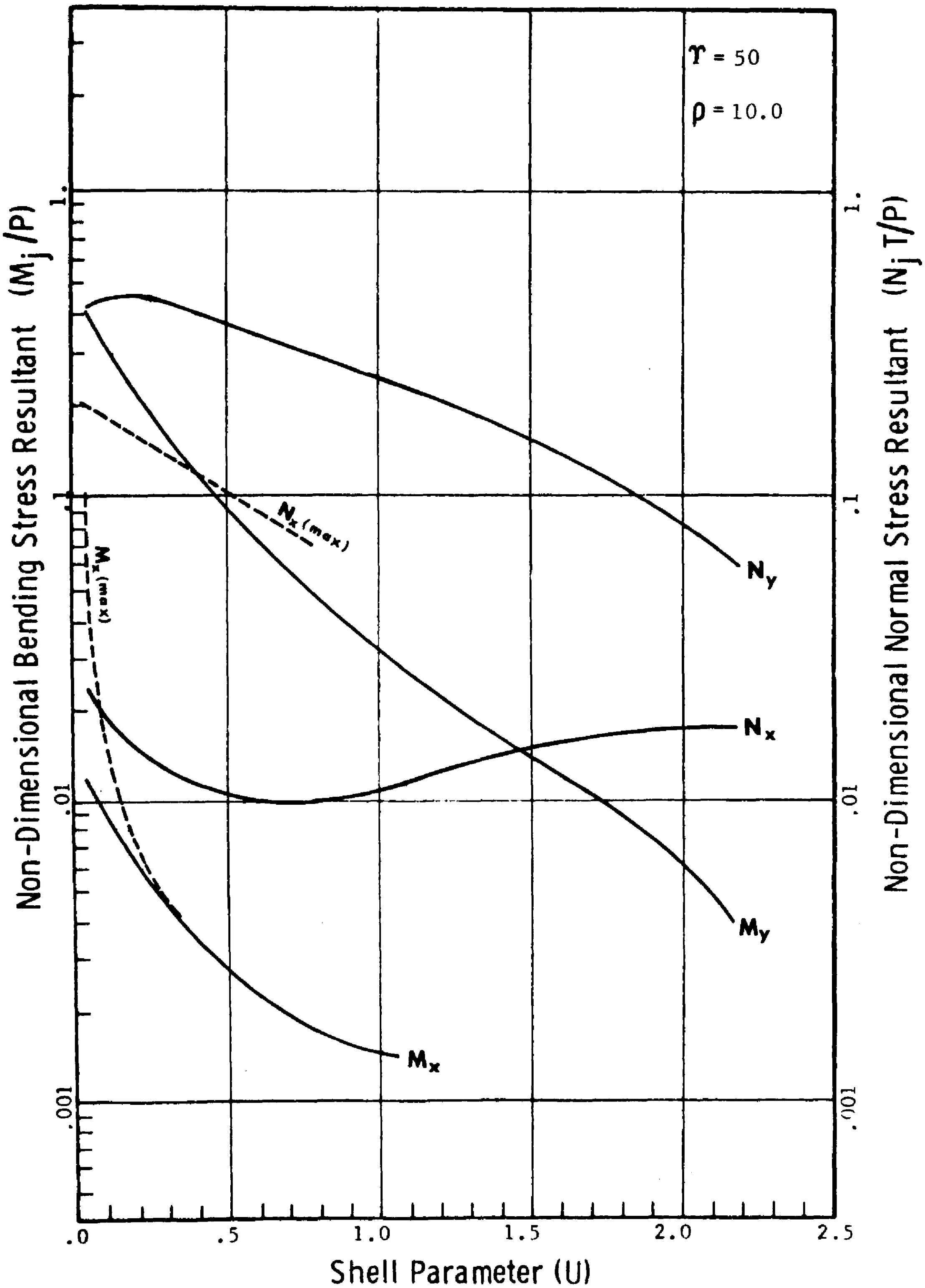

Figure B7.2.1.5-12 Non-Dimensional Stress Resultants for Radial Load (P) Hollow Attachment Υ = 50 and ρ = 10.0

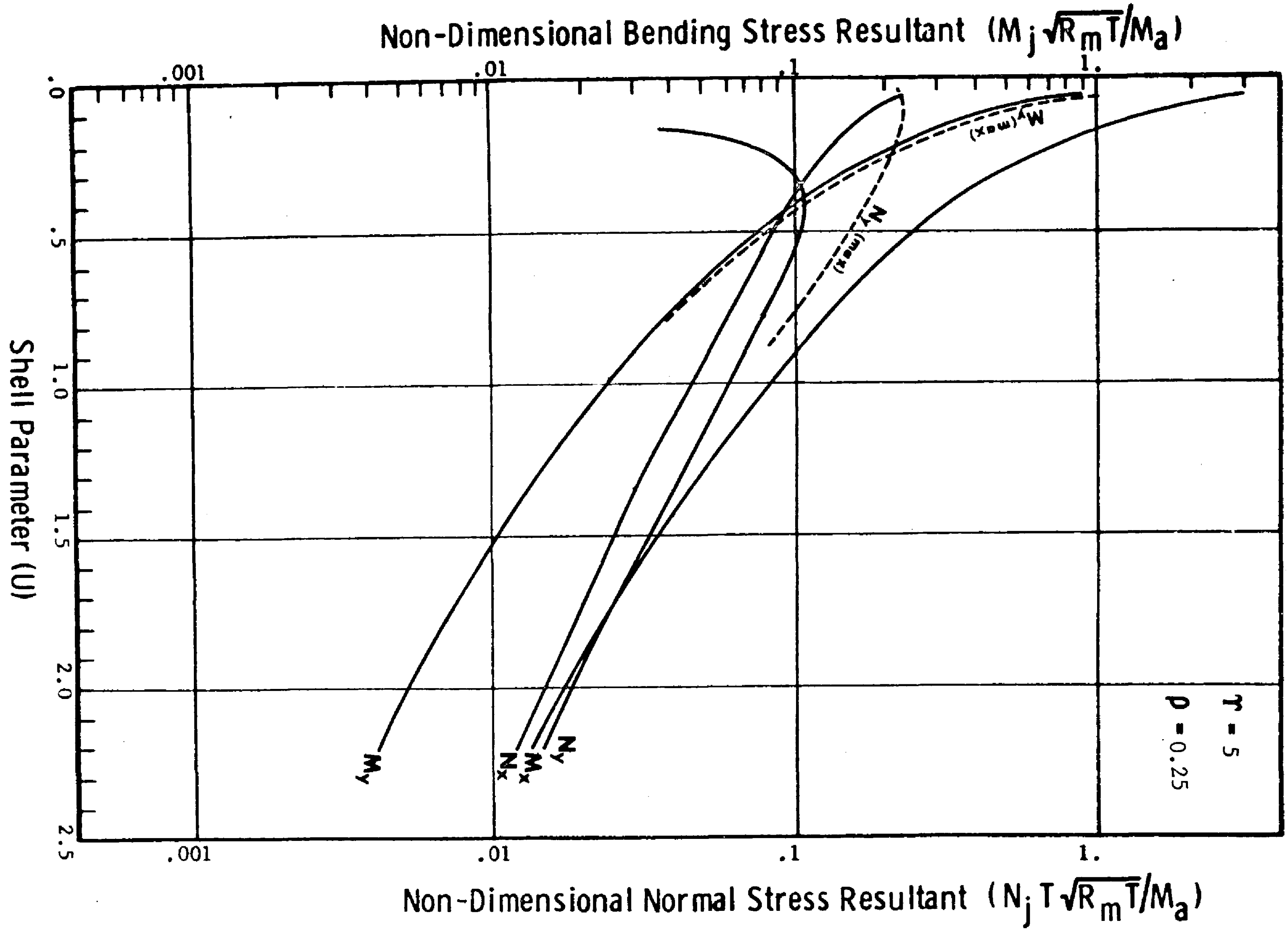

Figure B7.2.1.5-13 Non-Dimensional Stress Resultants for Overturning Moment (M_a) Hollow Attachment Υ = 5 and ρ = 0.25

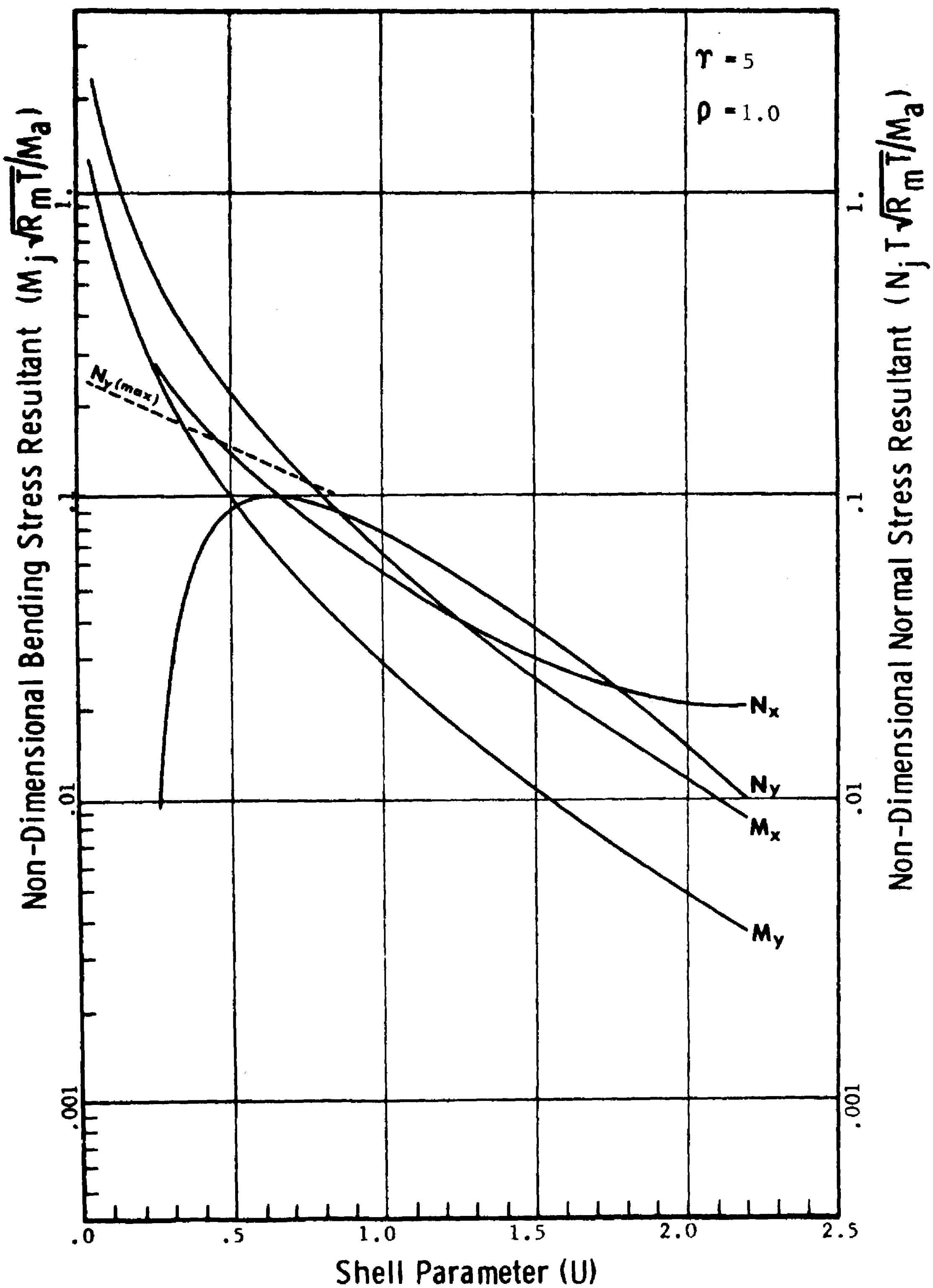

Figure B7.2.1.5-14 Non-Dimensional Stress Resultants for Overturning Moment (M_a) Hollow Attachment Υ = 5 and ρ = 1.0

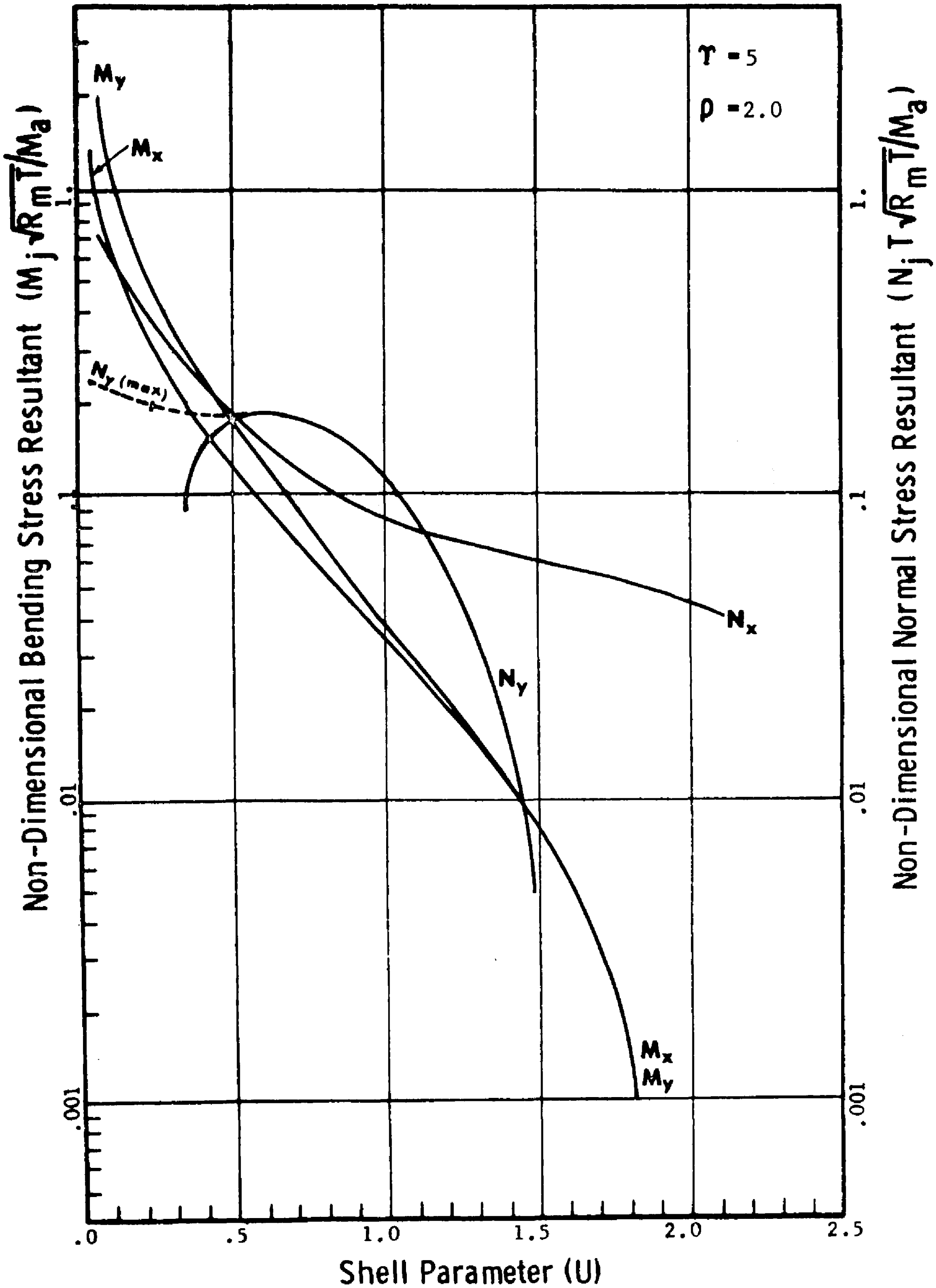

Figure B7.2.1.5-15 Non-Dimensional Stress Resultants for Overturning Moment (M_a) Hollow Attachment $\Upsilon = 5$ and $\rho = 2.0$

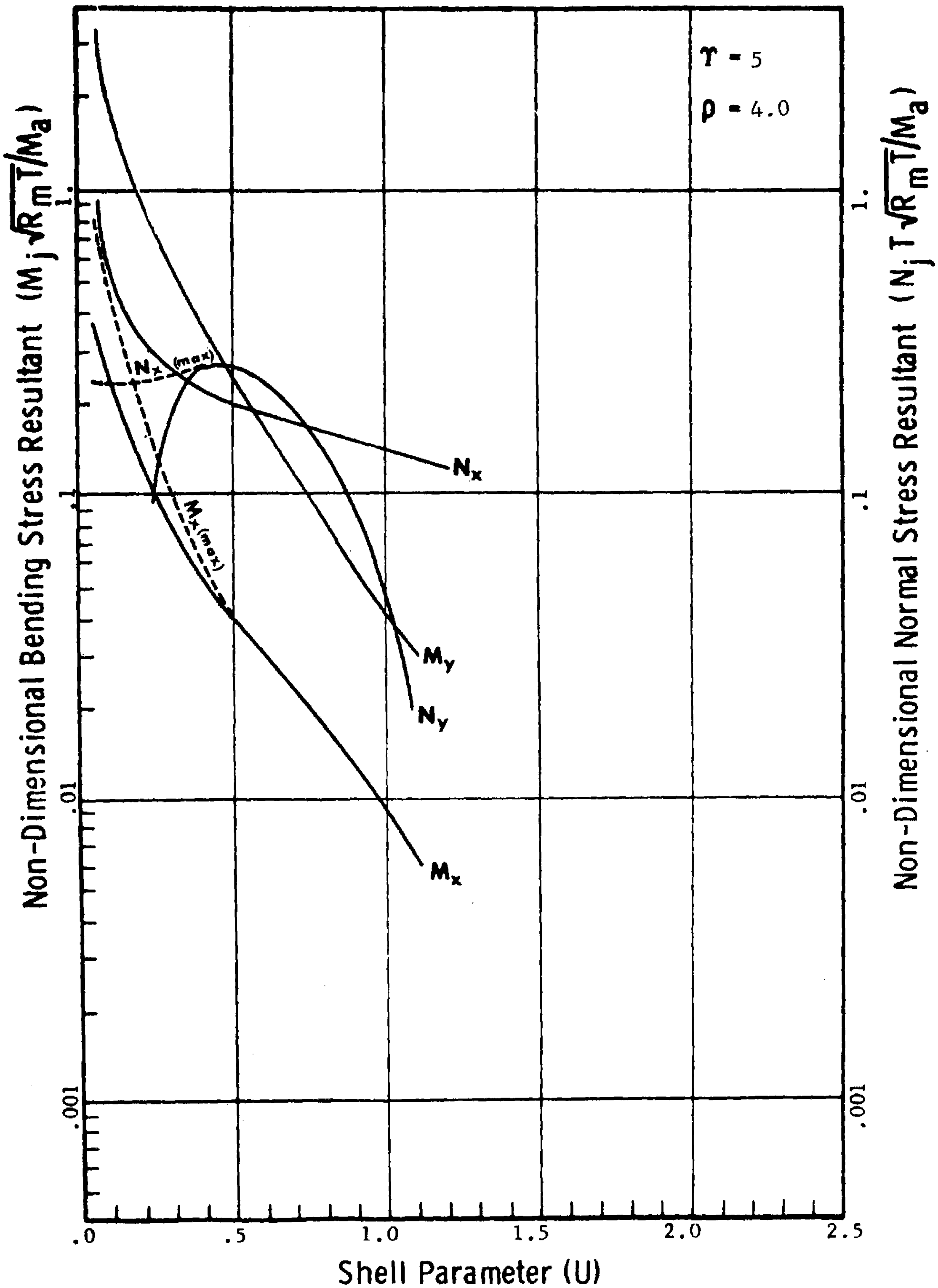

Figure B7.2.1.5-16 Non-Dimensional Stress Resultants for Overturning Moment (M_a) Hollow Attachment Υ = 5 and ρ = 4.0

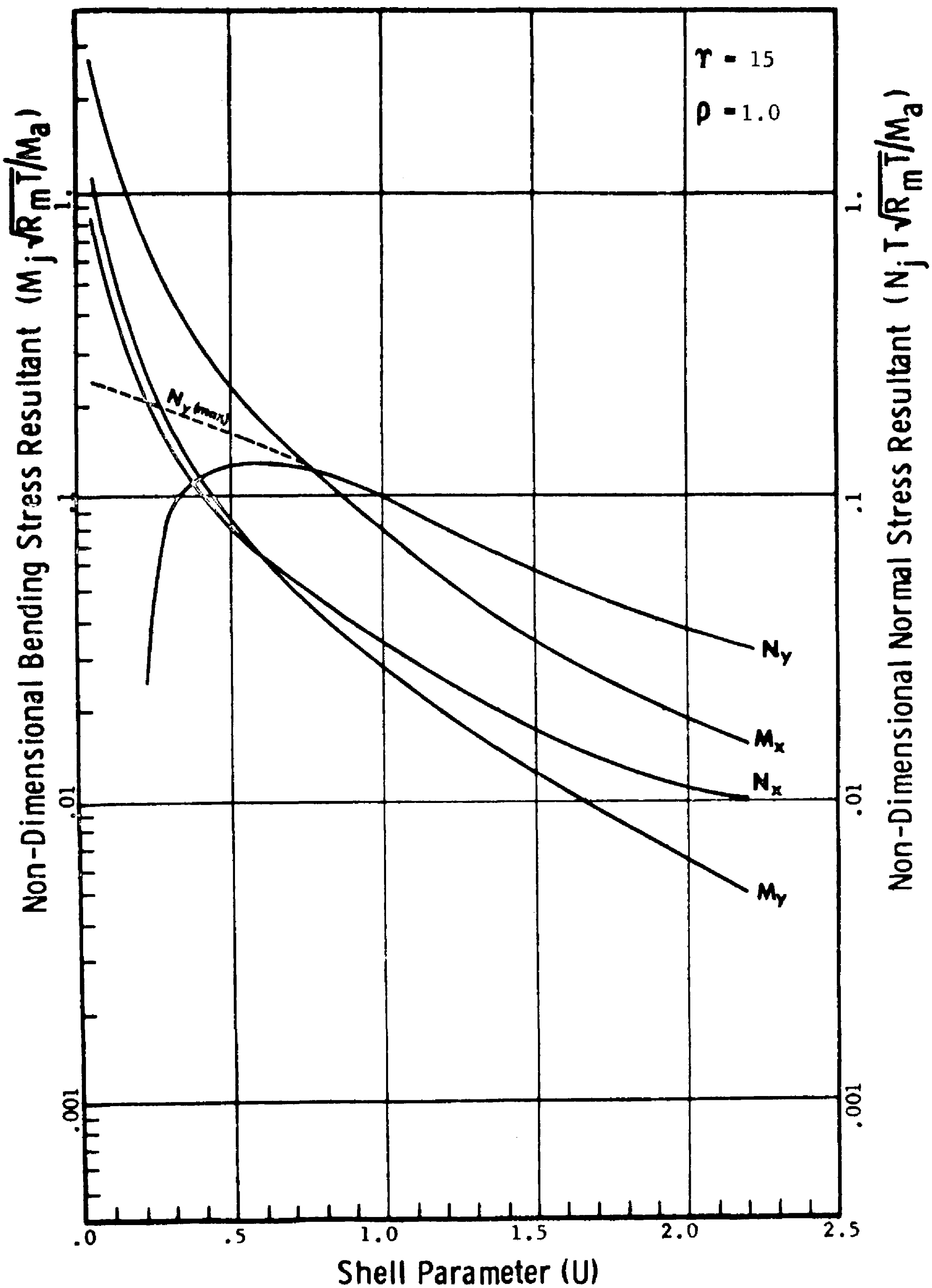

Figure B7.2.1.5-17 Non-Dimensional Stress Resultants for Overturning Moment (M_a) Hollow Attachment $\Upsilon = 15$ and $\rho = 1.0$

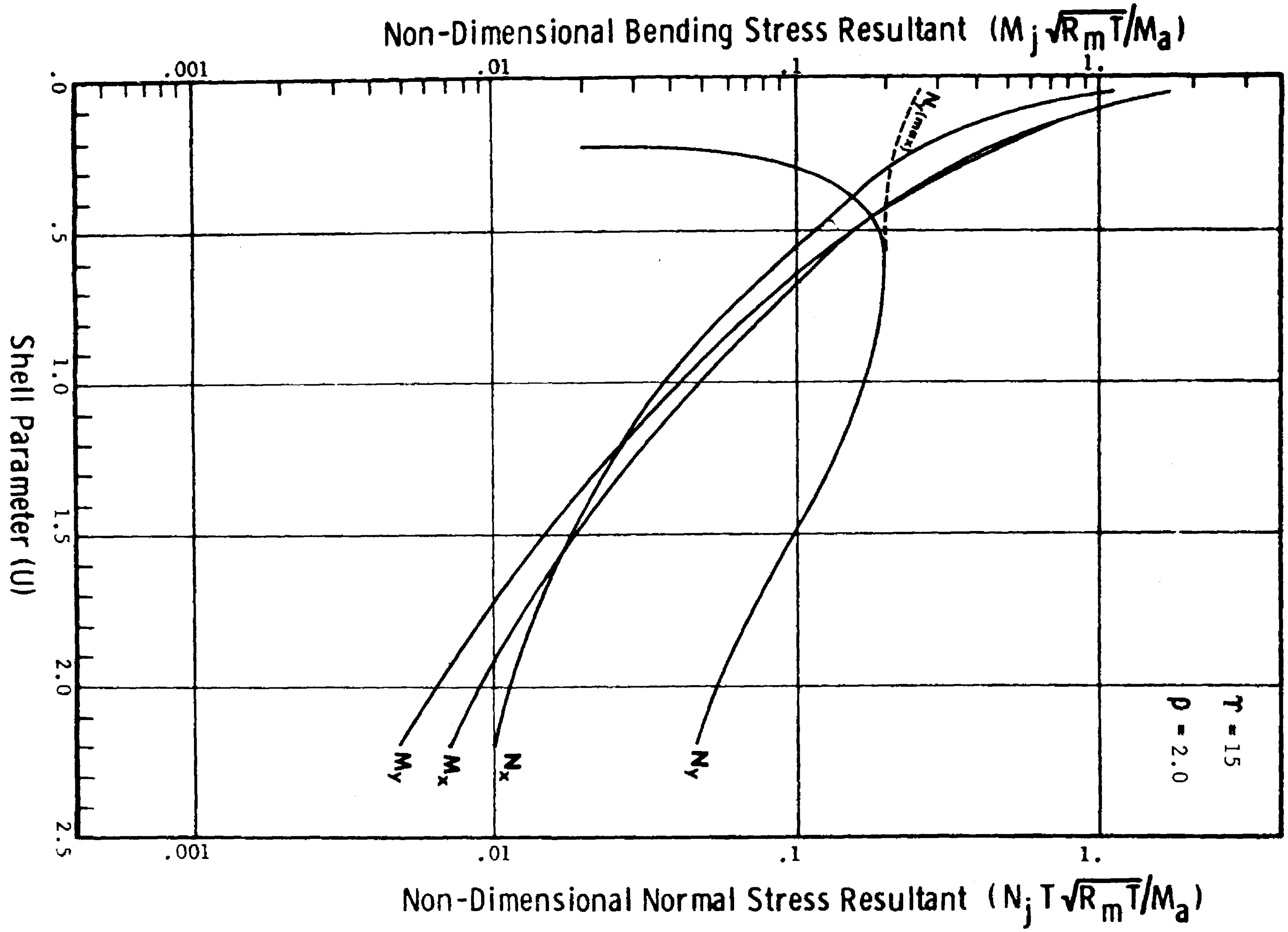

Figure B7.2.1.5-18 Non-Dimensional Stress Resultants for Overturning Moment (M_a) Hollow Attachment Υ = 15 and ρ = 2.0

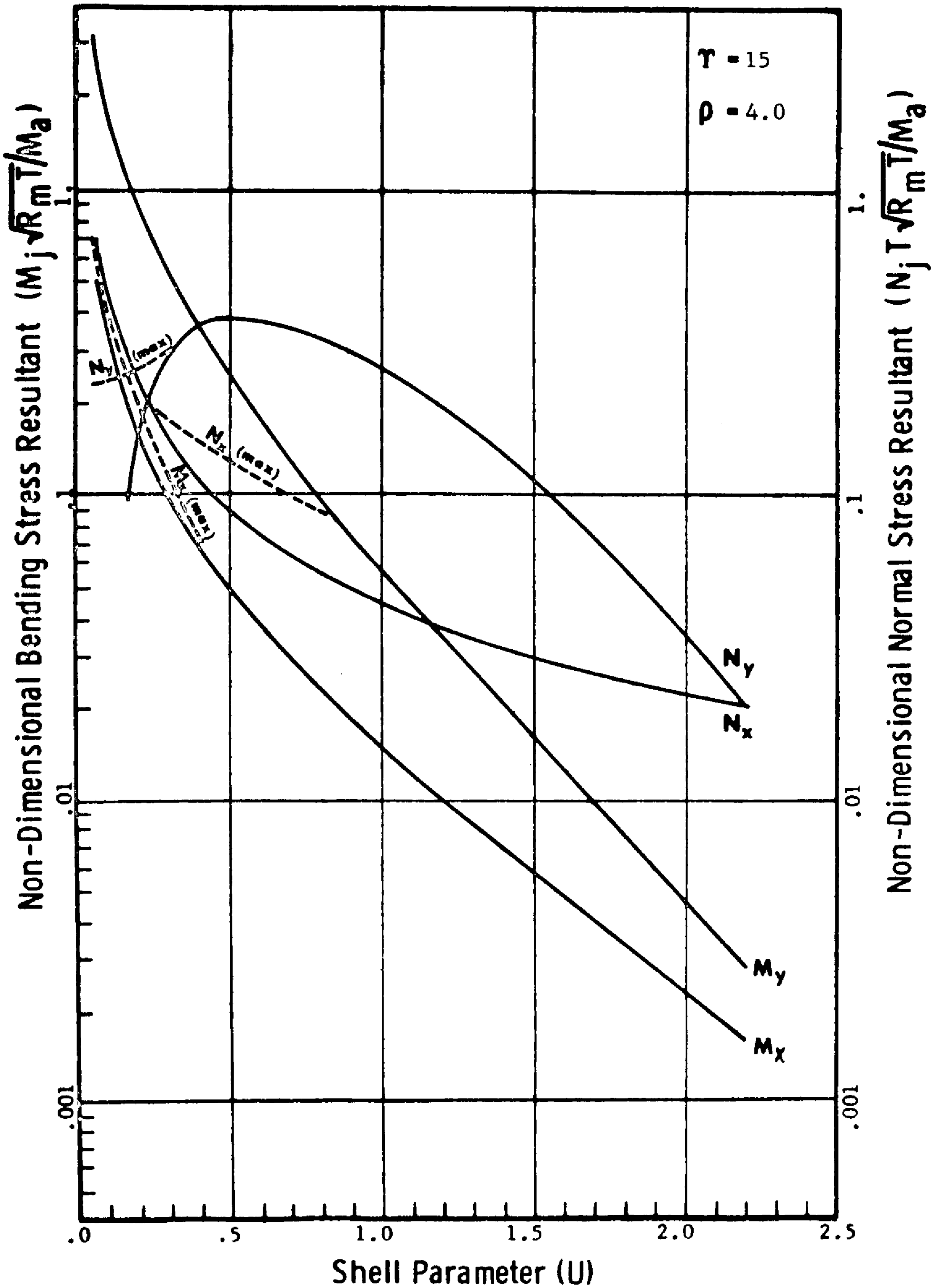

Figure B7.2.1.5-19 Non-Dimensional Stress Resultants for Overturning Moment (M_a) Hollow Attachment Υ = 15 and ρ = 4.0

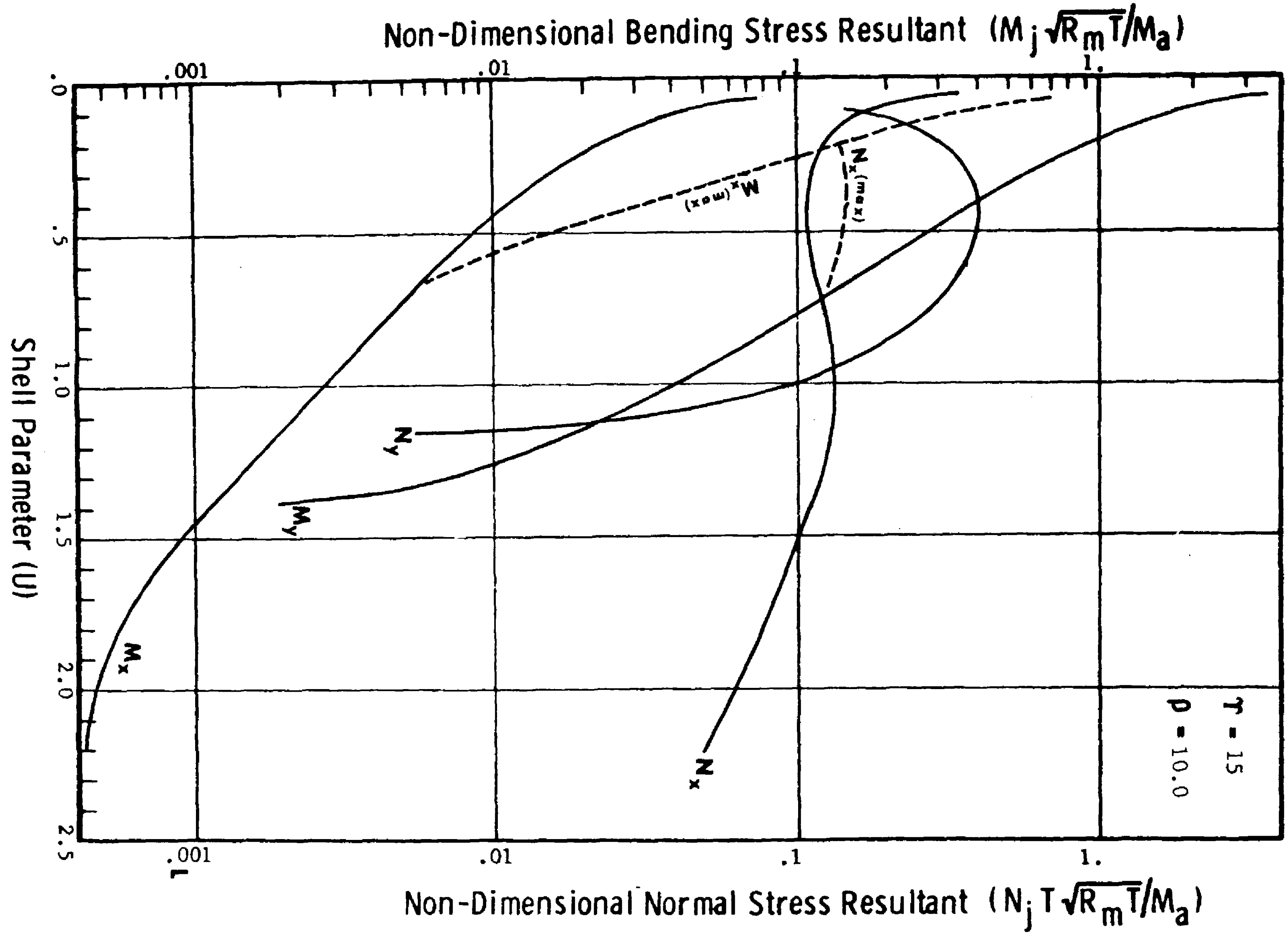

Figure B7.2.1.5-20 Non-Dimensional Stress Resultants for Overturning Moment (M_a) Hollow Attachment Υ = 15 and ρ = 10.0

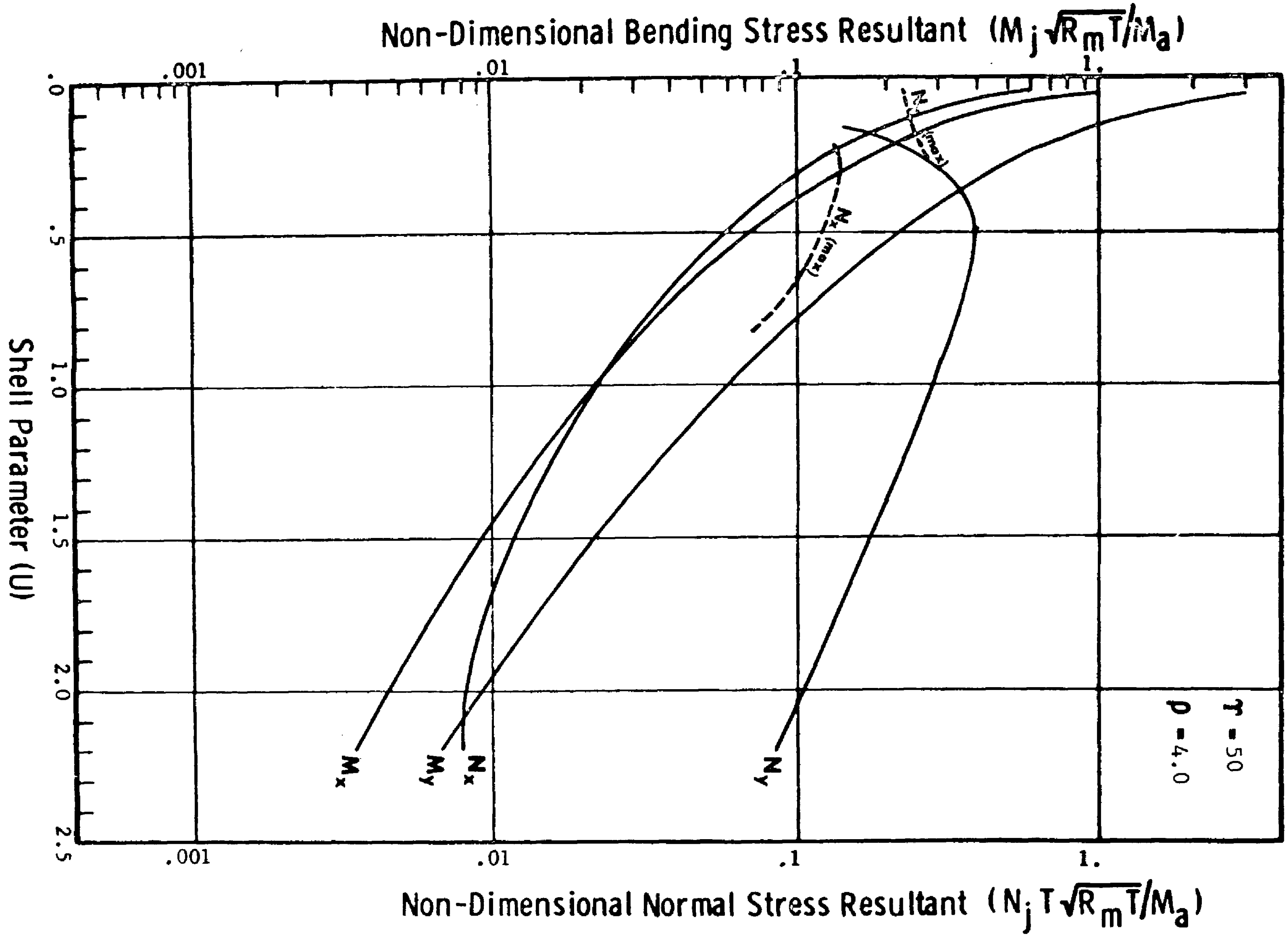

Figure B7.2.1.5-21 Non-Dimensional Stress Resultants for Overturning Moment (M_a) Hollow Attachment Υ = 50 and ρ = 4.0

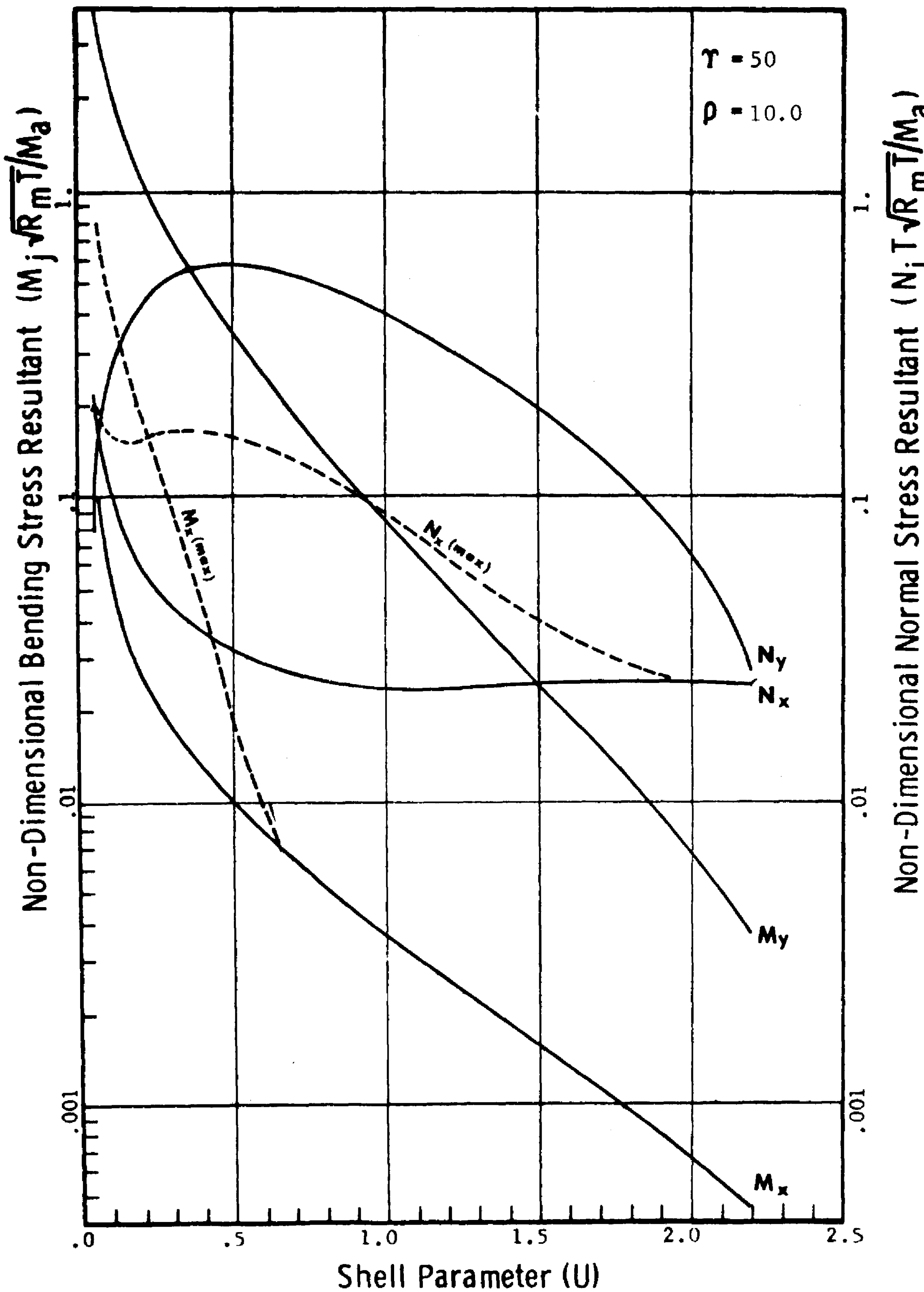

Figure B7.2.1.5-22 Non-Dimensional Stress Resultants for Overturning Moment (M_a) Hollow Attachment Υ = 50 and ρ = 10.0

B7.2.1.6 EXAMPLE PROBLEM

A spherical bulkhead with a welded hollow attachment is subjected to the force and moments shown in Figure B7.2.1.6-1. Shell and attachment geometry are shown in Figures B7.2.1.6-1 and B7.2.1.6-2.

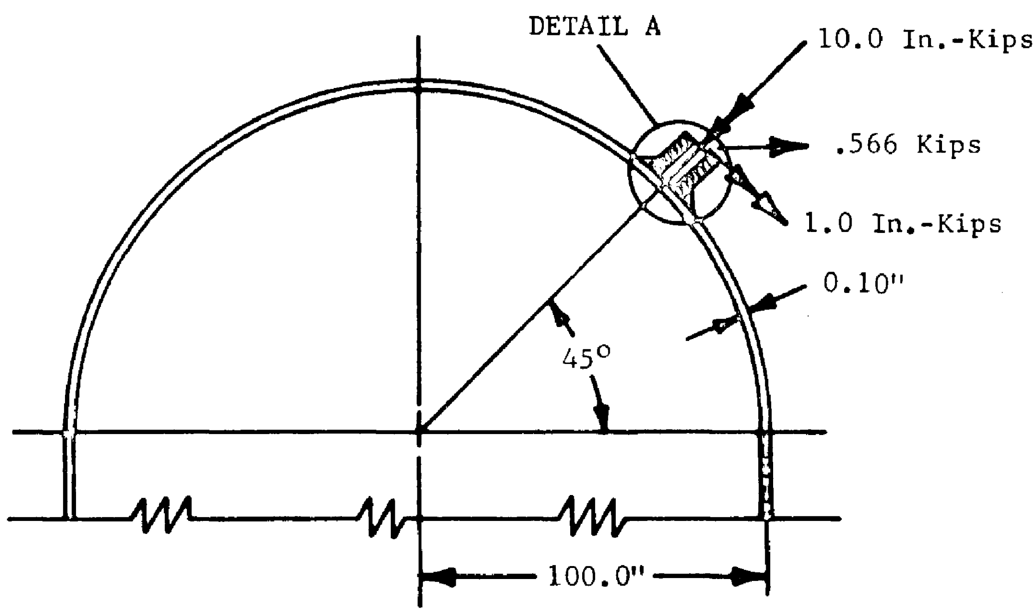

Fig. B7.2.1.6-1 Spherical Bulkhead

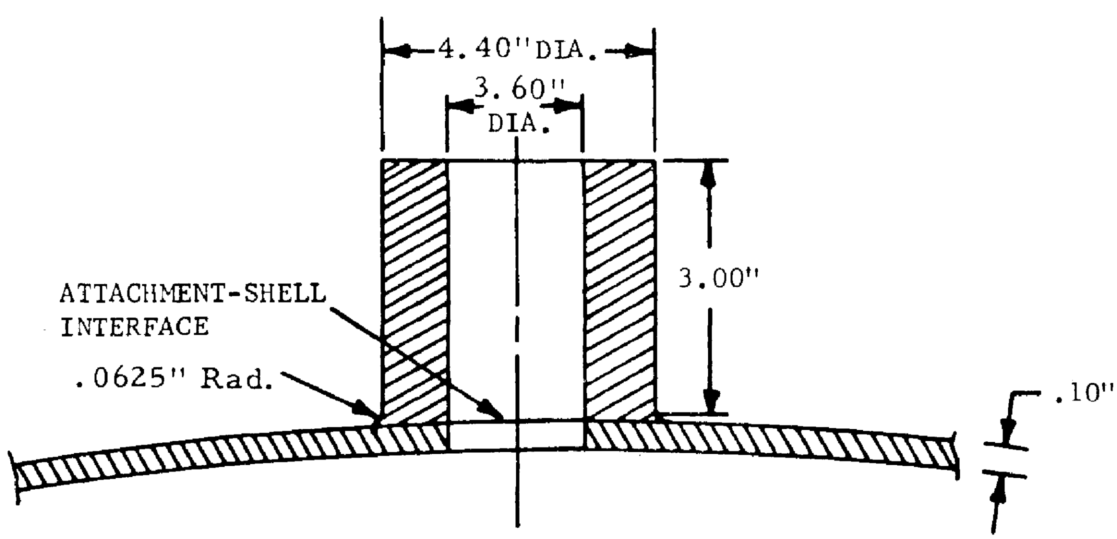

Fig. B7.2.1.6-2 Welded Hollow Attachment (Detail A)

1. Establish a local coordinate system (Figs. B7.2.1.1-1 and B7.2.1.6-3) on the center line of the attachment at the attachment-shell interface, so that the loading in Figure B7.2.1.6-1 is in the 2-3 plane.

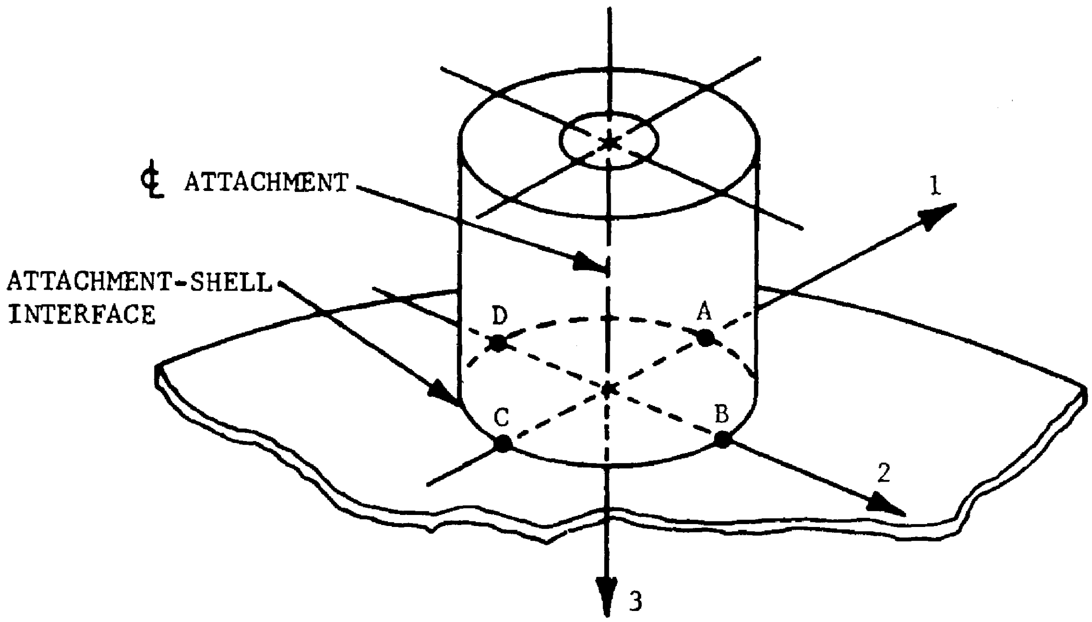

Fig. B7.2.1.6-3 Local Coordinate System

2. Resolve the load system into components (Figs. B7.2.1.1-1 and B7.2.1.6-4) and enter results on the appropriate stress calculation sheet (Figs. B7.2.1.2-2 for hollow attachments and B7.2.1.2-3 for solid attachments). Figure B7.2.1.6-5 shows the stress calculation sheet for the example problem.

3. Establish the appropriate shell geometric properties (Figure B7.2.1.1-2) and enter results on the stress calculation sheet. All dimensions are in inches.

$R_m = 100.0$

$T = 0.10$

$r_0 = 2.20$

$r_m = 2.00$

$t = 0.40$

$a = 0.0625$

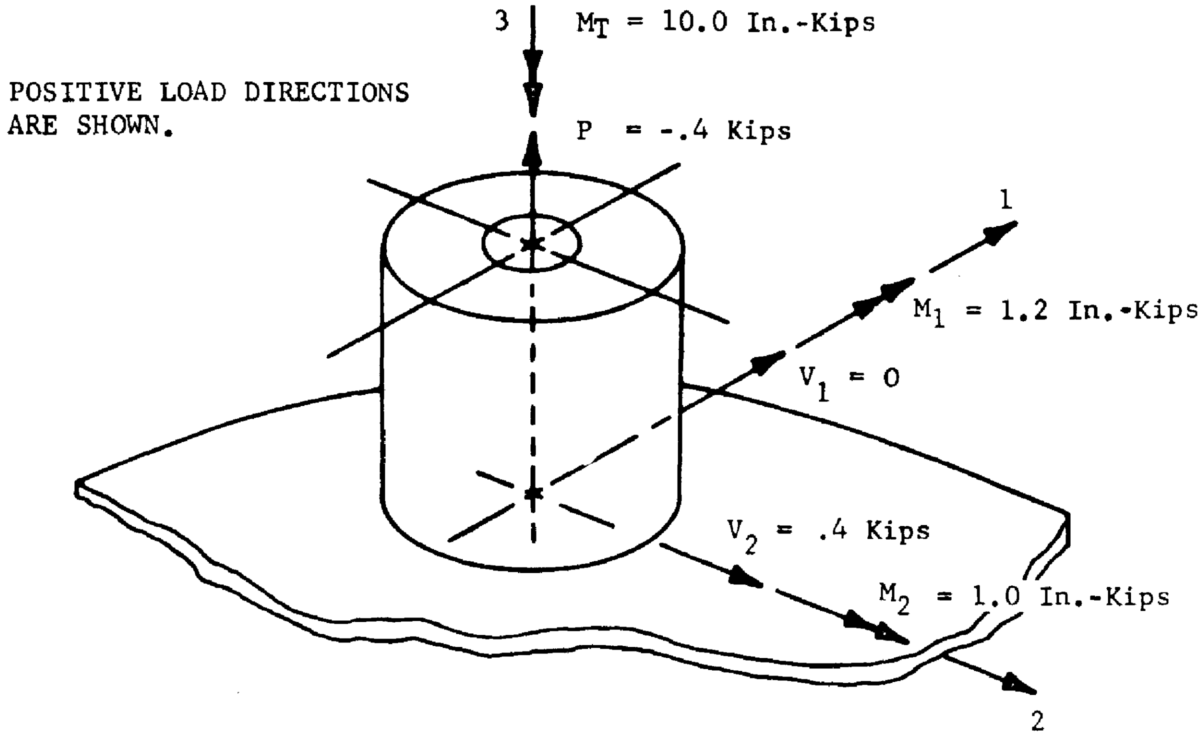

Fig. B7.2.1.6-4 Arbitrary Load System Components

4. Determine the appropriate parameters according to paragraph B7.2.1.2-II, and enter results on the stress calculation sheet.

$$U = r_0/(R_m T)^{\frac{1}{2}} = 2.20/(100 \times 0.1)^{\frac{1}{2}} = 0.695$$

For hollow attachment:

$$\Upsilon = r_m/t = 2.00/0.40 = 5.0$$

$$\rho = T/t = 0.10/0.40 = 0.25$$

For a brittle material (weld) at the attachment-to-shell juncture:

$$K_n = 1 + (T/5.6\,a)^{0.65} = 1 + (0.1/5.6 \times .0625)^{0.65} = 1.44$$

$$K_b = 1 + (T/9.4\,a)^{0.80} = 1 + (0.1/9.4 \times .0625)^{0.80} = 1.70$$

5. Determine the stresses according to paragraph B7.2.1.2-III through VI and enter results on the stress calculation sheet. The nondimensional stress resultants are obtained from Figure B7.2.1.5-3 (Hollow Attachment- $\Upsilon = 5$ and $\rho = 0.25$) for the radial load and from Figure B7.2.1.5-13 (Hollow Attachment - $\Upsilon = 5$ and $\rho = 0.25$) for the overturning moments.

STRESS CALCULATION SHEET FOR STRESSES IN SPHERICAL SHELLS CAUSED BY LOCAL LOADS (HOLLOW ATTACHMENT)

APPLIED LOADS	SHELL GEOMETRY	PARAMETERS
P = -.4 kips	T = .10	u = .695
V_1 = 0 "	t = .40	τ = 5.0
V_2 = .4 "	R_m = 100.	ρ = .25
M_T = 10.0 in-kips	r_m = 2.00	K_n = 1.44
M_1 = 1.2 "	r_o = 2.20	K_b = 1.70
M_2 = 1.0 "	a = .0625	

STRESS	LOAD	NON-DIMENSIONAL STRESS RESULTANT	ADJUSTING FACTOR	STRESS COMPONENT	STRESSES* A_i	A_o	B_i	B_o	C_i	C_o	D_i	D_o
MERIDIONAL STRESS (f_x)	P	$\frac{N_x T}{P}$ = (.061) .066	$\frac{K_n P}{T^2}$ = -57.6	$\frac{K_n N_x}{T}$ =	(+3.5) ∓3.2	(+3.5) ∓3.2	(+3.5) ∓3.2	(+3.5) ∓3.2	(+3.5) ∓3.2	(+3.5) ∓3.2	(+3.5) ∓3.2	(+3.5) ∓3.2
		$\frac{M_x}{P}$ = .064	$\frac{6K_b P}{T^2}$ = -408	$\frac{6K_b M_x}{T^2}$ =	∓26.1	±26.1	∓26.1	±26.1	∓26.1	±26.1	∓26.1	±26.1
	M_1	$\frac{N_x T\sqrt{R_m T}}{M_1}$ = .067	$\frac{K_n M_1}{T^2\sqrt{R_m T}}$ = 54.7	$\frac{K_n N_x}{T}$ =			-3.7	-3.7			+3.7	+3.7
		$\frac{M_x\sqrt{R_m T}}{M_1}$ = .15	$\frac{6K_b M_1}{T^2\sqrt{R_m T}}$ = 387.3	$\frac{6K_b M_x}{T^2}$ =			+58.1	-58.1			-58.1	+58.1
	M_2	$\frac{N_x T\sqrt{R_m T}}{M_2}$ = .067	$\frac{K_n M_2}{T^2\sqrt{R_m T}}$ = 45.6	$\frac{K_n N_x}{T}$ =	+3.1	+3.1			-3.1	-3.1		
		$\frac{M_x\sqrt{R_m T}}{M_2}$ = .16	$\frac{6K_b M_2}{T^2\sqrt{R_m T}}$ = 322.8	$\frac{6K_b M_x}{T^2}$ =	-48.4	+48.4			+48.4	-48.4		
	TOTAL MERIDIONAL STRESSES (f_x)				-68.2	81.1	31.8	-32.5	22.7	-22.2	-77.3	91.4
CIRCUMFERENTIAL STRESS (f_y)	P	$\frac{N_y T}{P}$ = .087	$\frac{K_n P}{T^2}$ = -57.6	$\frac{K_n N_y}{T}$ =	±5.0	±5.0	±5.0	±5.0	±5.0	±5.0	±5.0	±5.0
		$\frac{M_y}{P}$ = .019	$\frac{6K_b P}{T^2}$ = -408	$\frac{6K_b M_y}{T^2}$ =	∓7.8	±7.8	∓7.8	±7.8	∓7.8	±7.8	∓7.8	±7.8
	M_1	$\frac{N_y T\sqrt{R_m T}}{M_1}$ = (.11) .087	$\frac{K_n M_1}{T^2\sqrt{R_m T}}$ = 54.7	$\frac{K_n N_y}{T}$ =			(6.0) -4.8	(6.0) -4.8			(6.0) +4.8	(6.0) +4.8
		$\frac{M_y\sqrt{R_m T}}{M_1}$ = (.046) .045	$\frac{6K_b M_1}{T^2\sqrt{R_m T}}$ = 387.3	$\frac{6K_b M_y}{T^2}$ =			(18.6) +17.4	(18.6) 17.4			(18.6) 17.4	(18.6) +17.4
	M_2	$\frac{N_y T\sqrt{R_m T}}{M_2}$ = (.11) .087	$\frac{K_n M_2}{T^2\sqrt{R_m T}}$ = 46.6	$\frac{K_n N_y}{T}$ =	(5.0) +4.0	(5.0) +4.0			(5.0) -4.0	(5.0) -4.0		
		$\frac{M_y\sqrt{R_m T}}{M_2}$ = (.046) .045	$\frac{6K_b M_2}{T^2\sqrt{R_m T}}$ = 322.8	$\frac{6K_b M_y}{T^2}$ =	(15.5) 14.5	(15.5) +14.5			(15.5) +14.5	(15.5) -14.5		
	TOTAL CIRCUMFERENTIAL STRESS (f_y)				-14.3	33.3	11.0	-11.8	8.7	-7.7	-16.6	37.4
SHEAR STRESS (f_{xy})	V_1	0	$\pi r_o T$ = .69	$\frac{V_1}{\pi r_o T}$ = 0			- 0	- 0			+ 0	+ 0
	V_2	.40	$\pi r_o T$ = .69	$\frac{V_2}{\pi r_o T}$ = .6	-.6	-.6			+.6	+.6		
	M_T	10.0	$2\pi r_o^2 T$ = 3.04	$\frac{M_T}{2\pi r_o^2 T}$ = 3.3	-3.3	-3.3	+3.3	+3.3	-3.3	-3.3	+3.3	+3.3
	TOTAL SHEAR STRESS (f_{xy})				-3.9	-3.9	3.3	3.3	-2.7	-2.7	3.3	3.3
PRINCIPAL STRESSES**	f_{max}	$\frac{f_x + f_y}{2} + \sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2}$ ***			-68.5	81.4	32.3	-33.0	23.2	-22.9	-77.5	91.6
	f_{min}	$\frac{f_x + f_y}{2} - \sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2}$ ***			-14.0	33.0	10.5	-11.3	8.2	-7.0	-16.4	37.2
	$f_{xy max}$	$\pm\sqrt{\frac{(f_x - f_y)^2}{4} + f_{xy}^2}$			27.2	24.2	10.9	10.9	7.5	8.0	30.5	27.2

* IF LOAD IS OPPOSITE TO THAT SHOWN IN FIGURE B7.2.1.1-1 THEN REVERSE THE SIGN SHOWN.
** SEE SECTION A3.1.0.
*** CHANGE SIGN OF THE RADICAL IF $(f_x + f_y)$ IS NEGATIVE.

Figure B7.2.1.6-5 Stress Calculation Sheet (Example Problem)