SECTION B7.1

MEMBRANE ANALYSIS OF

TABLE OF CONTENTS

B7.1.0.0 MEMBRANE ANALYSIS OF THIN SHELLS OF REVOLUTION

In engineering applications, shells that have the form of surfaces of revolution find extensive application in various kinds of containers, tanks, and domes. Furthermore, this type of shell offers a convenient selection of coordinates.

Thin shells, in general, display large stresses and deflections when subjected to relatively small bending moments. Therefore, in the design of thin shells, the condition of bending stresses is avoided or minimized. If, in the equilibrium equations of such shells, all moment expressions are neglected, the resulting shell theory is called "membrane theory," and the state of stress is referred to as a "momentless" state of stress. There are two types of shells that comply with this membrane theory: (1) shells sufficiently flexible so that they are physically incapable of resisting bending, and (2) shells that are flexurally stiff but loaded and supported in a manner that avoids the introduction of bending strains.

The momentless state of stress in practical shell problems is difficult to achieve. However, with the comparison of the complete bending analysis and membrane analysis for a thin shell of revolution built in along its edges and having noncritical axisymmetric loading, the following conclusions can be made:

1. The stresses and deformations are almost identical for all locations on the shell except for a narrow strip of the shell surface adjacent to the boundary. This narrow strip is usually no wider than $\sqrt{Rt}$.
2. Except for the strip along the boundary, all bending moments, twisting moments, and vertical shears are negligible; this causes the entire bending solution to be practically identical to the membrane solution.

3. Boundary conditions along the supporting edge are very significant; however, local bending and shear decrease rapidly away from the boundary and may become negligible outside the narrow strip.

For cases where bending stresses cannot be neglected or when a more complete analysis is desired, see Section B7.3 for bending analysis.

Shells of revolution are frequently loaded internally or externally by forces having the same symmetry as the shell itself. This loading condition is referred to as axisymmetric loading and contributes significantly to the simplification of the analysis methods presented in this section.

B7.1.1.0 GENERAL

Before investigating the stresses and deflections of a shell of revolution, we must examine the geometry of such a surface. A surface of revolution is generated by the rotation of a plane curve about an axis in its plane. This generating curve is called a meridian. The intersections of the generated surface with planes perpendicular to the axis of rotation are parallel circles and are called parallels. For such surfaces, the lines of curvature are its meridians and parallels.

A convenient selection of surface coordinates is the curvilinear coordinate system ϕ and θ, where ϕ is the angle between the normal to the surface and the axis of rotation and θ is the angle determining the position of a point on the corresponding parallel, with reference to some datum meridian. (See Figure B7.1.1 - 1.) If the surface of revolution is a sphere, these coordinates are spherical coordinates used in geography; θ is the longitude and ϕ is the complement to the latitude; hence, we have the nomenclature of meridians and parallels.

Figure B7.1.1 - 1 shows a meridian of a surface of revolution. Let R be the distance of one of its points normal to the axis of rotation and R_1 its radius of curvature. In future equations, we will also need the length R_2, measured on a normal to the meridian between its intersection with the axis of rotation and the shell surface. Noting that $R = R_2 \sin\phi$, the surface of the shell of revolution is completely described by R_1 and R_2 which are functions of only one of the curvilinear coordinates, ϕ. R_0 will be the radius of curvature when $\phi = 0$.

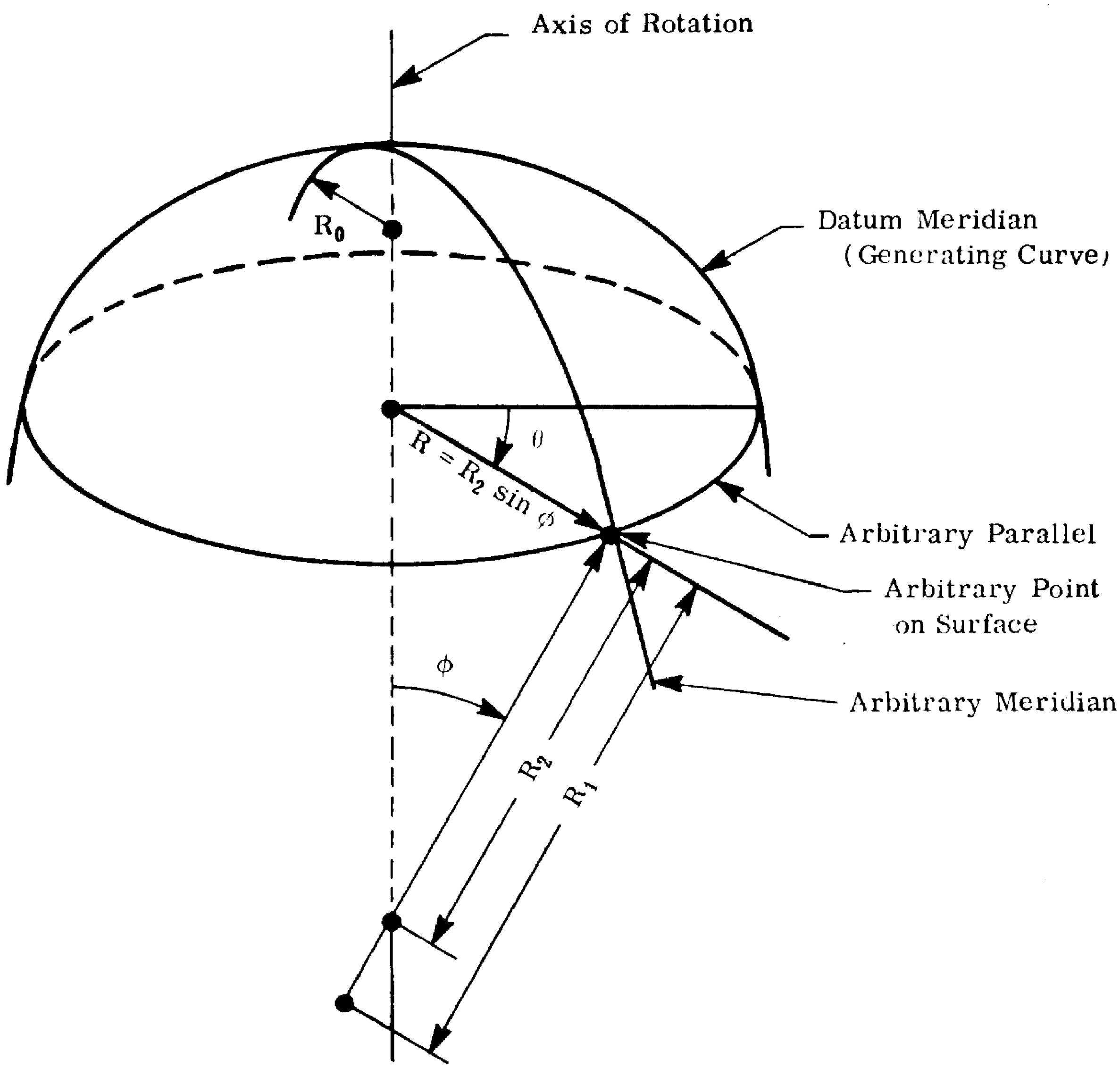


Fig. B7.1.1 - 1. Geometry of Surfaces of Revolution

The surface of revolution thus described will be that surface which bisects the thickness of the shell and will henceforth be referred to as the "middle surface" or "reference surface." By specifying the form of the middle surface and the thickness "t" of the shell at any point, the shell is entirely defined geometrically. Figure B7.1.1 - 2 shows an element of the middle surface of the shell.

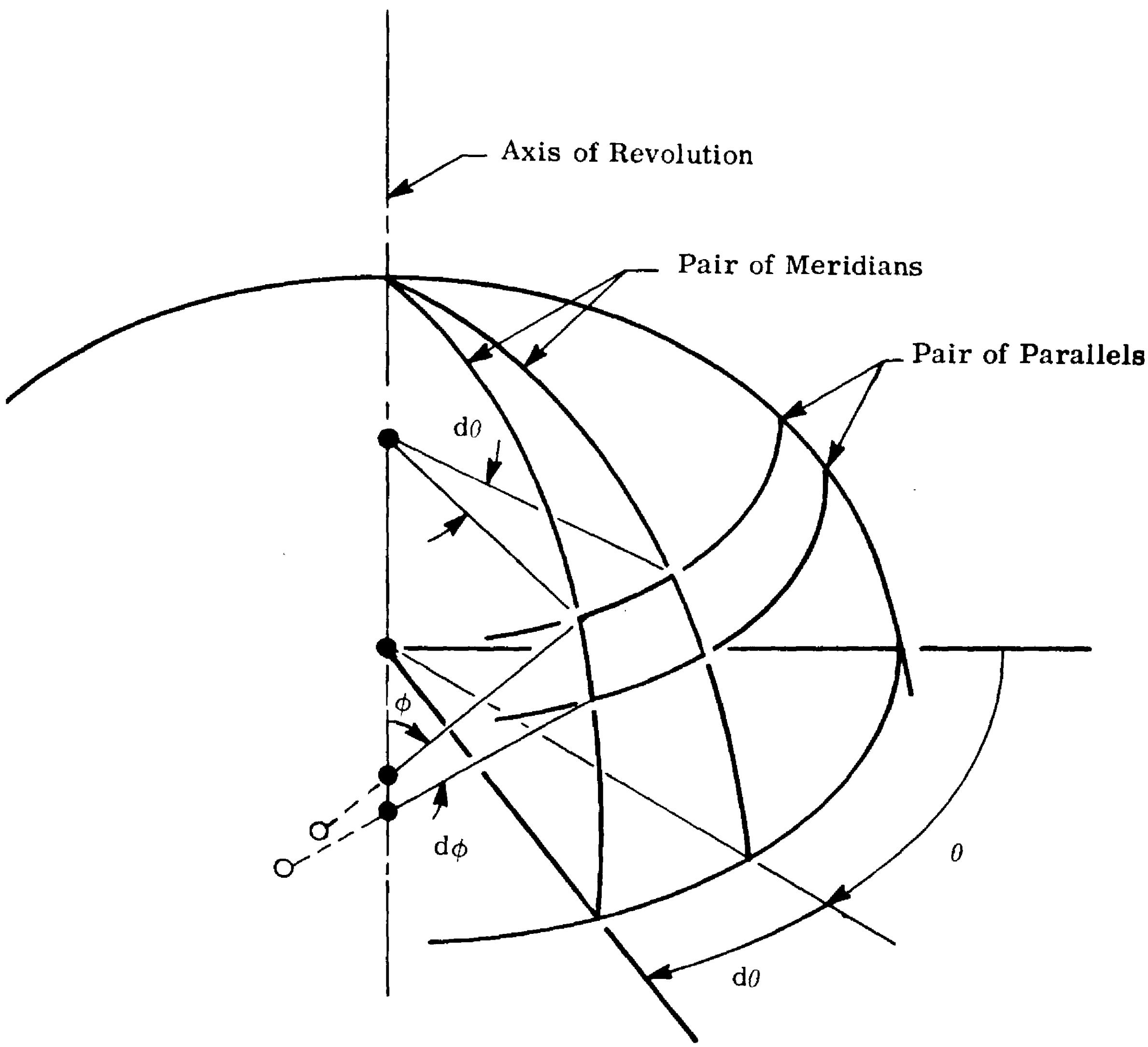


Fig. B7.1.1 - 2. Shell Element, Middle Surface

When the thickness of the shell is considered for analyzing the internal stresses, it becomes apparent that the radius of curvature "R" cannot be a principal radius of curvature; e.g., it is not normal to the shell surface (except when $R = R_2$ in the special case of circular cylinders). Henceforth, R_2 will be used as the principal radius of curvature of an element in the parallel direction. (See Figure B7.1.1 - 3.) The error introduced by this assumption will be negligible in all calculations. Note that R_1 is a principal radius of curvature in the meridional direction.

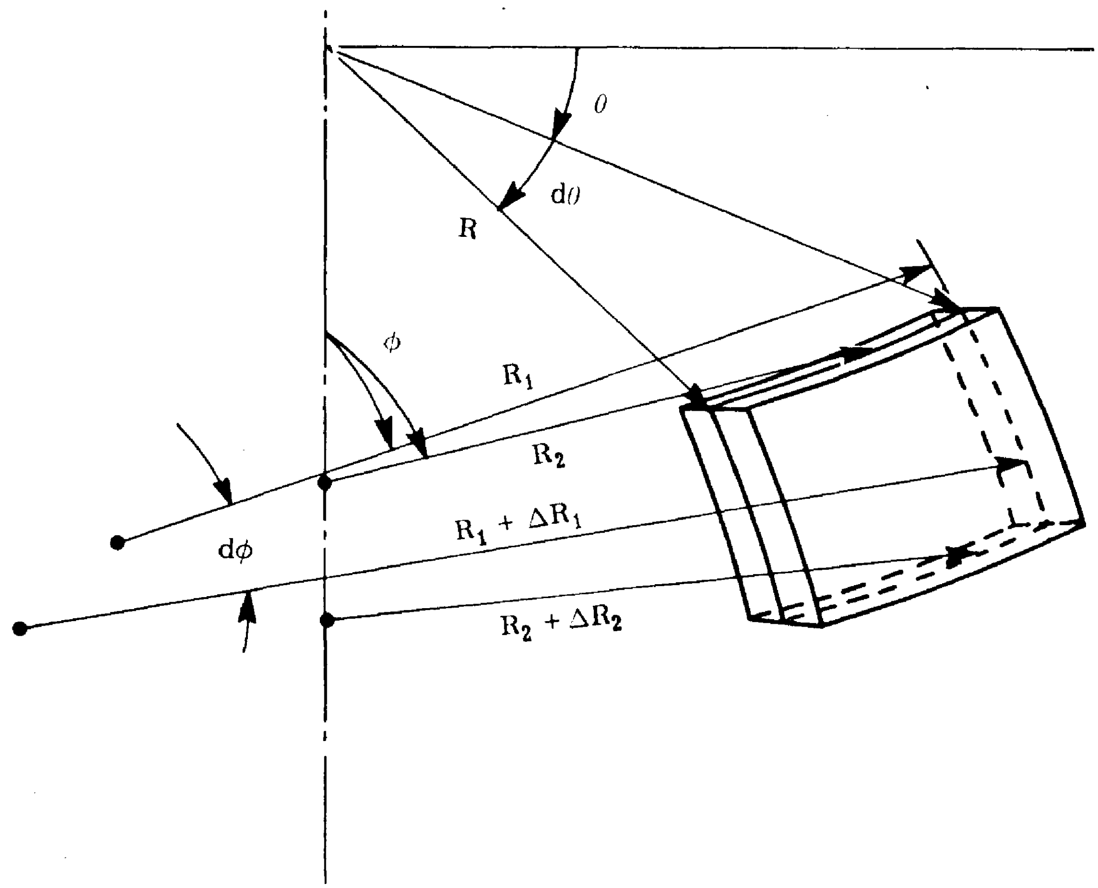


Fig. B7.1.1 - 3. Principal Radii of Curvature

Any element of a shell may have the usual internal stresses acting on the faces of the element. These stresses are indicated in Figure B7.1.1 - 4. For the analytical work that will follow, it is convenient to convert these stresses into the resulting forces and moments acting on the middle surface. In the section θ = constant (Figure B7.1.1 - 4), the total force normal to this section is by definition $N_\theta \, ds_\phi$. It is the resultant of σ_θ stresses acting on this area.

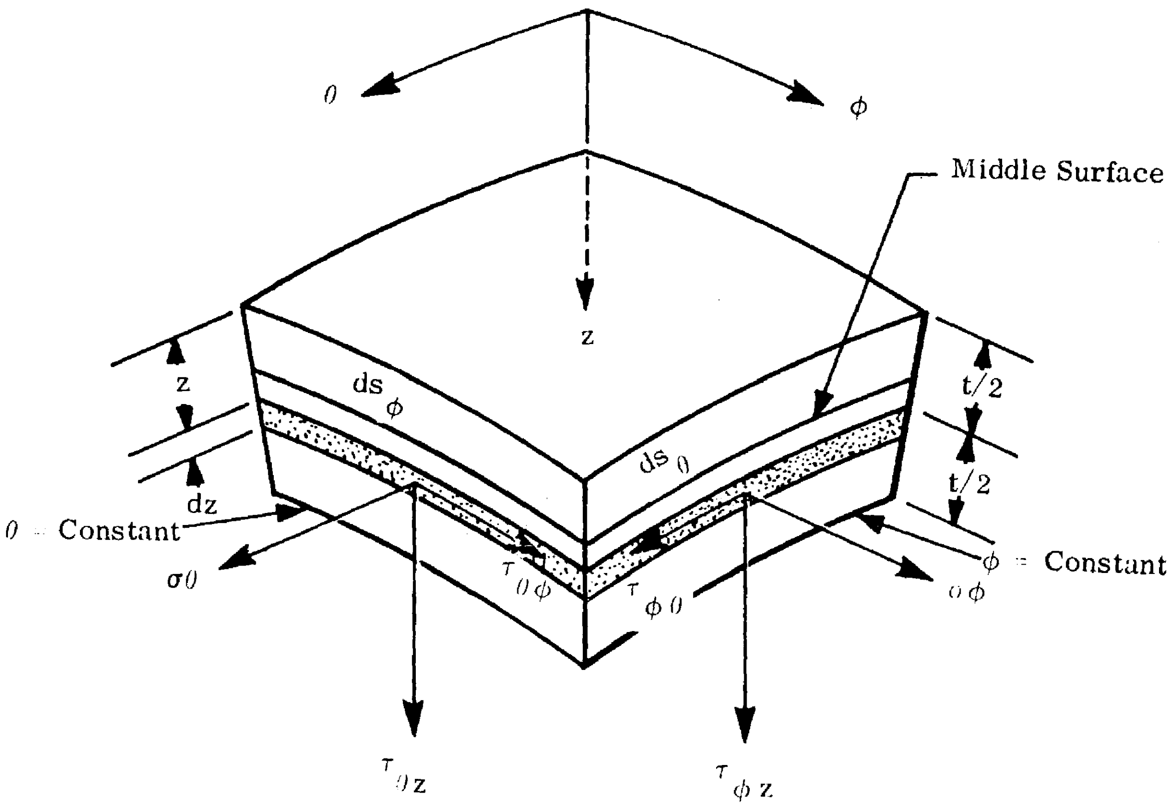


Fig. B7.1.1 - 4. Shell Stresses

Because of the curvature of the shell, its width is not simply ds_ϕ, but

$ds_\phi \dfrac{(R_1 - z)}{R_1}$, and the force transmitted through it is $\sigma_\theta \, ds_\phi \left(1 - \dfrac{z}{R_1}\right) dz$.

The total normal force for the element ds_ϕ t is found by integrating from $-\frac{t}{2}$ to $\frac{t}{2}$.

$$N_\theta ds_\phi = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \sigma_\theta ds_\phi \left(1 - \frac{z}{R_1}\right) dz \quad .$$

When ds_ϕ is dropped from both sides, we have the resulting normal force related to the normal stress. In a like manner, $\tau_{\theta\phi}$ and $\tau_{\theta z}$ must be integrated to obtain $N_{\theta\phi}$ and Q_θ. Altogether, we have

$$N_\theta = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \sigma_\theta \left(1 - \frac{z}{R_1}\right) dz$$

$$N_{\theta\phi} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \tau_{\theta\phi} \left(1 - \frac{z}{R_1}\right) dz$$

$$Q_\theta = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \tau_{\theta z} \left(1 - \frac{z}{R_1}\right) dz \quad .$$

Applying the same reasoning to the section ϕ = constant, we have

$$N_\phi = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \sigma_\phi \left(1 - \frac{z}{R_2}\right) dz$$

$$N_{\phi\theta} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \tau_{\phi\theta} \left(1 - \frac{z}{R_2}\right) dz$$

and

$$Q_{\phi} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \tau_{\phi z} \left(1 - \frac{z}{R_2}\right) dz \quad .$$

Note the different radii of curvature for sections θ = constant and ϕ = constant. (Refer to Figure B7.1.1 - 2.)

If the stresses are not distributed uniformly across the thickness, bending and twisting moments may result. From θ = constant (Figure B7.1.1 - 3), the bending moment is

$$M_{\theta} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \sigma_{\theta} \left(1 - \frac{z}{R_1}\right) z dz$$

and the twisting moment is

$$M_{\theta\phi} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \tau_{\theta\phi} \left(1 - \frac{z}{R_1}\right) z dz \quad .$$

In like manner, when ϕ = constant,

$$M_\phi = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \sigma_\phi \left(1 - \frac{z}{R_2}\right) zdz \quad \text{and} \quad M_{\phi\theta} = \int_{-\frac{t}{2}}^{+\frac{t}{2}} \tau_{\phi\theta} \left(1 - \frac{z}{R_2}\right) zdz .$$

N_θ, N_ϕ, $N_{\theta\phi}$, $N_{\phi\theta}$, Q_θ, Q_ϕ, M_θ, M_ϕ, $M_{\theta\phi}$, and $M_{\phi\theta}$ describe the forces and moments acting on the sides of a rectangular shell element. The fact that the shell element is not necessarily rectangular will be considered when writing the equations of equilibrium in Section B7.1.1.4. Since these ten quantities are all results of stresses, a common name for the group as a whole is "stress resultants." Figure B7.1.1 - 5 shows these stress resultants acting on the middle surface of the shell element. According to membrane theory being considered in the chapter, resultant moments and resultant transverse shearing forces cannot exist. Also, in the assumption of thin shell theory, the quantities $\frac{z}{R_1}$ and $\frac{z}{R_2}$ are very small compared to unity; thus, the only unknowns are the three quantities N_θ, N_ϕ, and $N_{\theta\phi} = N_{\phi\theta}$. Three equilibrium equations can be written for these three unknowns; hence, the problem becomes statically determinate if the forces acting on the shell are known.

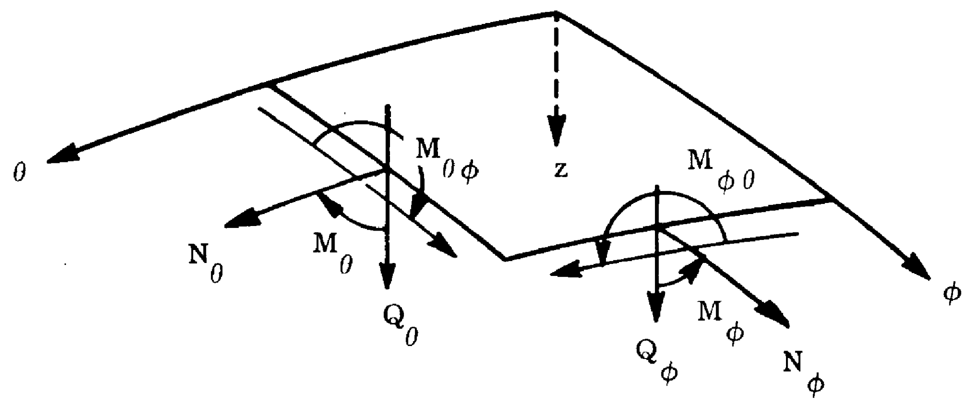


Fig. B7.1.1 - 5. Stress Resultants

B7.1.1.1 NOTATIONS

ϕ	Angle in vertical plane (measured from axis of rotation) defining the location of a point on the meridian
θ	Angle in horizontal plane that controls the location of a point on the shell
R	Radius of a point on the shell measured perpendicular to axis of rotation
R_1	Radius of curvature of meridian at any point
R_2	Radial distance between point on the shell and the axis of rotation
R_0	Radius of curvature when $\phi = 0$
t	Shell thickness
z	Coordinate in direction of surface normal
σ_ϕ, σ_θ	Internal normal stresses
$\tau_{\theta z}$, $\tau_{\theta\phi}$, $\tau_{\phi\theta}$	Inplane shear stresses
N_θ	Circumferential inplane force per unit length at $\theta =$ constant.
$N_{\theta\phi}$	Shear per unit length acting at $\theta =$ constant
Q_θ	Transverse shear at $\theta =$ constant
N_ϕ	Meridional inplane force per unit length at $\phi =$ constant
$N_{\phi\theta}$	Shear per unit length acting at $\phi =$ constant
Q_ϕ	Transverse shear at $\phi =$ constant

M_θ	Bending moment per unit length at section $\theta =$ constant
$M_{\theta\phi}$	Twisting moment per unit length at section $\theta =$ constant
M_ϕ	Bending moment per unit length at section $\phi =$ constant
$M_{\phi\theta}$	Twisting moment per unit length at section $\phi =$ constant
P_z, P_θ, P_ϕ	Loading components in radial, circumferential, and meridional directions, respectively
P	Vertical load
C	Constant of integration
ϕ_0	Angle defining opening in shell of revolution
u	Displacement in the direction of the tangent to the meridian
$\bar{u}$	Displacement in the vertical direction
v	Displacement in the direction tangent to parallel
w	Displacement in the direction normal to surface
$\bar{w}$	Displacement in the horizontal direction
E	Young's modulus
μ	Poisson's ratio
ϵ_θ	Strain component in circumferential direction
ϵ_ϕ	Strain component in meridional direction
a	Radius of sphere or major axis length of ellipsoid

ρ	Specific weight of liquid
h	Height of liquid head
b	Minor axis length of ellipsoid
n	Constant defining the shape of a Cassini dome
x	Coordinate along length of cylinder or along generatrix of cone surface
x_0	Distance from apex of cone to upper edge of cone measured along generatrix
α	Cone angle
s	Arc length

B7.1.1.2 SIGN CONVENTIONS

In general, the sign conventions for stresses, displacements, loads, coordinates, etc., are given in the various figures in Section B7.1.1.0. The following is a list of appropriate figures.

Coordinates	Figure B7.1.1 - 1
Stress Resultants	Figure B7.1.1 - 5
Stresses	Figure B7.1.1 - 4
Loads	Figure B7.1.1.4 - 4
Displacements	Figure B7.1.1.4 - 4, Figure B7.1.1.4 - 5

B7.1.1.3 LIMITATIONS OF ANALYSIS

The limitations and assumptions of Section B7.1 are as follows:

1. The analysis is limited to thin shells. A thin shell is usually defined as a shell where the t/R relation can be neglected in comparison to unity. However, this definition is artificial and arbitrary unless those values which are negligible in comparison to unity are defined. For example, if it is assumed that the usual error of five percent is permissible, then the range of thin monocoque shells will generally be dictated by the relation $t/R < 1/20$. The great majority of shells commonly used are in the $1/1000 < t/R < 1/50$ range. This means that they belong to the thin-shell family. If an error of 20 to 30 percent is permissible, the theory of thin shells can be used with caution even when $t/R \leq 1/3$.
2. Flexural strains are zero or negligible compared to direct axial strain.
3. The deflections, rotations, and strains are small. (See Section B7.0 for detailed definition.)
4. The shell is homogeneous, isotropic, and monocoque and is a shell of revolution.
5. It is assumed that Hooke's Law holds (stress is a linear function of strain) and the stresses are within the elastic range.
6. The boundaries of the shell must be free to rotate and to deflect normal to the shell middle surface.

7. Abrupt discontinuities must not be present in shell shape, thickness, elastic constants, or load distribution.
8. Linear elements normal to the unstrained middle surface remain straight during deformation, and their extensions are negligible.
9. Transverse shear strains are zero throughout the thickness.
10. Surface stresses and body forces are negligible.
11. Only nonshallow shells are considered (See Section B7.0.)

B7.1.1.4 EQUATIONS

I GENERAL

The equations presented in this section are for the membrane or primary solution of the shell. The effects of boundary conditions (secondary solutions) not compatible with membrane theory will be treated in Section B7.3 on bending theory. Because the bending and membrane theories give practically the same results except for a strip adjacent to the boundary, the effects of moments and shears near boundaries can be calculated by using bending theory and can be superimposed over the membrane solution. The results thus obtained will be almost identical to those obtained by using the complete, exact bending theory.

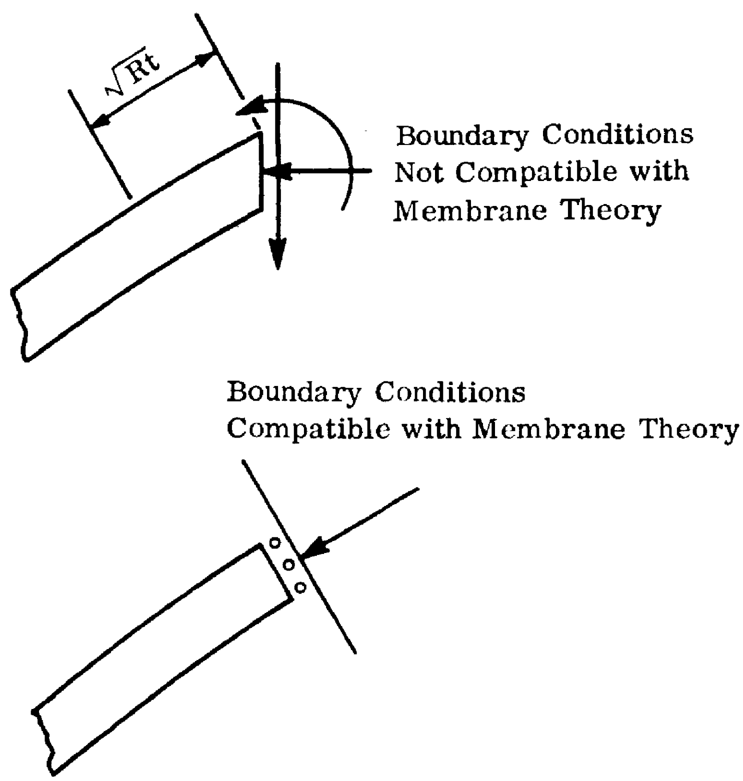


Fig. B7. 1. 1. 4 - 1. Boundary Conditions

B7.1.1.4 EQUATIONS

II EQUILIBRIUM EQUATIONS

The membrane solution is begun by considering the equilibrium of the middle surface of the shell element, cut by two meridians and two parallels (Figure B7.1.1.4 - 2a). The conditions of its equilibrium will furnish three equations in three unknowns, adequate to determine the three unknown stress resultants: the meridional force N_ϕ, the hoop force N_θ, and the shear $N_{\theta\phi} = N_{\phi\theta}$.

Beginning with the forces parallel to a tangent to the meridian, the shear transmitted by one edge of the element is $N_{\theta\phi} R_1 d\phi$, and on the opposite edge it is $\left(N_{\theta\phi} + \frac{\partial N_{\theta\phi}}{\partial \theta} d\theta \right) R_1 d\phi$. Only their difference, $\frac{\partial N_{\theta\phi}}{\partial \theta} R_1 d\theta d\phi$, enters the equilibrium condition. In the same way, we have the difference in the two meridional forces. Bearing in mind that both the force N_ϕ and the length $R d\theta$ vary with ϕ, we have $\frac{\partial}{\partial \phi}(R N_\phi) d\phi d\theta$. The hoop forces also contribute. The two forces $N_\theta R_1 d\phi$ on either side of the element lie in the plane of a parallel circle where they include an angle $d\theta$. They, therefore, have resultant force $N_\theta R_1 d\phi d\theta$ situated in that plane and pointing towards the axis of the shell. Resolving this force into normal and tangential components shows that $N_\theta R_1 d\phi\, d\theta \cos\phi$ (Figure B7.1.1.4 - 2b) enters the condition of equilibrium. Finally, considering the component of some external force, $P_\phi R R_1 d\theta d\phi$, the equilibrium equation reads:

$$\frac{\partial N_{\theta\phi}}{\partial \theta} R_1 d\theta d\phi \quad \frac{\partial}{\partial \phi}(R N_\phi) d\phi\, d\theta - N_\theta R_1 d\phi\, d\theta \cos\phi + P_\phi R R_1 d\theta d\phi = 0 .$$

Noting that all terms contain $d\theta d\phi$ gives:

$$\frac{\partial}{\partial \phi}(RN_{\phi}) + R_1 \frac{\partial N_{\theta\phi}}{\partial \theta} - R_1 N_{\theta} \cos\phi + P_{\phi} RR_1 = 0 \quad . \qquad (1)$$

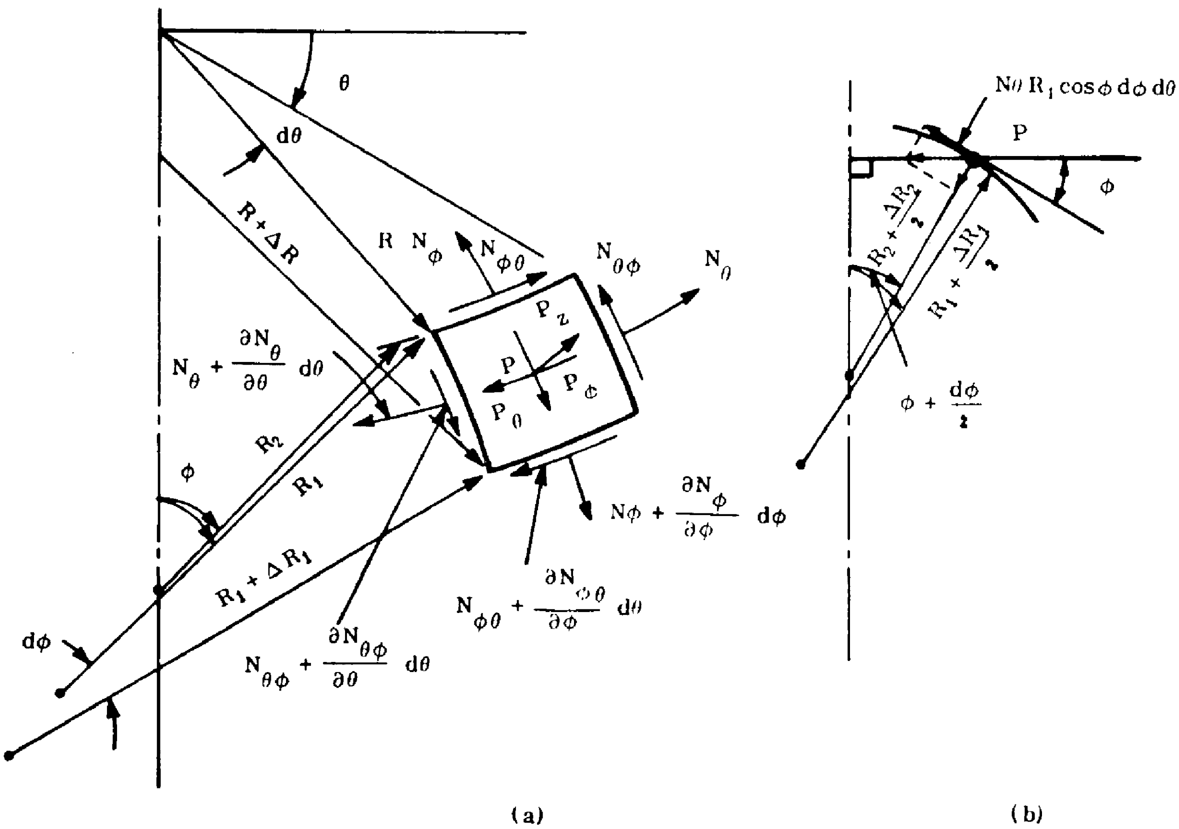


Fig. B7.1.1.4 - 2. Equilibrium of Shell Element

By similar reasoning, we obtain an equation for the forces in the direction of the tangent to a parallel circle.

$$\frac{\partial}{\partial \phi}(RN_{\phi\theta}) + R_1\frac{\partial N_\theta}{\partial \theta} + R_1N_{\theta\phi} \cos \phi + P_\theta RR_1 = 0 \quad . \tag{2}$$

The third equation is derived from forces perpendicular to the middle surface of the shell.

$$N_\theta R_1 \sin\phi + N_\phi R - P_z RR_1 = 0 \quad .$$

Dividing by RR_1 and using the geometric relation $R = R_2 \sin \phi$, we arrive at the third equation of equilibrium.

$$\frac{N_\phi}{R_1} + \frac{N_\theta}{R_2} = P_z \quad . \tag{3}$$

The problem of determining stresses under unsymmetrical loading reduces to the solution of equations (1), (2), and (3) for given values of the load P_ϕ, P_θ, and P_z.

However, it was stated previously that only axisymmetric loading would be considered in this section. For this type of loading, the stresses are independent of θ and $N_{\theta\phi} = N_{\phi\theta} = 0$. Therefore, the equations of equilibrium reduce to:

$$\frac{d}{d\phi}(RN_\phi) - R_1N_\theta \cos \phi = -P_\phi RR_1 \tag{4}$$

$$\frac{N_\phi}{R_1} + \frac{N_\theta}{R_2} = P_z \quad . \tag{3}$$

B7.1.1.4 EQUATIONS

III STRESS RESULTANTS

By solving equation (3) for N_θ and substituting the results into equation (4), we obtain a first order differential equation for N_ϕ that may be solved by integration. N_θ can then be obtained by equation (3).

$$N_\phi = \frac{1}{R_2 \sin^2\phi} \left[\int R_1 R_2 (P_z \cos\phi - P_\phi \sin\phi) \sin\phi \, d\phi + C \right]$$

$$N_\theta = R_2 \left(P_z - \frac{N_\phi}{R_1} \right) .$$

The constant of integration "C" represents the effect of loads applied above a parallel circle $\phi = \phi_0$. $2\pi C$ is the resultant of these forces. If the shell is closed, the loading will degenerate to the concentrated radial force P_z at the vertex of the shell. (See Figure B7.1.1.4 - 3a.)

If the shell has an opening, the angle ϕ_0 defines the opening and the loading (lantern type loading, Figure B7.1.1.4 - 3b) results in the following:

$$N_\phi = - \frac{P}{2\pi R_2 \sin^2\phi} , \qquad N_\theta = \frac{P}{2\pi R_1 \sin^2\phi} .$$

These loads may be treated as additive loads because of the loaded opening at the vertex of the shell. Bending stresses will be introduced at ϕ_0 but will tend to dissipate rapidly with increasing ϕ .

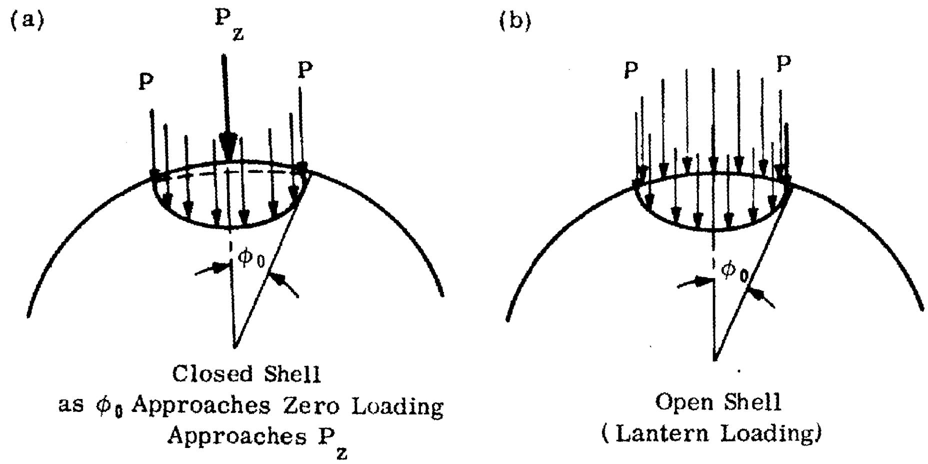


Fig. B7.1.1.4 - 3. Loading above $\phi = \phi_0$

B7.1.1.4 EQUATIONS

IV STRESS, STRAIN, AND DISPLACEMENT

Once the stress resultants, N_ϕ and N_θ, are obtained, stresses, strains, and displacements are readily obtained by the usual methods. For the symmetrically loaded membrane shell, the loading component and displacement in the circumferential direction are zero (Figure B7.1.1.4 - 4).

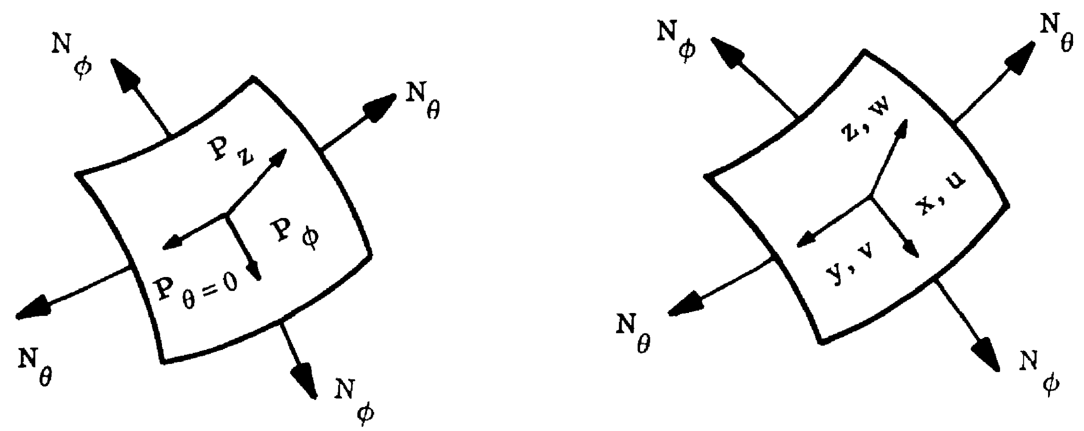


Fig. B7.1.1.4 - 4. Loads and Displacements

P_z = Radial component of loading acting on differential element

P_ϕ = Component of loading acting in X direction (tangential to meridian)

$P_\theta = 0$ = Component of loading acting in Y direction (tangential to parallel)

w = Small displacement of a point in the Z direction (normal to surface)

u = Small displacement in X direction (tangential to meridian)

v = 0 = Displacement in Y direction (tangential to parallel)

Because of assumptions of membrane theory and axisymmetric loading (e.g., $t/R << 1$, all moments $\cong 0$, and all shearing forces $\cong 0$) the normal stresses can be expressed simply as:

$$\sigma_\phi = \frac{N_\phi}{t} \quad , \quad \sigma_\theta = \frac{N_\theta}{t} \quad .$$

The strain components can be found either from σ_ϕ and σ_θ or N_ϕ and N_θ :

$$\epsilon_\phi = \frac{1}{Et}(N_\phi - \mu N_\theta) \quad , \quad \epsilon_\theta = \frac{1}{Et}(N_\theta - \mu N_\phi)$$

where E = Young's modulus

t = Thickness of shell

μ = Poisson's ratio .

The displacement components are computed next, thereby completing the solution of the shell problem. The general solution for u is

$$u = \sin\phi \left[\int \frac{f(\phi)}{\sin\phi} d\phi + C\right]$$

where C is a constant of integration to be determined from support conditions and

$$f(\phi) = R_1\epsilon_\phi - R_2\epsilon_\theta = \frac{1}{Et}\left[R_1(N_\phi - \mu N_\theta) - R_2(N_\theta - \mu N_\phi)\right]$$
$$= \frac{1}{Et}\left[N_\phi(R_1 + \mu R_2) - N_\theta(R_2 + \mu R_1)\right] \quad .$$

The displacement w can then be found from the equation

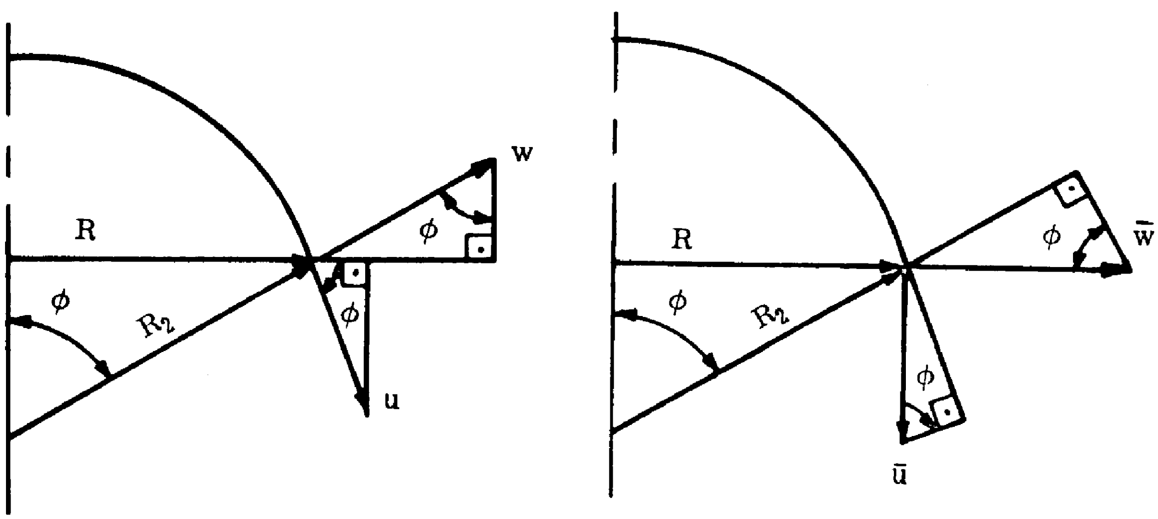
$$w = u\cot\phi - R_2\epsilon_\theta \quad .$$

Because the interaction process of two or more shells is often required, displacements are often calculated in terms of $\bar{u}$ and $\bar{w}$, the vertical and horizontal displacements. (See Figure B7.1.1.4 - 5.) Note that when $R_2 = R$, $u = \bar{u}$ and $w = \bar{w}$. The displacement $\bar{u}$ can be found in a manner similar to the solution for u. The general solution for $\bar{u}$ is

$$\bar{u} = R\epsilon_\theta \cot\phi - \int \frac{f(\phi)}{\sin\phi} d\phi + C$$

where $f(\phi) = R_1 \epsilon_\phi - R_2 \epsilon_\theta$ and C is again a constant of integration determined from support conditions. The horizontal displacement is simply

$$\bar{w} = R\epsilon_\theta \quad .$$



(u and w known)

$$\bar{w} = w\sin\phi + u\cos\phi$$
$$\bar{u} = -w\cos\phi + u\sin\phi$$

($\bar{u}$ and $\bar{w}$ known)

$$w = \bar{w}\sin\phi - \bar{u}\cos\phi$$
$$u = \bar{w}\cos\phi + \bar{u}\sin\phi$$

Fig. B7.1.1.4 - 5. Displacements

These formulas (Figure B7.1.1.4 - 5) can be used to convert from one form of displacement to the other, depending on the given solution and the requirements of the user.

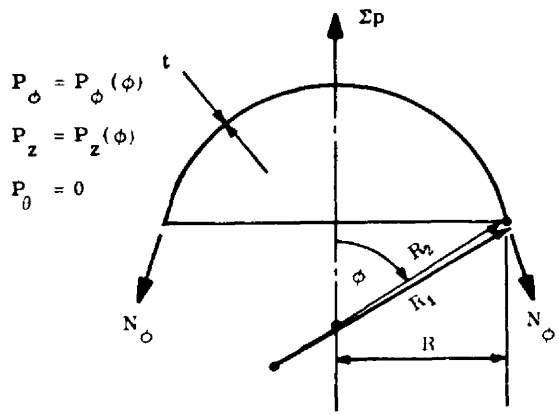
B7.1.1.4 EQUATIONS

V SUMMARY

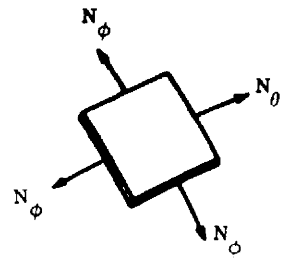
Application of the solutions presented in this section can be classified conveniently for the two following cases: general shell of revolution with general axisymmetric load distribution, and general shell of revolution subjected to uniform pressure. Table B7.1.1.4 - 1 presents a summary of solutions for these general cases.

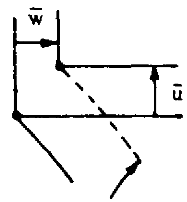
The remainder of this section presents practical shell of revolution problems with various types of axisymmetric loading. Based on the various shell geometries, solutions for N_ϕ and N_θ are presented with the force-displacement relationships. The stresses can be calculated directly using the equations in Section B7.1.1.4 - IV.

Table B7.1.1.4 - 1. Summary of Equations, Axisymmetrically Loaded Shells of Revolution, Linear Membrane Theory

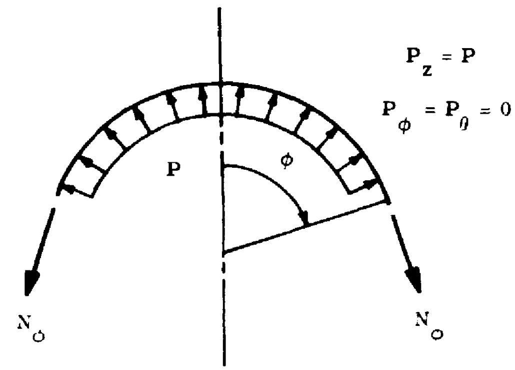


General Shell of Revolution
General Axisymmetric Loading





Deflected Position



General Shell of Revolution
Uniform Pressure

N_ϕ =	$\frac{1}{R_2 \sin^2\phi}\left[\int R_1 R_2 (P_z \cos\phi - P_\phi \sin\phi) \sin\phi \, d\phi + C\right]$	N_ϕ	$\frac{R_2 P}{2}$
N_θ =	$R_2\left(P_z - \frac{N_\phi}{R_1}\right)$	N_θ	$\frac{R_2 P}{2}\left(2 - \frac{R_2}{R_1}\right)$
$\sigma_\phi, \sigma_\theta$ =	$\frac{N_\phi}{t}, \frac{N_\theta}{t}$	$\sigma_\phi, \sigma_\theta$	$\frac{N_\phi}{t}, \frac{N_\theta}{t}$
$\epsilon_\phi, \epsilon_\theta$ =	$\frac{1}{Et}(N_\phi - \mu N_\theta), \frac{1}{Et}(N_\theta - \mu N_\phi)$	$\epsilon_\phi, \epsilon_\theta$	$\frac{1}{Et}(N_\phi - \mu N_\theta), \frac{1}{Et}(N_\theta - \mu N_\phi)$
$\bar{w}$ =	$R\epsilon_\theta$	$\bar{w}$	$R\epsilon_\theta$
$\bar{u}$ =	$\bar{w}\cot\phi - \int \frac{R_1\epsilon_\phi - R_2\epsilon_\theta}{\sin\phi} d\phi + C$	$\bar{u}$	$\bar{w}\cot\phi - \int \frac{R_1\epsilon_\phi - R_2\epsilon_\theta}{\sin\phi} d\phi + C$