SECTION B4.8

SHEAR BEAMS

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DEFINITIONS OF SYMBOLS

t thickness of web plate

t_u thickness of attached stiffener leg

b spacing of intermediate stiffeners, or width of unstiffened web plate

b_c clear web plate distance between stiffeners

d_c clear depth of web plate

α_e effective aspect ratio;

$= b/d_c$ for unstiffened web plates, or web plates reinforced by single-sided stiffeners;

$= b_c/d_c$ for web plates reinforced by double-sided stiffeners

D flexural rigidity of unit width of plate $= Et^3/12(1-u^2)$

E Young's modulus

μ Poisson's ratio

I moment of inertia of stiffener about base of stiffener (next to web)

$\gamma = \dfrac{EI}{Db}$

γ_L limiting value of γ

K_s critical shear-stress coefficient

K_L limiting value of K_s

η_s, η_B plasticity coefficients which account for the reduction of modulus of elasticity for stress above the elastic limit; within the elastic range, $\eta = 1$

A_f area of tension or compression flange

DEFINITIONS OF SYMBOLS (Continued)

C_r rivet factor $= \dfrac{p - D_r}{p}$

D_r rivet diameter

f_b applied bending stress

f_s applied web shear stress

$F_{s_{cr}}$ critical (or initial) buckling stress in shear

M applied bending moment

p rivet spacing

q applied web shear flow

S, V applied transverse shear on beam

A parameter used for type of shear beam selection

h height of beam between centroids of flanges

B_T = EI_T, flexural rigidity of transverse stiffeners

γ_T = B_T/Db, nondimensional flexural rigidity parameter for transverse stiffeners

C_T torsional rigidity of transverse stiffeners

B_L = EI_L, flexural rigidity of longitudinal stiffeners

γ_L = $\dfrac{B_L}{Dd_c}$, nondimensional stiffeners parameter for longitudinal stiffeners

C_L torsional rigidity of longitudinal stiffeners

Γ_L = C_L/Dd_c

DEFINITIONS OF SYMBOLS (Concluded)

Γ_T = C_T/Dd_c

G modulus of rigidity

K_{LH} limiting value of K_s for web reinforced by vertical stiffeners and a central horizontal stiffener

m distance from edge of plate to longitudinal stiffener

$F_{B_{cr}}$ critical (or initial) buckling stress in bending

I_o stiffener moment of inertia for longitudinal stiffener

4.8.0 SHEAR BEAMS

The analysis and design of a metal beam composed of flange members riveted or welded to web members are common problems in aerospace structural design.

Shear beams denote a particularly efficient type of beam. In shear beams the moment resistance is provided by the flanges, which are concentrated near the extreme fibers, and the shear resistance is provided by the thin web connecting the tension and compression caps.

The analysis and design of shear beams as structural components are generally based upon the web response to the applied shear loads. If buckling of the web is inhibited within the design ultimate load, the beam is known as a shear-resistant beam. If, however, the web is allowed to buckle after some application of load causing shear to be resisted in part by tension-field action, the beam is known as a tension-field beam.

The type of shear beam most suitable for a particular design application may depend on many factors. One of the most common factors is based on economy of weight. H. Wagner [1] offers the following criterion, based upon the economy of weight:

$$A = \frac{\sqrt{V}}{h} \quad ,$$

where

V = shear load, lb

and

h = depth of web, in.,

with the recommendation that when $A < 7$ the tension-field web is best, and when $A > 11$ the shear-resistant web is best. When $7 < A < 11$, there is little choice between the two; factors other than weight will then determine the type of web to be used.

The criterion above should not be adhered to rigidly, however, because new data and design techniques have become available that have resulted in reduced weight designs for shear-resistant beams.

Shear-resistant beams and tension-field beams will be discussed in Paragraphs 4.8.1 and 4.8.2 respectively.

4.8.1 PLANE-STIFFENED SHEAR RESISTANT BEAMS

As previously stated, a shear beam whose web is so designed that it does not buckle under the applied loads is referred to as a shear-resistant beam. The analysis of this beam is primarily one of stability. That is, with the exception of the tension flanges, the web, the compression flange, and the stiffeners are all designed from a stability standpoint rather than from a material allowable stress standpoint.

The stability of the shear web can always be increased by increasing its thickness, but such a design will not always be economical with respect to the weight of the material used. A more economical solution is obtained by keeping the thickness of the plate as small as possible (just thick enough to fulfill strength requirements) and increasing the stability by introducing stiffeners. The weight of such stiffeners will usually be less than the additional weight introduced by an adequate increase in the thickness of the plate.

The design criteria of shear-resistant beams may be stated as follows:

1. Local buckling of the web between the stiffeners under combined shear, bending, axial, and transverse stresses must not occur.
2. Elements of the transverse stiffeners must not buckle locally under the transverse stresses.
3. Elements of the flange must not buckle locally under the longitudinal stresses.

If the criteria above are not met, the procedures of analysis that follow are not applicable.

Design analysis techniques for shear resistant beams are given in the following paragraphs.

4.8.1.1 Stability of Web Panel

The critical buckling stress, $F_{s_{cr}}$, of a web panel of height d_c, width b, and thickness t, is given by

$$\frac{F_{s_{cr}}}{\eta_s} = \frac{K_s \pi^2 E}{12(1-\mu^2)} \left(\frac{t}{d_c}\right)^2 \qquad \text{for } d_c \leq b \tag{1.a}$$

or

$$\frac{F_{s_{cr}}}{\eta_s} = \frac{K_s \pi^2 E}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2 \qquad \text{for } b < d_c \quad , \tag{1.b}$$

where K_s is a function of the aspect ratio, d_c/b, and the edge restraint offered by the stiffeners and flanges.

Figure 1 shows how the value of the critical shear stress coefficient, K_s, increases with decreasing values of the aspect ratio, b/d_c ($d_c \leq b$), for a number of different edge conditions. This figure indicates quite clearly why vertical stiffeners are so effective in increasing the buckling stress of rectangular webs.

In view of the importance of obtaining the correct design of intermediate stiffeners, a number of theoretical and experimental investigations have been made to determine the relationship between the size and spacing of intermediate stiffeners, and the buckling stress of the stiffened web plate. These investigations have also included the effects of stiffener torsional rigidity, stiffener thickness and rivet location, central longitudinal stiffness, and single-sided or double-sided stiffness. The procedure for the design and analysis of these effects will be given below.

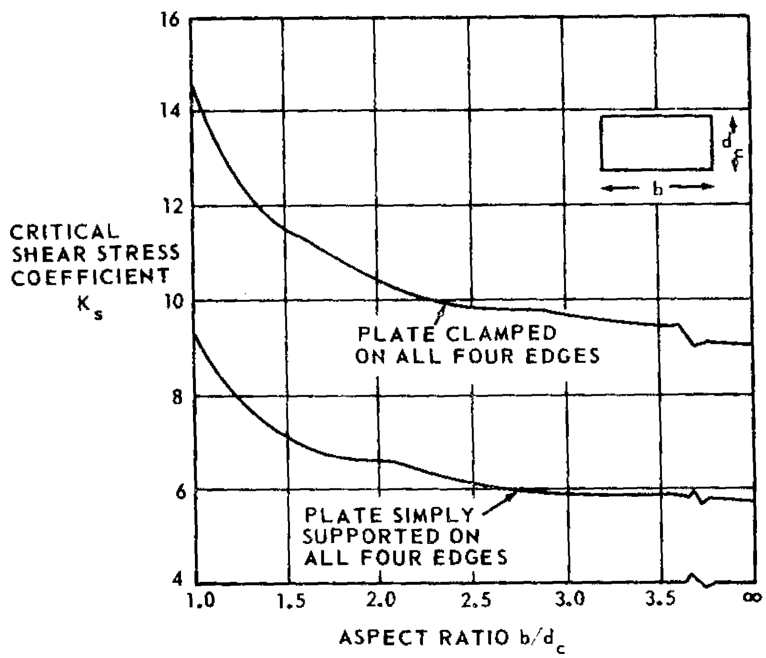


FIGURE 1. K_s VERSUS ASPECT RATIO FOR DIFFERENT EDGE RESTRAINTS

I. Transverse Stiffeners – Flexural Rigidity Only

Theoretical work has been performed [2-4] to ascertain the relationship between K_s and the nondimensional parameter γ $(= \frac{EI}{Db})$ for various aspect ratios and boundary conditions. Figure 2 is a typical plot showing the relationship of these parameters. It will be noted that points of discontinuity occur on the K_s/γ curves. These points of discontinuity denote where changes in the buckle pattern occur. It can also be seen that when a certain value of γ is reached, there is no appreciable increase in K_s. This value of γ is called γ_L or limiting value of γ, since higher values would result in an inefficient design.

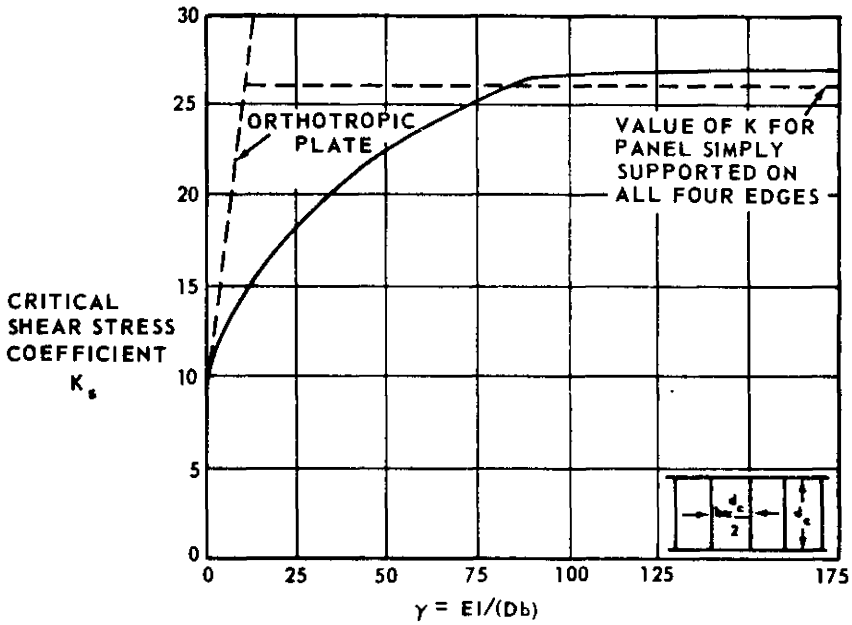


FIGURE 2. THE THEORETICAL K/γ RELATIONSHIPS DERIVED BY STEIN AND FRALICH FOR INFINITELY LONG PLATES SIMPLY SUPPORTED AT THE EDGES AND REINFORCED BY STIFFENERS $0.5d_c$ APART.

For design purposes, the following relationships should be used. However, it should be noted that these relationships are valid only for stiffeners whose thickness is equal to or greater than the thickness of the web plate.

$$\gamma_L = 27.75\,(\alpha_e)^{-2} - 7.5 \text{ for double-sided stiffeners,} \tag{2.a}$$

$$\gamma_L = 21.5\,(\alpha_e)^{-2} - 7.5 \text{ for single-sided stiffeners,} \tag{2.b}$$

$$K_L = 7.0 + 5.6\ (\alpha_e)^{-2} \text{ for both single- and double-sided stiffeners,} \quad (3)$$

where

$$\alpha_e = b_c/d_c \text{ for double-sided stiffeners } (b_c \geq d_c) \text{ and}$$

$$\alpha_e = b/d_c \text{ for single-sided stiffeners } (b \geq d_c) \quad .$$

II. Transverse Stiffeners — Effect of Stiffener Thickness and Rivet Location

Investigations have been carried out to investigate fully the various parameters that affect the behavior of single-sided stiffeners having attached legs thinner than the web-plate [5]. The parameters t_u/t and c/t were studied in these investigations to evaluate their effect on K_s, where t, t_u, and c are as shown in Figure 3. It was shown that the primary influence on K_s was the value c and that the t_u/t variation had little effect on K_s. Figure 4 shows the variation of K_s with the parameter $(t_u/t)(t/c)^{1/2}$. From the figure it can be seen that for the stiffener to provide a value of K_s equal to K_L, it is necessary that

$$(t_u/t)(t/c)^{1/2} \geq 0.27 \ . \quad (4)$$

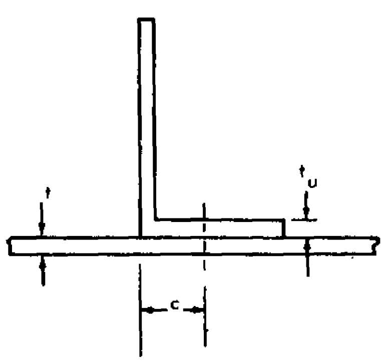


FIGURE 3. VALUES OF t, t_u, AND c

This equation can be used to determine the position of the rivet for fully effective stiffeners for various values of t_u/t. For values of $(t_u/t)(t/c)^{1/2}$ less than 0.27 the value of K_s should be reduced as shown in Figure 4.

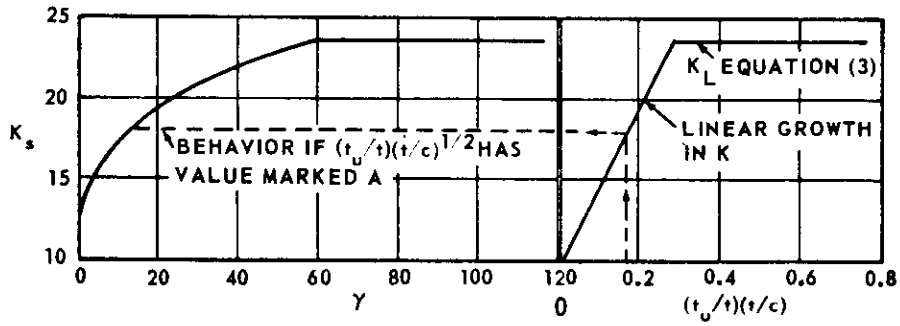


FIGURE 4. VARIATION OF K_s WITH THE STIFFENER THICKNESS AND RIVET LOCATION PARAMETER

III. Transverse Stiffeners — Flexural and Torsional Rigidity

Theoretical results have been obtained [4, 6-8] that provide relationships between K_s and the flexural rigidity of the stiffeners for various values of the ratio of torsional rigidity to flexural rigidity for simply supported or clamped longitudinal edges. It was assumed that the stiffeners were symmetrically disposed about the midplane of the web plate as shown in Figure 5.

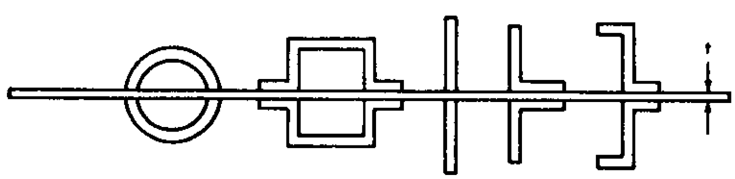


FIGURE 5. VARIOUS SHAPES OF STIFFENERS

Figures 6 and 7 give K_s/γ relationships for b = d and b = d/2 with the longitudinal edges simply supported. Figures 8 and 9 give K_s/γ relationships for b = d and b = d/2 with the longitudinal edges clamped. The maximum value of C_T/B_T plotted is for a closed circular tube. This value is 0.769.

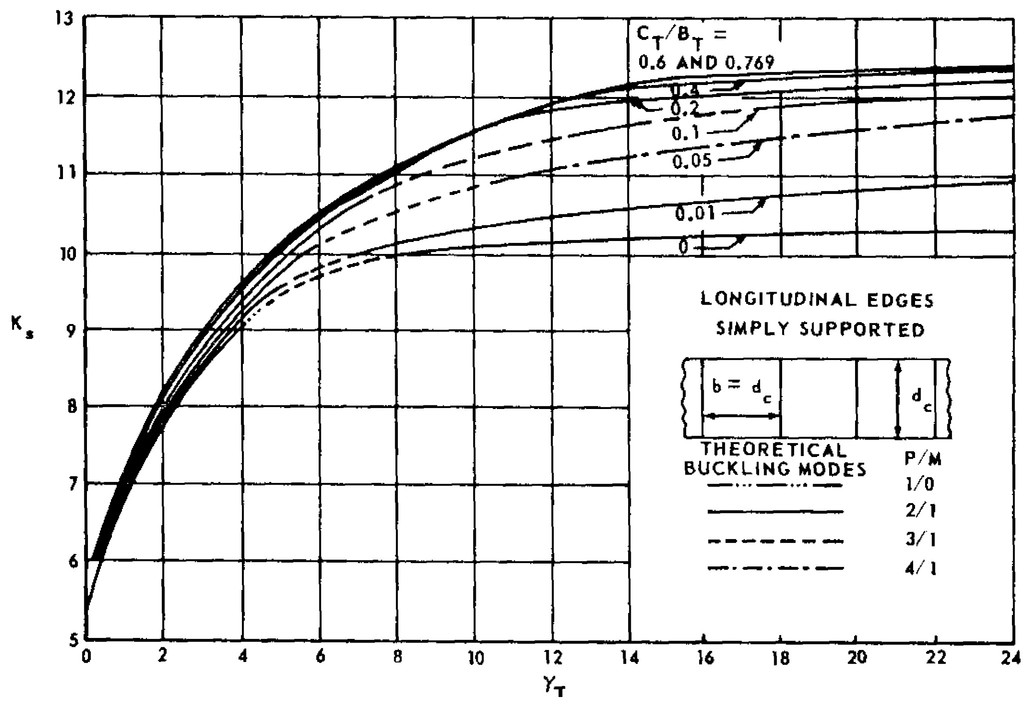


FIGURE 6. K_s VERSUS γ_T RELATIONSHIPS FOR b = d_c

From these figures it is shown that very significant increases in the buckling resistance, K_s, are obtainable by using closed-section stiffeners in place of the open-section stiffeners so frequently used. For example, by using a thin-walled circular tube for the stiffeners (C_T/B_T = 0.769) the gain in K_L

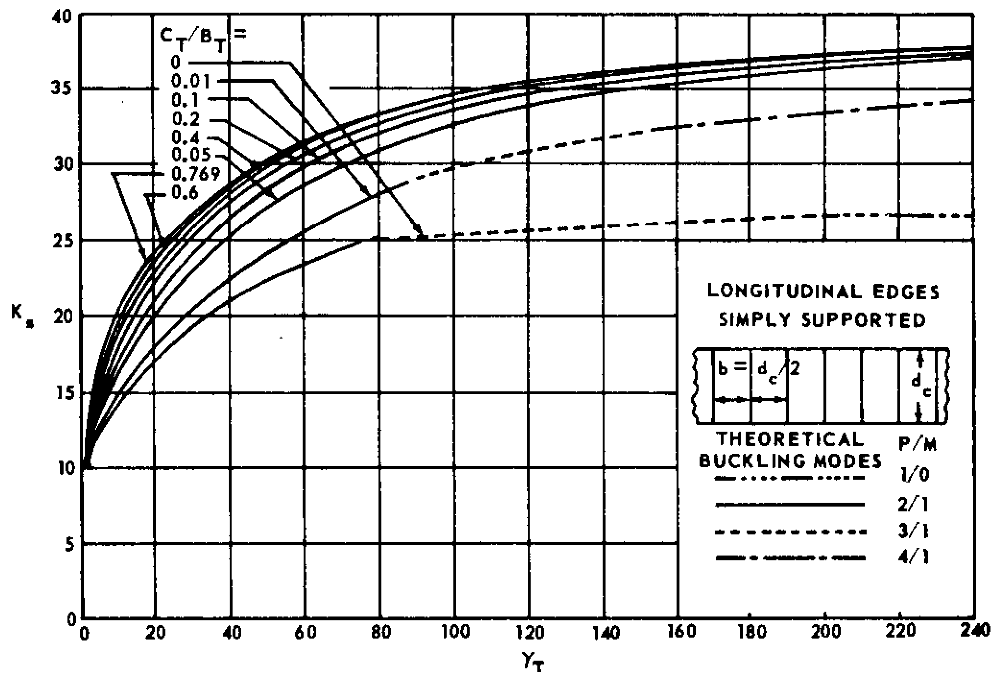


FIGURE 7. K_S VERSUS γ_T RELATIONSHIPS FOR $b = d_c/2$

is 25 percent and 60 percent for $\alpha = 1$ and $\alpha = 2$, respectively, when the longitudinal edges are simply supported. For the case of clamped edges, the gains are 13 percent and 43 percent for $\alpha = 1$ and $\alpha = 2$, respectively. Thus, if a minimum weight design is desired, consideration should be given to closed-section stiffeners.

IV. Transverse and Central Longitudinal Stiffeners

The use of deep beams with webs having a high depth-to-thickness ratio, may make it desirable to employ both vertical and horizontal stiffening. When

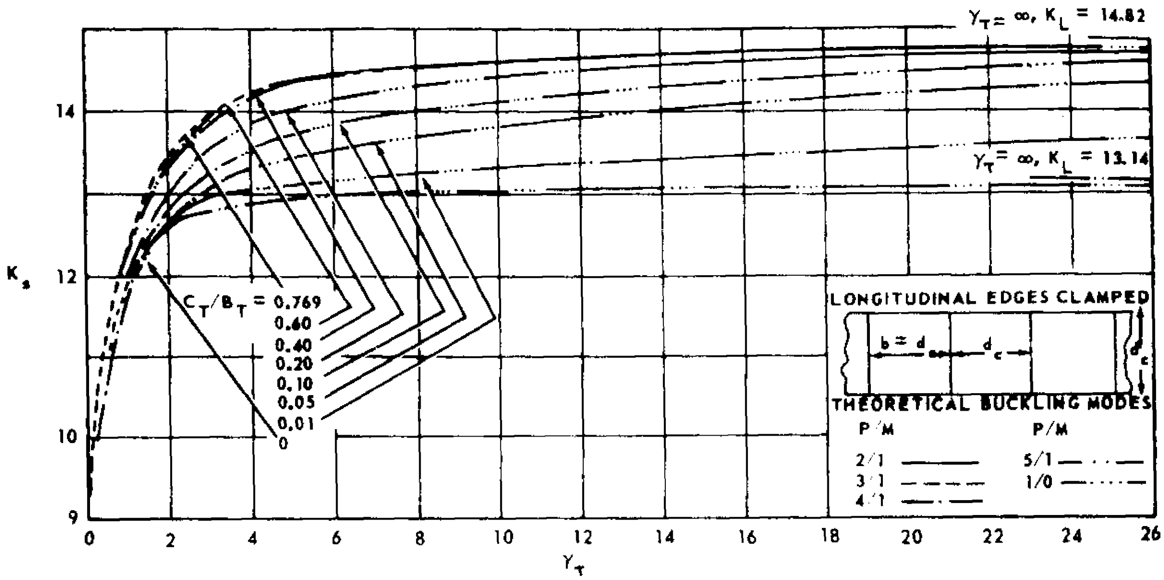


FIGURE 8. K, γ_T RELATIONSHIPS; ASPECT RATIO α = 1.0

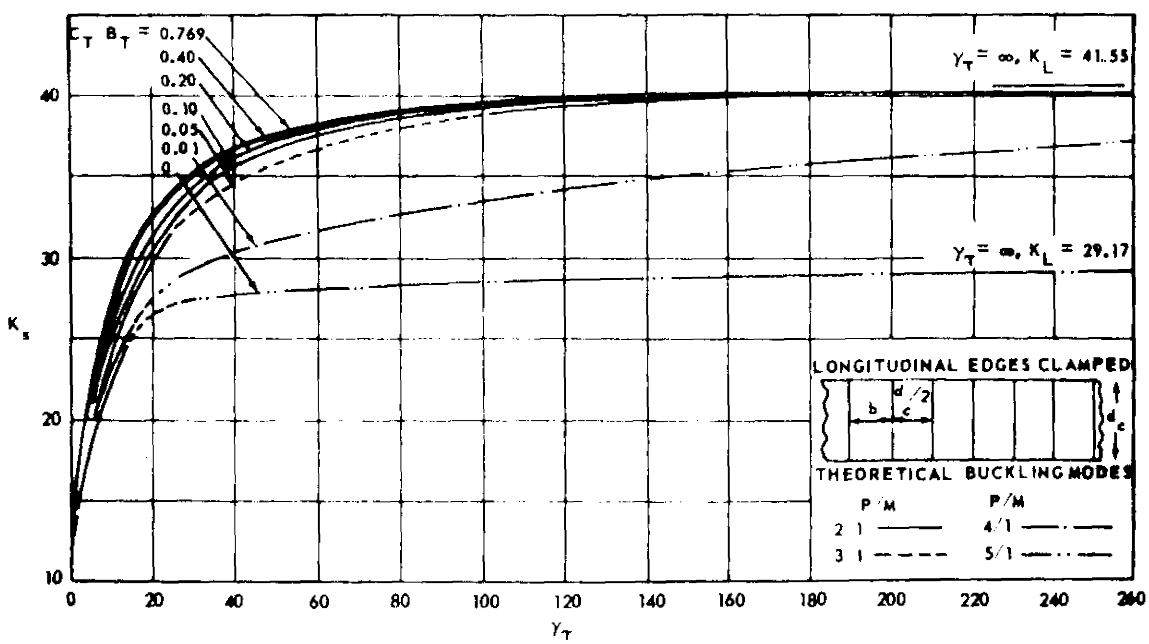


FIGURE 9. K, γ_T RELATIONSHIPS; ASPECT RATIO α – 2.0

a web is subjected to shear, the most effective position for a single horizontal stiffener is at middepth. This combination of vertical and horizontal stiffening can result in more economical designs than are possible when only vertical stiffeners are employed. For example, the weight of stiffening required to achieve a given buckling stress with horizontal and vertical stiffening can be as little as 50 percent of the weight required when only vertical stiffeners are used.

If neither of the transverse or central longitudinal stiffeners has torsional rigidity and the vertical stiffeners have a rigidity equal to or greater than EI_{LV}, then the value of γ_{LH} necessary to produce the limiting value of K_{LH} is given by

$$\gamma_{LH} = 11.25\ (b/d_c)^2 \tag{5}$$

and

$$K_{LH} = 29 + 4.5(b/d_c)^{-2} \quad . \tag{6}$$

Additional weight savings can be achieved if torsionally strong stiffeners are used in either the transverse or longitudinal direction. For example, studies in Reference 6 have shown gains in the value of K_L (which is proportional to the weight of the web) of up to 25 and 60 percent for α equal to one and two, respectively, by using closed-section stiffeners in the transverse direction only. Investigators have given parameter studies on the following in References 4 and 9:

1. Transverse and longitudinal stiffeners of closed tubular cross section.
2. Transverse stiffeners of closed tubular cross section; longitudinal stiffener possessing only flexural rigidity.

3. Transverse stiffeners possessing only flexural rigidity, the longitudinal stiffeners being of closed tubular cross section.

Figure 10 enables one to make an assessment of the benefits that result from using torsionally strong stiffeners when a central longitudinal stiffener is used in conjunction with a system of equally spaced transverse stiffeners. When $\alpha \leq 1$, it is evident that little increase is obtained in the buckling resistance by using torsionally strong transverse stiffeners, but that a considerable increase in buckling resistance will be obtained by using a torsionally strong longitudinal stiffener. However, as α increases (that is, as the transverse stiffeners are

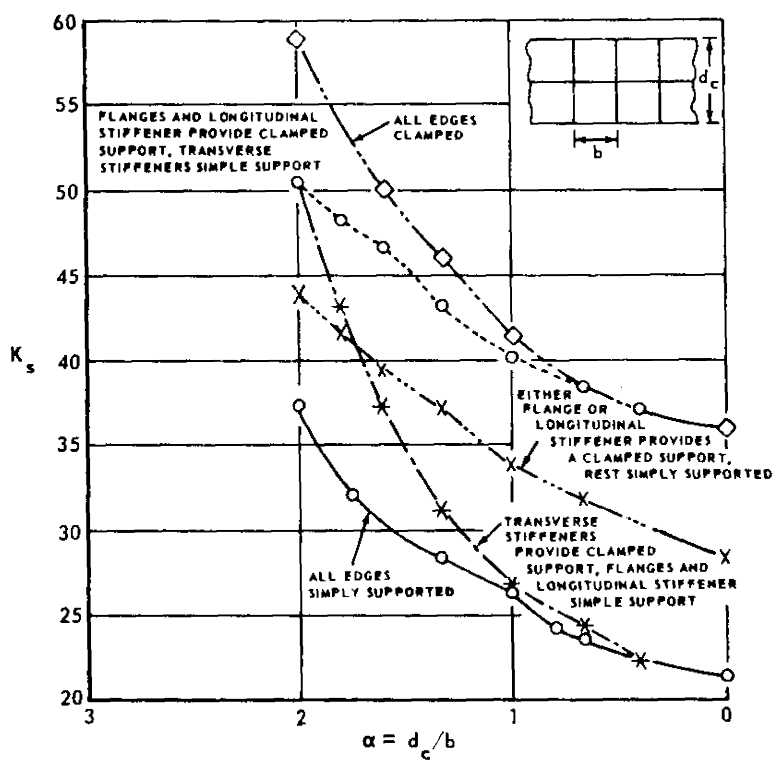


FIGURE 10. K_S VERSUS ASPECT RATIO FOR VARIOUS EDGE RESTRAINTS

more closely spaced), the increase in K resulting from the use of torsionally strong stiffeners becomes more significant; until, when $\alpha = 1.7$, the buckling resistance obtained with torsionally strong transverse stiffeners and a longitudinal stiffener without torsional rigidity is equal to that obtained with a torsionally strong longitudinal stiffener and transverse stiffeners possessing only flexural rigidity. Thus, it will be readily seen that is is necessary to know the relationships between K_S and the stiffener properties for the cases enumerated above. Figures 11, 12, and 13 give typical relationships between K_S and γ_T for b = d and different positions of the torsionally strong stiffener. Curves for other values of the aspect ratio can be obtained from Reference 4. The points

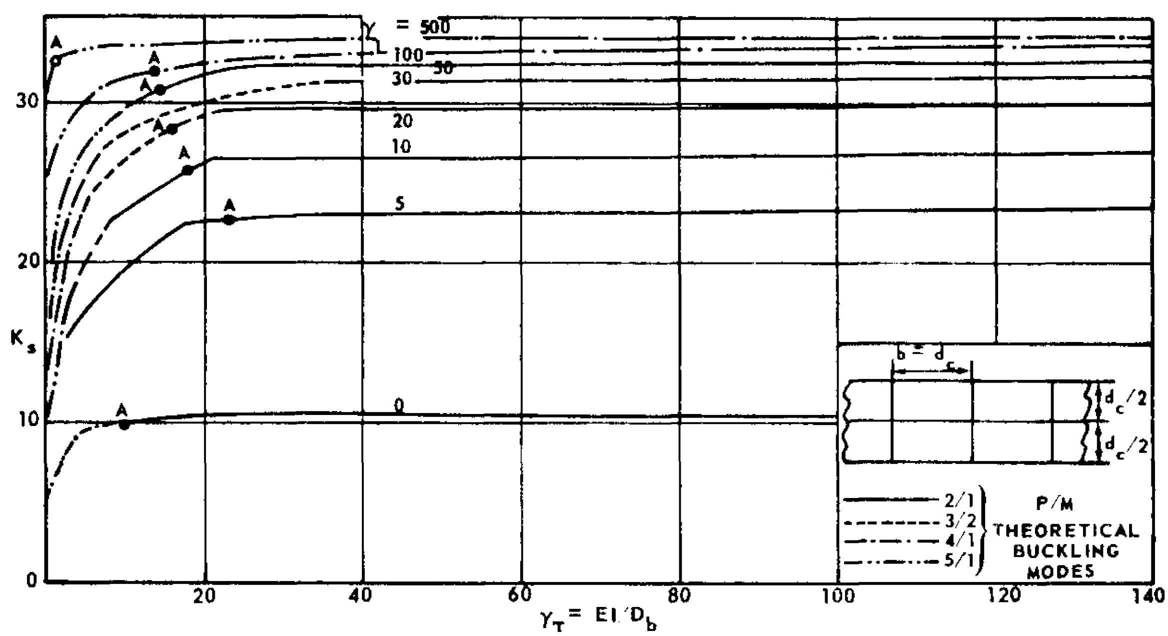


FIGURE 11. K_S, γ_T RELATIONSHIPS; LONGITUDINAL STIFFENER WITH TORSION ($C_L = 0.769\ B_L$); TRANSVERSE STIFFENERS WITH ONLY FLEXURAL RIGIDITY ($C_T = 0$); ASPECT RATIO $\alpha = 1.0$

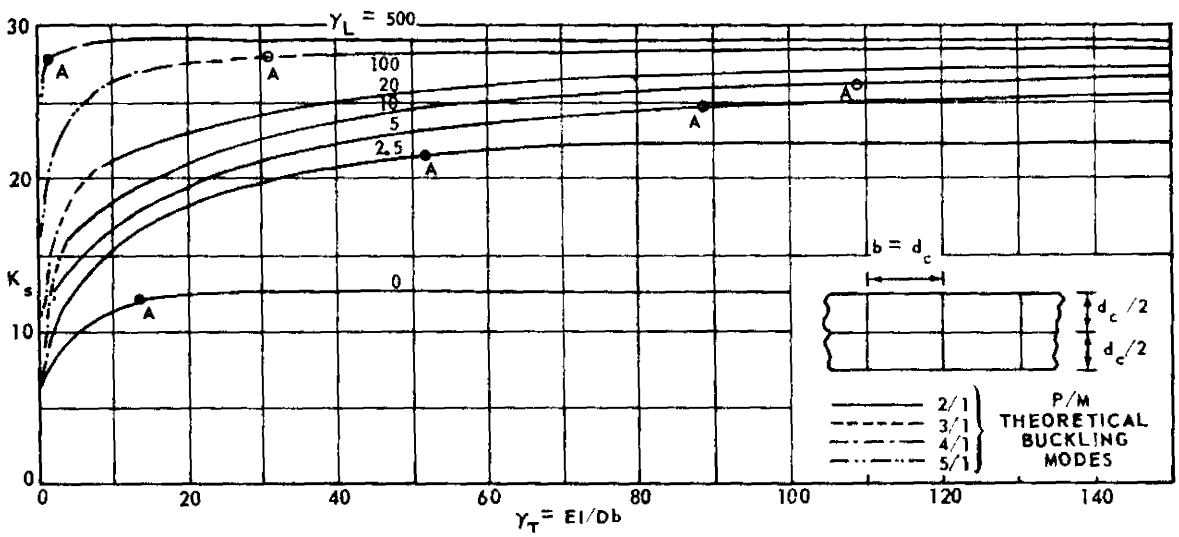


FIGURE 12. K_S, γ_T RELATIONSHIPS; LONGITUDINAL EDGES SIMPLY SUPPORTED; TRANSVERSE STIFFENERS WITH TORSIONAL RIGIDITY (C_T = 0.769 B_T); LONGITUDINAL STIFFENERS WITH ONLY FLEXURAL RIGIDITY (C_L = 0); ASPECT RATIO α = 1.0

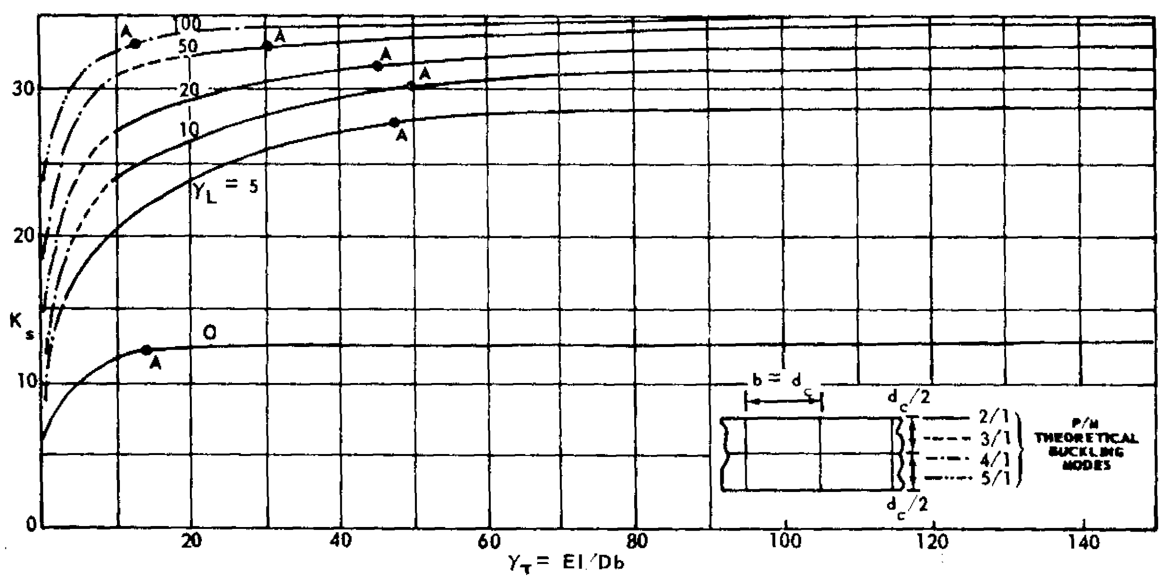


FIGURE 13. K_S, γ_T RELATIONSHIPS; SIMPLY SUPPORTED LONGITUDINAL EDGES; ALL STIFFENERS WITH TORSIONAL RIGIDITY (C_T = 0.769 B_T, C_L = 0.769 B_L); ASPECT RATIO α = 1.0

marked A on the curves of Figures 11, 12, and 13 are values of γ_T which would give 95 percent of the limiting value of K_s. This is suggested as a good cutoff point for an efficient design. If greater values of γ_T are chosen, only a small increase in the value of K_s would occur. Thus, the extra increase in γ_T would not be very beneficial from a weight standpoint.

V. Longitudinally Stiffened Web Plates in Longitudinal Compression

In deep beams, it is often economical to stiffen the web plate by longitudinal stiffeners in locations where the longitudinal compressive stresses resulting from bending are high. Two positions of the stiffener will be considered here: (1) The stiffener located at the longitudinal center line of the web, that is, at the neutral axis (Fig. 14a) and (2) the stiffener located in the compressive region at a distance from the edge of the plate (Fig. 14b). In case 1, the stiffener itself does not carry compressive stresses.

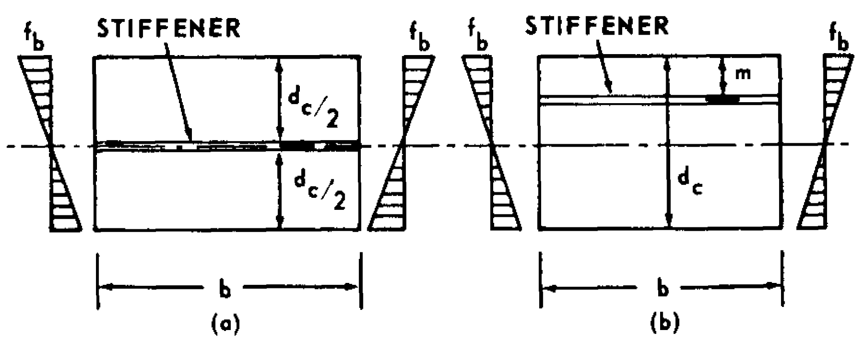


FIGURE 14. POSITIONS OF LONGITUDINAL STIFFENERS

Adding this longitudinal stiffener results in another structural part and more assembly cost; therefore, such construction is not widely used although it is a structural arrangement that will save structural weight under certain conditions of beam depth, span, and external loading.

A. Stiffener at the Centerline

For a stiffener at the centerline, the largest practical value of the stiffener moment of inertia is

$$\frac{I_o}{\eta_B} = 0.92 t^3 d_c \tag{7}$$

With this value of I_o, the critical bending stress $F_{B_{cr}}$ is

$$\frac{F_{B_{cr}}}{\eta_B} = \frac{35.6\,\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{d_c}\right)^2 \quad \text{for } \alpha \geq 2/3 \quad . \tag{8}$$

For values of $F_{B_{cr}}$ for stiffener moment of inertia less than I_o, see Reference 10.

B. Stiffeners Located Between Compression Edge and Neutral Axis

The increase in buckling strength that can be obtained by a stiffener at the centerline of the web amounts to only 50 percent of the unstiffened plate in the inelastic range. Stiffeners at the centerline are therefore not very effective in improving the stability of web plates in case of pure bending stresses.

For a stiffener spaced at a distance $m = d_c/4$, the critical buckling stress is given by

$$\frac{F_{B_{cr}}}{\eta_B} = \frac{101\,\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{d_c}\right)^2 \qquad (\alpha \geq 0.4), \tag{9}$$

if $\gamma \geq \gamma_o$.

Where $\gamma = \dfrac{EI}{\eta_B D d_c}$,

$$\gamma_o = \frac{EI_o}{Dd_c} = (12.6 + 50\,\delta)\,\alpha^2 - 3.4\,\alpha^3 \qquad (\alpha \leq 1.6) \text{ and}$$

$$\delta = \frac{A}{\eta d_c t} \ , \quad \alpha = b/d_c \ .$$

Comparison of the results obtained above for a stiffener at the centerline of the plate with the results obtained for a plate stiffened in the compression region shows that the reinforcement in the latter case is much more effective.

Limited numerical results have been obtained for plates reinforced by a longitudinal stiffener located at a distance $m = d_c/5$ from the compression edge of the plate. This information is plotted in Figures 15 and 16. The largest value of the buckling strength of the plate stiffener system corresponds to $K = 129$ and is larger than in the case of a stiffener located at the distance $d_c/4$ from the compression edge.

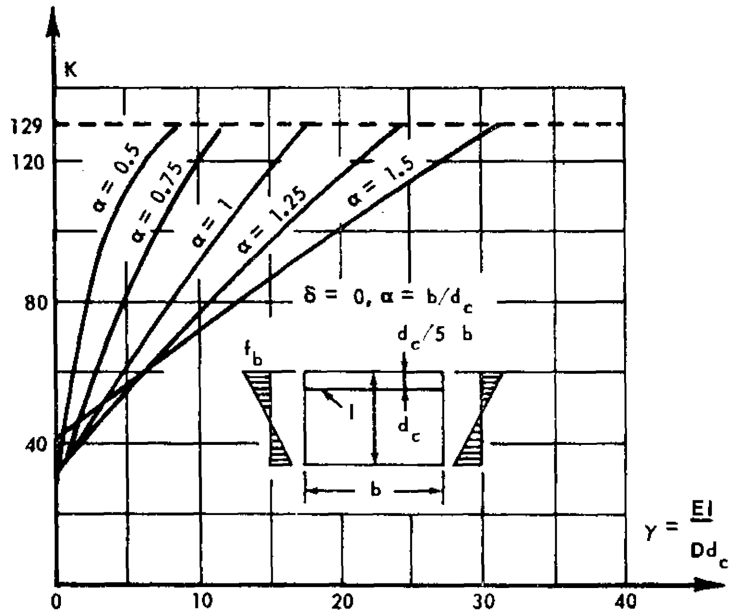


FIGURE 15. K VERSUS γ FOR VARIOUS VALUES OF α, $\delta = 0$

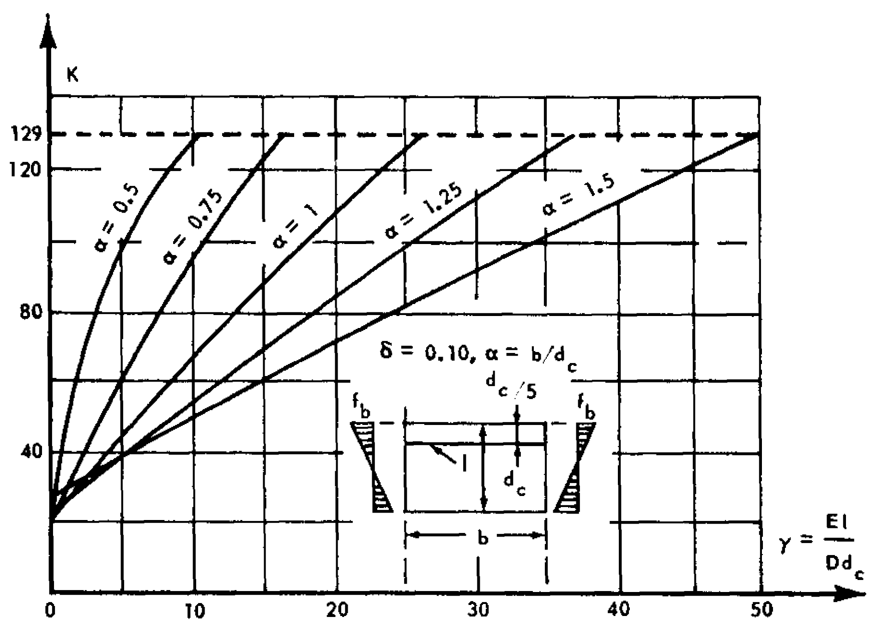


FIGURE 16. K VERSUS γ FOR VARIOUS VALUES OF α, $\delta = 0.10$

VI. Combined Stresses

The above mentioned bending, shear, and possibly axial and transverse stresses that act upon the web should be interacted by the following equations.

$$R_s^2 + R_{c_L} \leqq 1 \tag{10}$$

$$R_s^2 + R_{c_T} \leqq 1 \tag{11}$$

$$R_s^2 + R_B^2 \leqq 1 \tag{12}$$

$$R_B^{1.75} + R_{c_L} \leqq 1 \tag{13}$$

where

$$R_s = \frac{f_s}{F_{s_{cr}}} \ , \quad R_B = \frac{f_b}{F_{B_{cr}}} \ , \quad R_c = \frac{f_c}{F_{C_{cr}}} \ ,$$

and the subscript, L, indicates longitudinal and the subscript, T, indicates transverse. The critical values of $F_{C_{cr}}$ should be obtained from Section C2.1.1.

If the interaction equations above are not satisfied, an iteration of the design must be performed.

4.8.1.2 Flange Design

The beam flanges are designed for tensile and compressive normal forces. The ultimate allowable stress for the tension flange is equal to F_{tu} of the material, reduced by the attachment efficiency factor. For riveted or bolted connections, the efficiency factor is the ratio of the net area to the gross area of the cap.

Compression flanges should be designed for column stability. The loads on the compression flange can be a combination of normal force, longitudinal shear at the web-flange connection, and transverse forces. Specific analysis techniques are available in Section B4.4.0.

4.8.1.3 Rivet Design

I. Web-to-Stiffener

Although no exact information is available on the strength required of the attachment of the stiffeners to the web, the data in Table B4.8-I are recommended.

Table B4.8-I. Rivets: Web-to-Stiffener

Web Thickness (in.)	Rivet Size	Rivet Spacing (in.)
0.025	AD 3	1.00
0.032	AD 4	1.25
0.040	AD 4	1.10
0.051	AD 4	1.00
0.064	AD 4	0.90
0.072	AD 5	1.10
0.081	AD 5	1.00
0.091	AD 5	0.90
0.102	DD 6	1.10
0.125	DD 6	1.00
0.156	DD 6	0.90
0.188	DD 8	1.00

II. Stiffeners-to-Flange

No information is available on the strength required of the attachment of the stiffeners to the flange. It is recommended that one rivet the next size larger than that used in the attachment of the stiffeners to the web or two rivets the same size be used whenever possible.

III. Web-to-Flange

The rivet size and spacing should be designed so that the rivet allowable (bearing or shear) divided by $q \times p$, the applied web shear flow times the rivet spacing, gives the proper margin of safety. For a good design and to avoid undue stress concentration, the rivet factor, C_r, should not be less than 0.6.

4.8.1.4 Design Approach

The design of stiffened shear-resistant beams is a trial-and-error method. Assuming q, b, h, and E are known, the first step is to assume a reasonable value of t and compute $f_s = q/t$. The problem is to find the moment of inertia of a stiffener required to develop an initial buckling stress, $F_{s_{cr}}$, in the web greater than f_s by the desired margin of safety. The procedure is to choose the desired $F_{s_{cr}}$, and $F_{s_{cr}}/\eta_s$ if required. K_s is then found from equation (1) as a function of $F_{s_{cr}}/\eta$ and d/t or h/t. Then, depending on the type of stiffening arrangement used, the required I of the stiffener is obtained from Paragraph 4.8.1.1. Particular attention should be given to stiffener properties that provide an efficient design. For a minimum-weight design, consideration should be given to longitudinal stiffeners and/or torsionally strong stiffeners as discussed in Paragraph 4.8.1.1. Attention should also be given to stiffener thickness and rivet location as discussed in Paragraph 4.8.1.1-II.

4.8.1.5 Stress Analysis Procedure

The stress analysis procedure for the web of a stiffened shear resistant beam is straightforward and easy to apply. Since q, b, h, E, t, f_s, and I are all known, the first step is to obtain K_s from the appropriate curve in Paragraph 4.8.1.1, depending upon the aspect ratio, torsional rigidity, stiffener thickness, etc. Then $F_{s_{cr}}$ can be obtained from equation (1). If necessary, values of η_s can be obtained from Section C2.0. Then the margin of safety for the web is

$$\text{M.S.} = \frac{F_{s_{cr}}}{f_s} - 1. \qquad (14)$$

4.8.1.6 Other Types of Web Design

The web designs discussed previously require a large number of parts (stiffeners) to achieve lightness. To keep manufacturing costs low, the number of parts must be kept to a minimum. The problem is one of weight trade-off versus manufacturing expense.

Three types of shear-resistant, nonbuckling webs are frequently used in design to save the expense of stiffeners. Actually, the web in most cases is as light as, or lighter than, a web with separate stiffeners. There is a general limitation, however, in that a stiffener must be provided wherever a significant load is introduced into the beam. The web types are:

I. Web with formed vertical beads at a minimum spacing

II. Web with round lightening holes having 45 degree formed flanges at various spacing

III. Web with round lightening holes having formed beaded flanges and vertical formed beads between holes.

The webs with holes, II and III, also provide built-in access space for the many hydraulic and electrical lines that are sometimes required.

Procedures for the design of these beams should be obtained from Reference 11.

4.8.2 PLANE TENSION-FIELD BEAMS

If web buckling occurs after some application of load, the shear load beyond buckling is resisted in part by pure tension-field action of the web, and in part by shear-resistant action of the web (Fig. 17). This action of the web is defined as an incomplete tension field, or partial tension field.

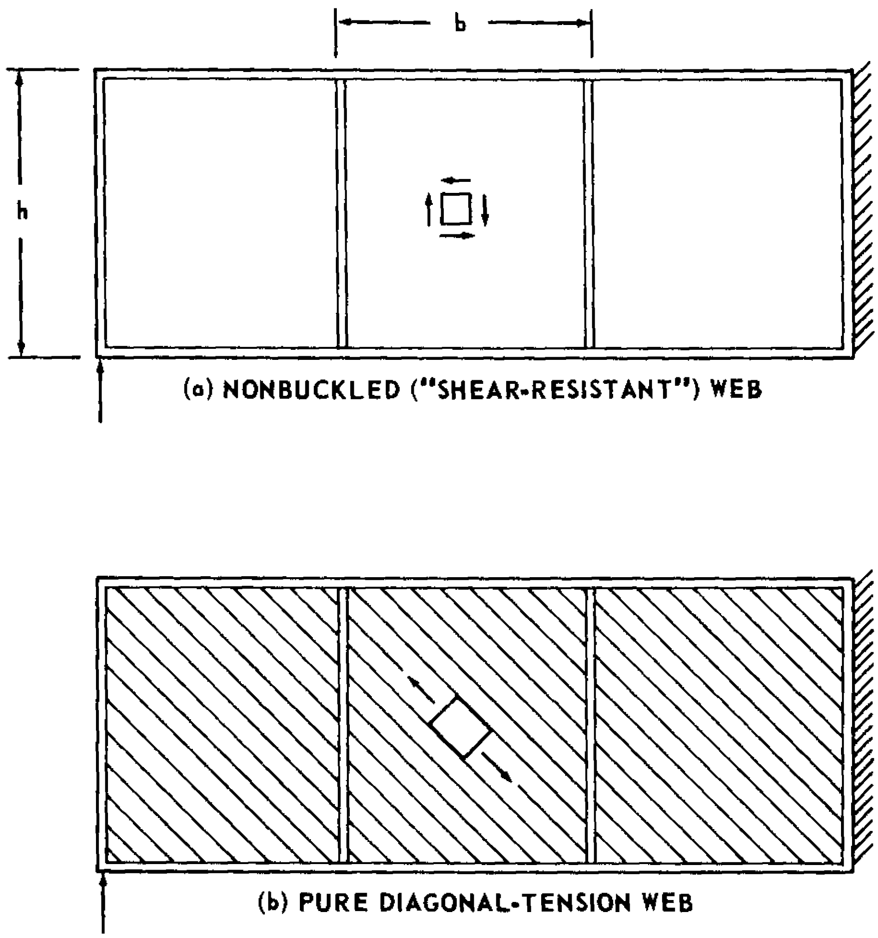


FIGURE 17. STATE OF STRESS IN A BEAM WEB

The theory of pure tension-field beams was published by Wagner [1]. In this theory, after initial web buckling, the total shear load is resisted by pure tension-field action of the web. Such behavior is essentially nonexistent in practice, and will not be discussed here.

Kuhn, Peterson, and Levin [12], developed a semiempirical analysis for partial tension-field beams. Correlation with experimental results indicates that the analysis is conservative for the beams within the range of beams tested. The experimental verification for the analysis was restricted to 2024S-T and 7075S-T aluminum alloy beams. As a consequence, the Kuhn analysis is limited to beams of these alloys. Extension of the analysis to include other alloys is not documented, and the designer must exercise considerable caution in attempting an extension of Kuhn's work in the analysis of beams fabricated from other alloys.

4.8.2.1 General Limitations and Symbols

The methods of analysis and design given herein are believed to furnish reasonable assurance of conservative strength predictions, provided that normal design practices and proportions are used. The most important points are:

I. The uprights should not be "too thin"; keep

$t_u/t > 0.6$.

II. The upright spacing should be in the range

$0.2 < b/h < 1.0$.

III. The method of analysis presented here is applicable only to beams with webs in the range $60 < h/t < 1500$.

When $h/t < 115$, the portal frames effect and the effect of unsymmetrical flanges must be taken into account by using Reference 13.

SYMBOLS (in addition to those given in the front matter)

A_f	area of tension or compression flange
A_u	actual area of upright (stiffener)
A_{ue}	effective area of upright
C_1, C_2, C_3	stress concentration factors
F_{co}	column yield stress (the column stress at $\frac{L'}{\rho} = 0$)
F_c	allowable column stress
F_{max}	ultimate allowable compressive stress for natural crippling
F_o	ultimate allowable compressive stress for forced crippling
F_s	ultimate allowable web shear stress
I_f	average moment of inertia of beam flanges
I_s	required moment of inertia of upright about its base
I_u	moment of inertia of upright about its base
M_{sb}	secondary bending moment in the flange
P	applied shear load
P_u	upright end load
e	distance of upright centroid to web
f_{cent}	compressive stress at centroidal axis of upright
f_f	compressive stress in flange because of the distributed vertical component of the diagonal tension
f_u	average lengthwise compressive stress in upright

f_{sb}	secondary bending stress in flange because of the distributed vertical component of the diagonal tension
$f_{u_{max}}$	maximum compressive stress in upright
k	diagonal tension factor
b	spacing of uprights
h	effective depth of beam centroid of compression flange to centroid of tension flange
q_r	rivet shear load, web-to-flange and web splices
α	angle of diagonal tension
ρ	radius of gyration of upright with respect to its centroidal axis parallel to web (no portion of web to be included)
k_{ss}	critical shear stress coefficient
R_d, R_h	restraint coefficients
α_{PDT}	angle of diagonal tension for pure tension field beam
N	applied transverse load
Q'	static moment of cross section
h_u	height of stiffener
L_e	effective stiffener length
ω	load-per-inch acting normal to end bay stiffener

4.8.2.2 Analysis of Web

The web shear flow can be closely approximated by

$$q = \frac{V}{h} . \tag{15}$$

From this, it follows that the web shear stress is

$$f_s = q/t \quad . \tag{16}$$

The critical buckling stress of the web is given by

$$\frac{F_{s_{cr}}}{\eta} = k_{ss} \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{b}\right)^2 \left[R_h + 1/2(R_d - R_h)\left(\frac{b}{d_c}\right)^3\right] \qquad b < d_c$$

(17.a)

and

$$\frac{F_{s_{cr}}}{\eta} = k_{ss} \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{d_c}\right)^2 \left[R_d + 1/2(R_h - R_d)\left(\frac{d_c}{b}\right)^3\right] \qquad b > d_c .$$

(17.b)

The value of k_{ss} is obtained from Figure 18. The values R_h and R_d, the restraint coefficients, are given in Figure 19. Figure 20 provides $F_{s_{cr}}$ for the case $\eta \neq 1$. When R_h is very small, the value of the critical shear stress calculated from the equation above may be less than the value computed disregarding the presence of stiffeners. In this case, the stiffeners are disregarded and the latter value is used.

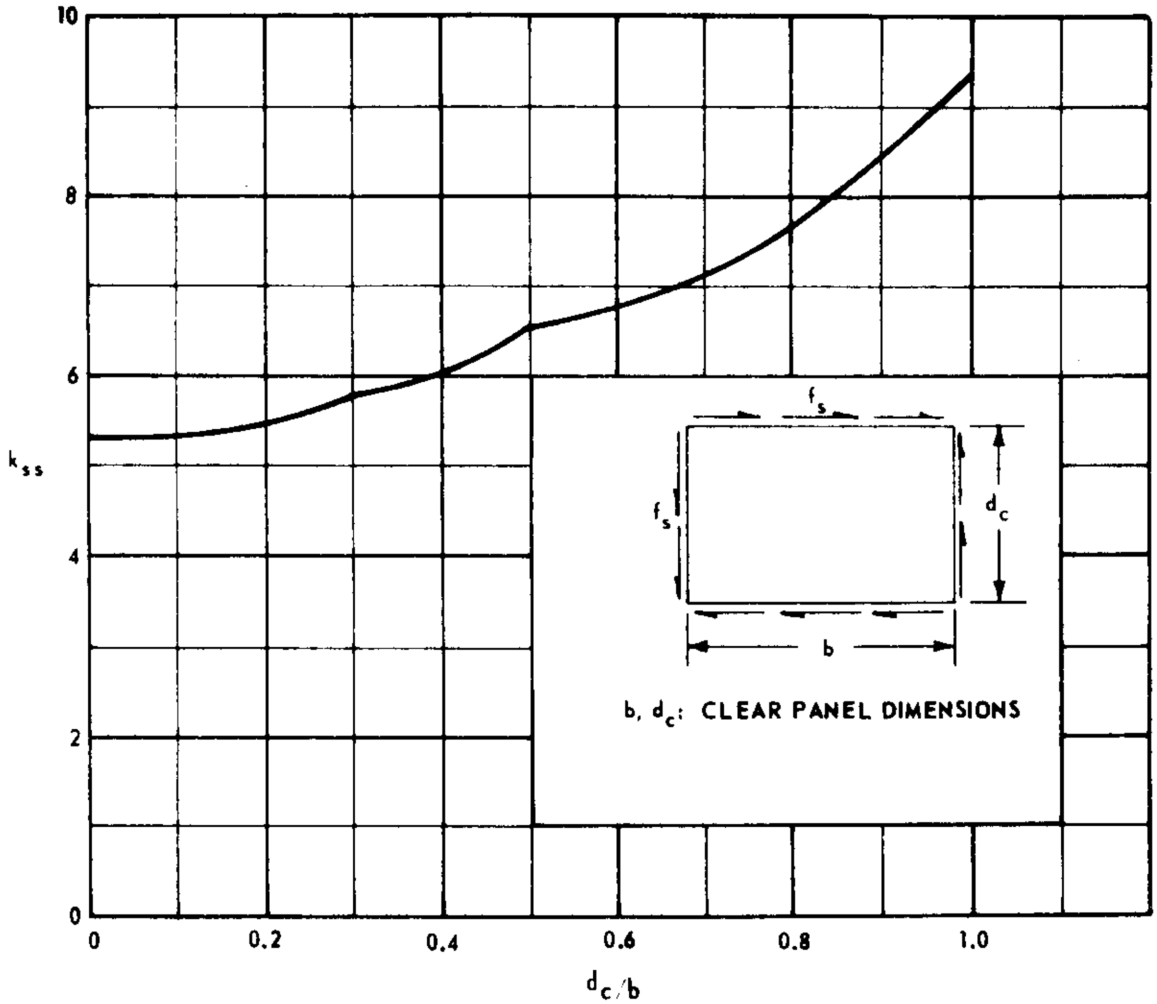


FIGURE 18. k_{ss} VERSUS d_c/b

The loading ratio, $f_s/F_{s_{cr}}$, is used to determine the tension field factor, k. It may be calculated by

$$k = \tanh\left(0.5 \log_{10} f_s/F_{s_{cr}}\right) \qquad f_s > F_{s_{cr}} , \tag{18}$$

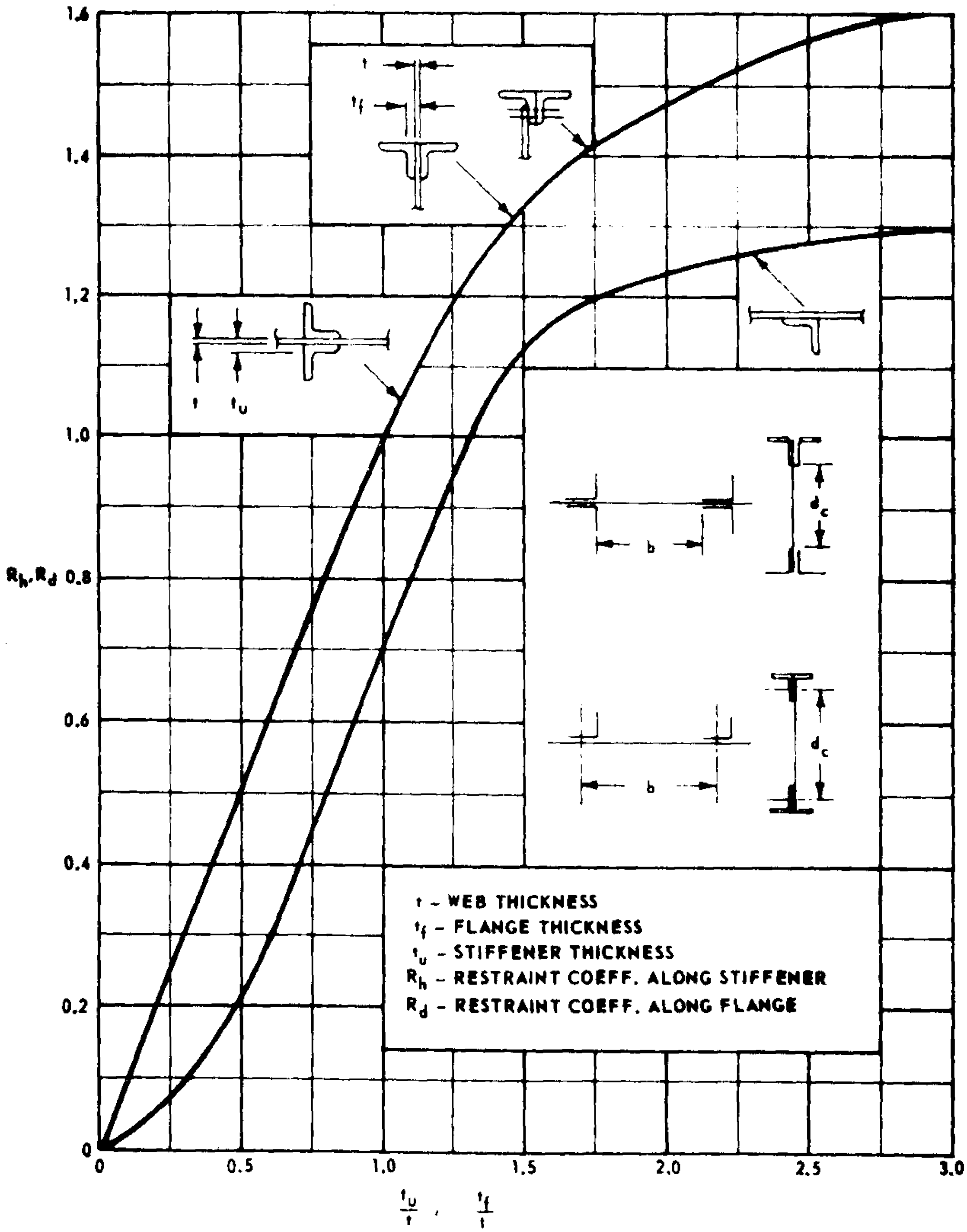


FIGURE 19. EDGE RESTRAINT COEFFICIENTS FOR WEB BUCKLING STRESS

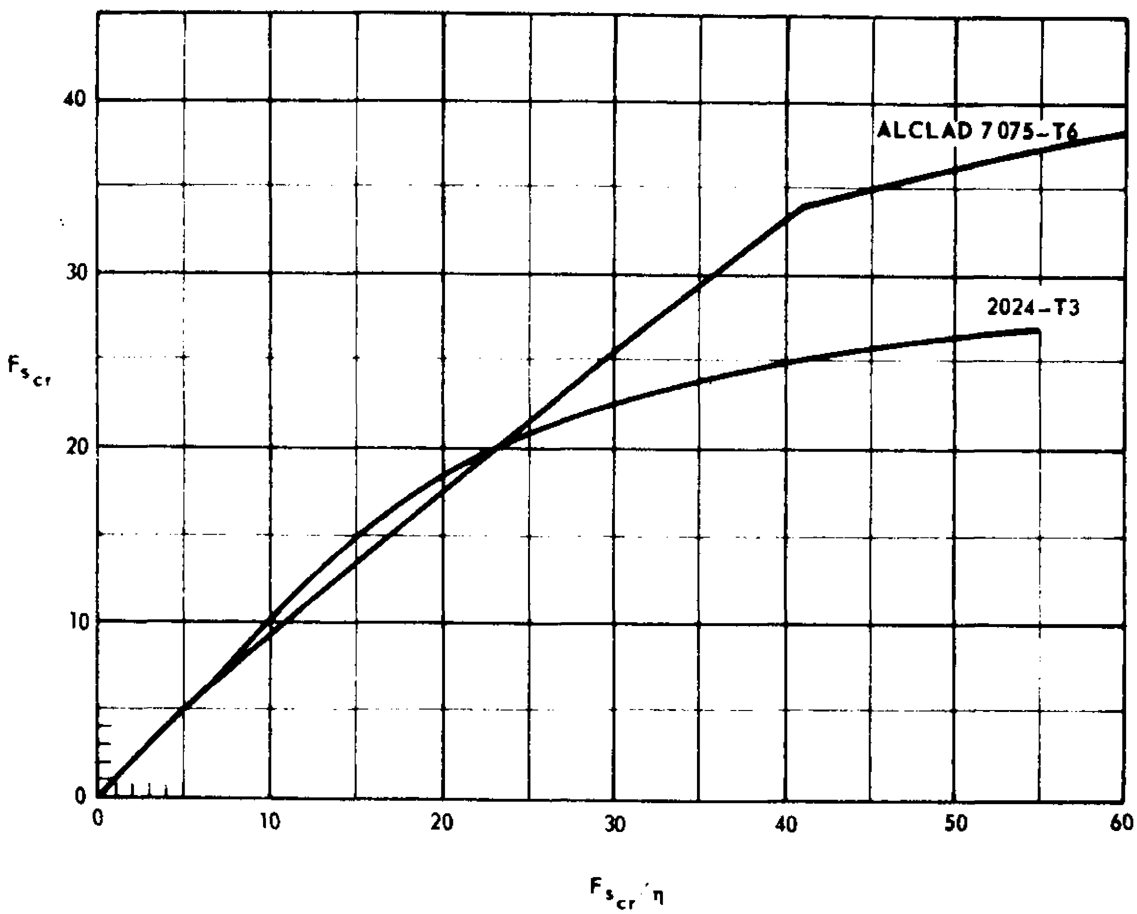


FIGURE 20. VALUES OF $F_{s_{cr}}$ WHEN $\eta \neq 1$

or it may read from Figure 21. For values of $f_s \leq F_{s_{cr}}$, the web is in the unbuckled state.

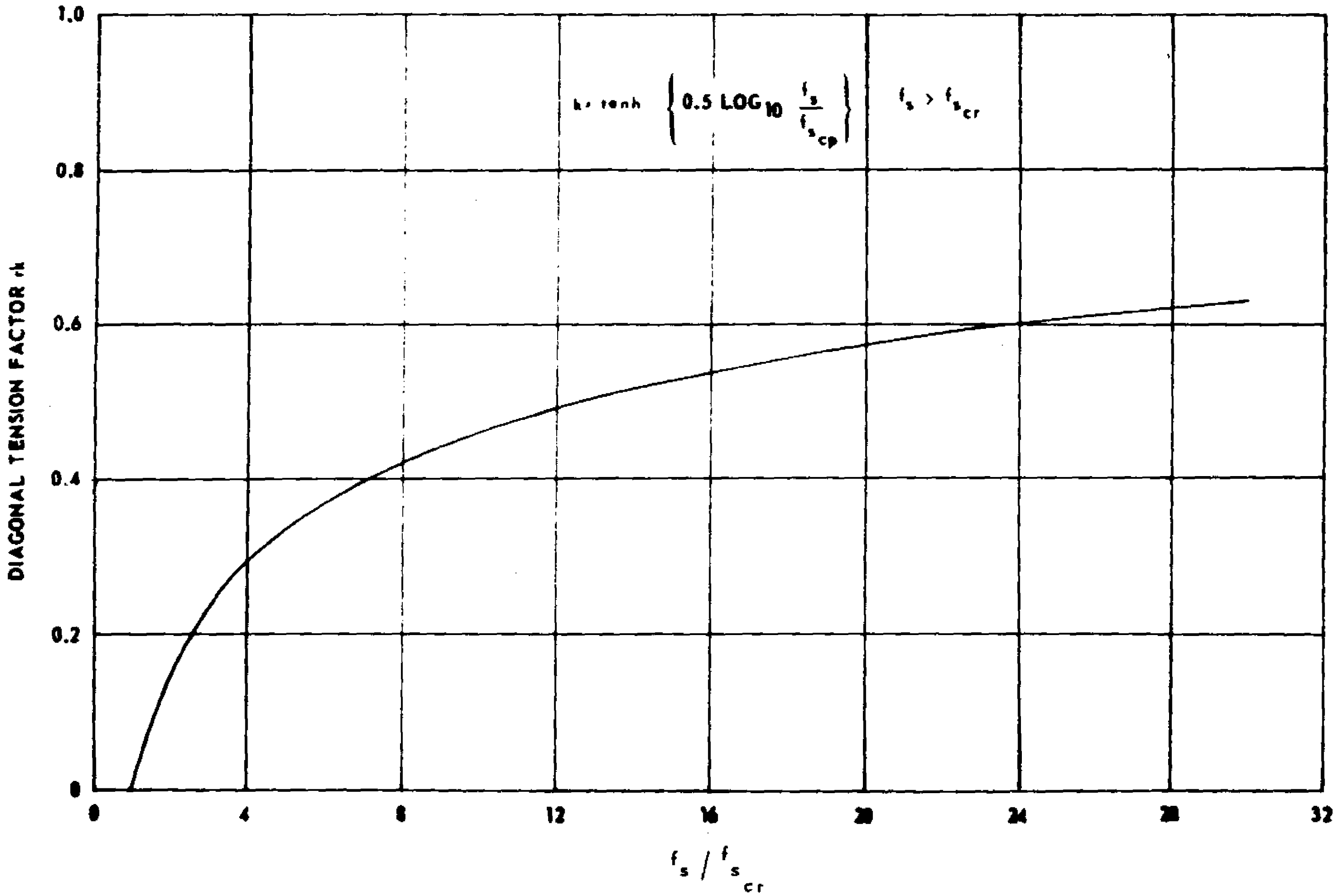


FIGURE 21. TENSION FIELD FACTOR VERSUS LOADING RATIO

The angle of the diagonal tension is then obtained from Figure 22, which shows the variation of tan α as a function of k and tb/A_{ue}. For double stiffeners, A_{ue} is equal to the cross-sectional area of the stiffeners. For single stiffeners,

$$A_{ue} = \frac{A_u}{1 + (e/\rho)^2} \quad . \tag{19}$$

It is recommended that the diagonal tension factor at ultimate load be limited to a maximum value,

$$k_{max} = 0.78 - (t - 0.012)^{1/2} \quad , \tag{20}$$

to avoid excessive wrinkling and permanent set at limit load, thereby inviting fatigue failure.

The average web shearing stress, f_s, may be appreciably smaller than the maximum web stress. The maximum web stress is given by

$$f_{s_{max}} = f_s (1 + k^2C_1)(1 + kC_2) \quad , \tag{21}$$

where C_1 and C_2 are empirical coefficients obtained from Figure 23. C_1 is a correction factor accounting for α differing from 45 degrees. C_2 is the stress-concentration factor arising from the flange flexibility.

The web allowable stress, $F_{s_{all}}$, is given in Figure 24 as a function of k and α_{PDT}, the angle that the buckles would assume if the web could reach the

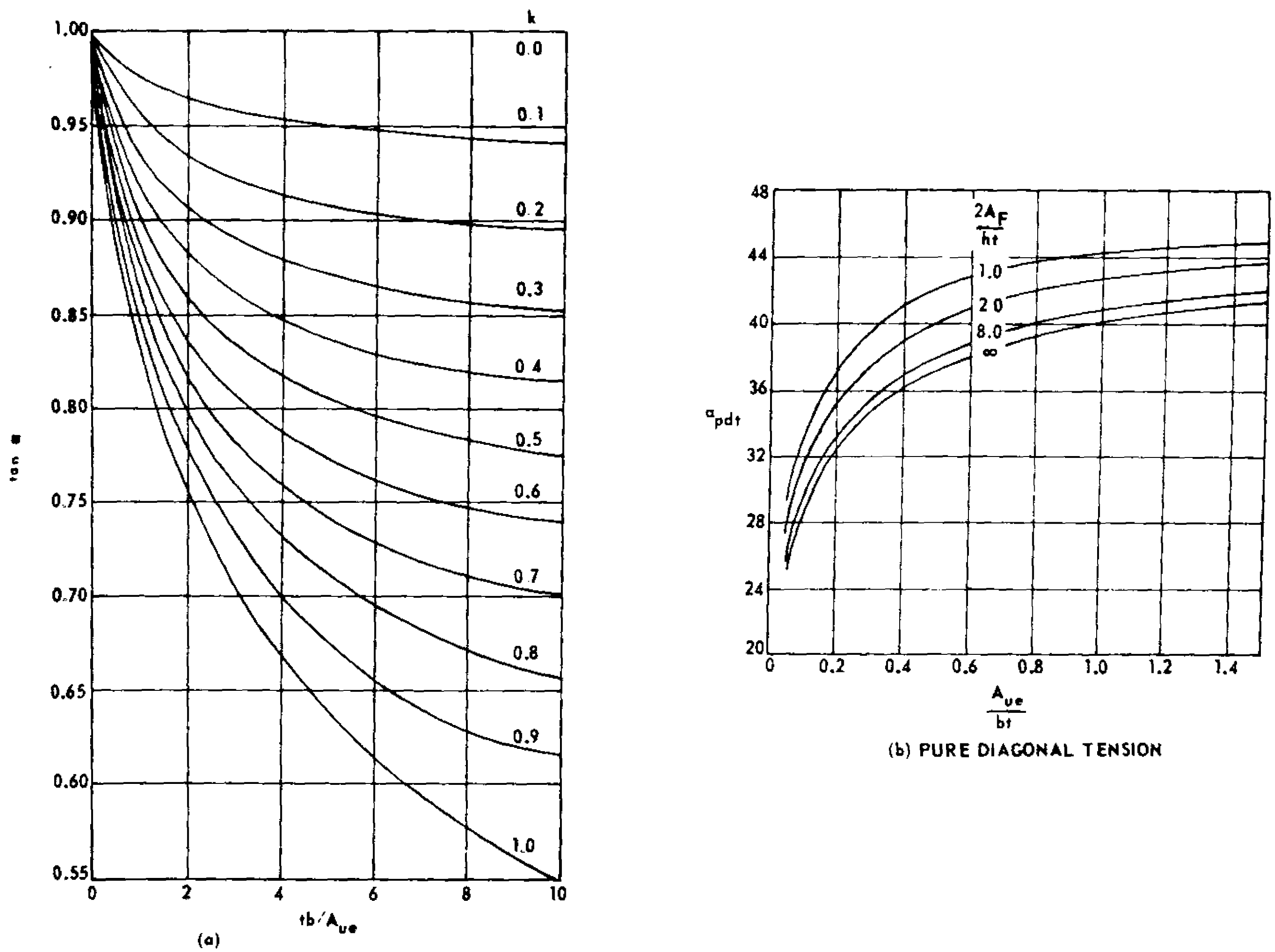


FIGURE 22. ANGLE OF DIAGONAL TENSION

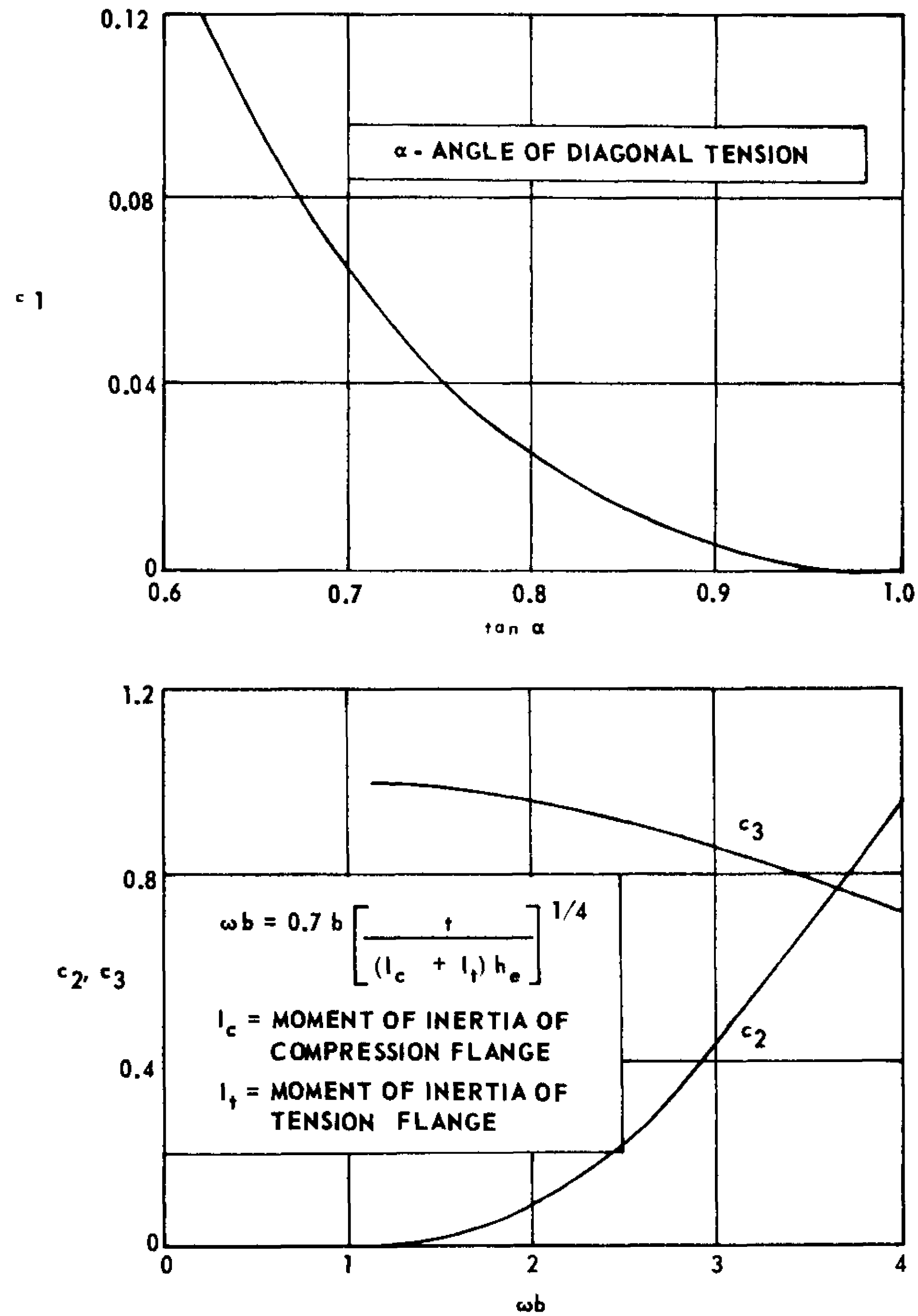
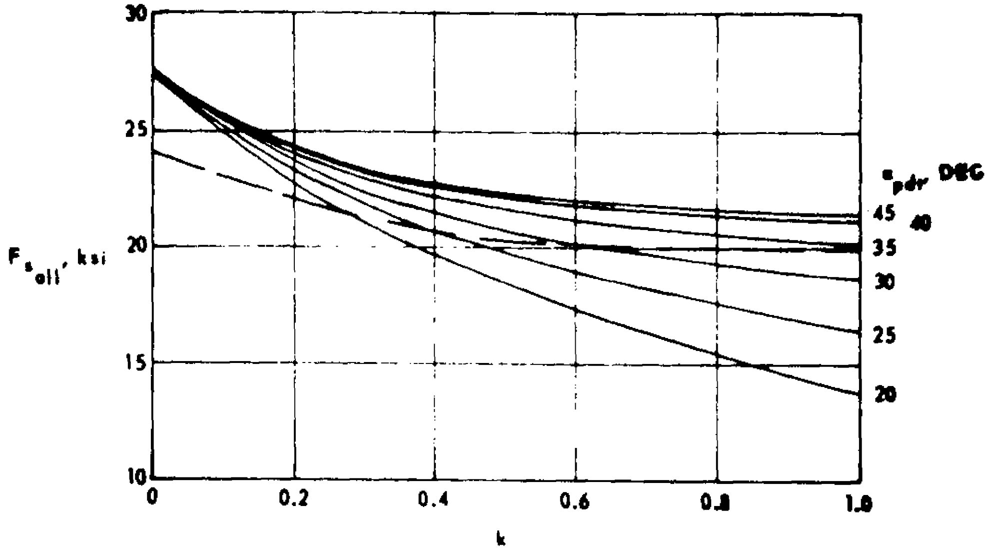
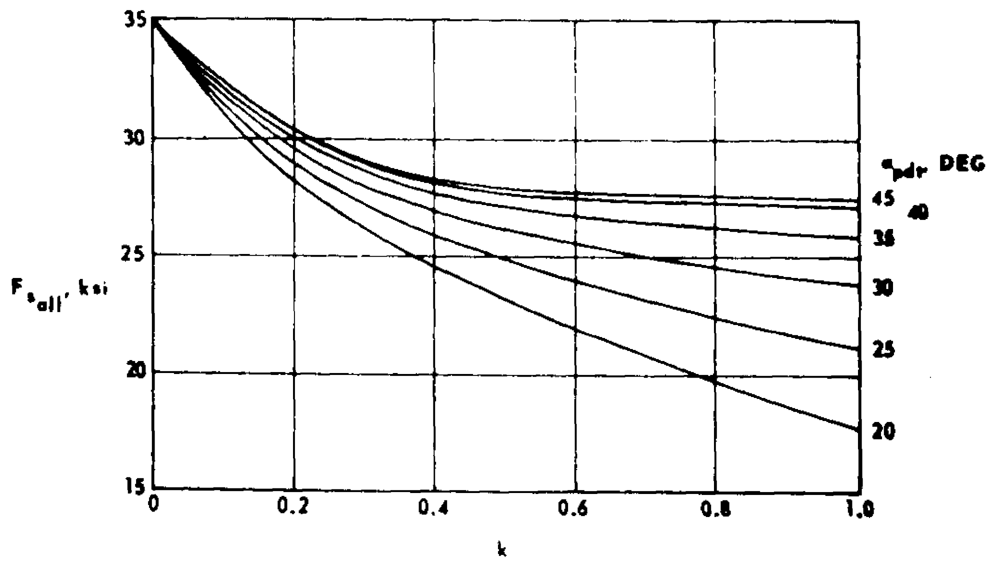


FIGURE 23. EMPIRICAL COEFFICIENTS FOR MAXIMUM SHEAR STRESS IN WEB AND FOR SECONDARY BENDING MOMENT IN FLANGES



(a) 2024-T3 ALUMINUM ALLOY. $f_{t_{ult}}$ = 62 ksi
DASHED LINE IS ALLOWABLE YIELD STRESS



(b) ALCLAD 7075-T6 ALUMINUM ALLOY. $f_{t_{ult}}$ = 72 ksi

FIGURE 24. BASIC ALLOWABLE VALUES OF $f_{s_{max}}$

state of pure diagonal tension without rupturing. The values of $F_{s_{all}}$ have been established by tests and may be called "basic allowable." For different connections, they are applied as follows:

I. Bolts, just snug, heavy washers under bolt heads, or web plates sandwiched between flange angles; use basic allowables.

II. Bolts, just snug, bolt heads bearing directly on sheet; reduce basic allowables 10 percent.

III. Rivets, assumed to be tight; increase basic allowables 10 percent.

IV. Rivets, assumed to be loosened in service; use basic allowables.

The allowable stresses given are valid if the allowable bearing stresses on the sheet or rivets are not exceeded. They are not valid for countersunk rivets. For webs of unusual dimensions arranged unsymmetrically with respect to the flange, use Figure 25 to obtain $F_{s_{all}}$.

4.8.2.3 Analysis of Stiffeners

Stiffener loads result from the web diagonal tension and the transverse load not carried by the web. The stiffener load is given by

$$P_u = ktbf_s \tan\alpha + Nb \left[1 - \frac{F_{s_{cr}} tb}{f_s (td + A_u)} \right] , \qquad (22)$$

where N is positive for a transverse compressive load. This load is resisted by the stiffener and the effective web. The effective width of the web working with the stiffener may be assumed to be given by

$$\frac{b_e}{b} = 0.5(1-k) . \qquad (23)$$

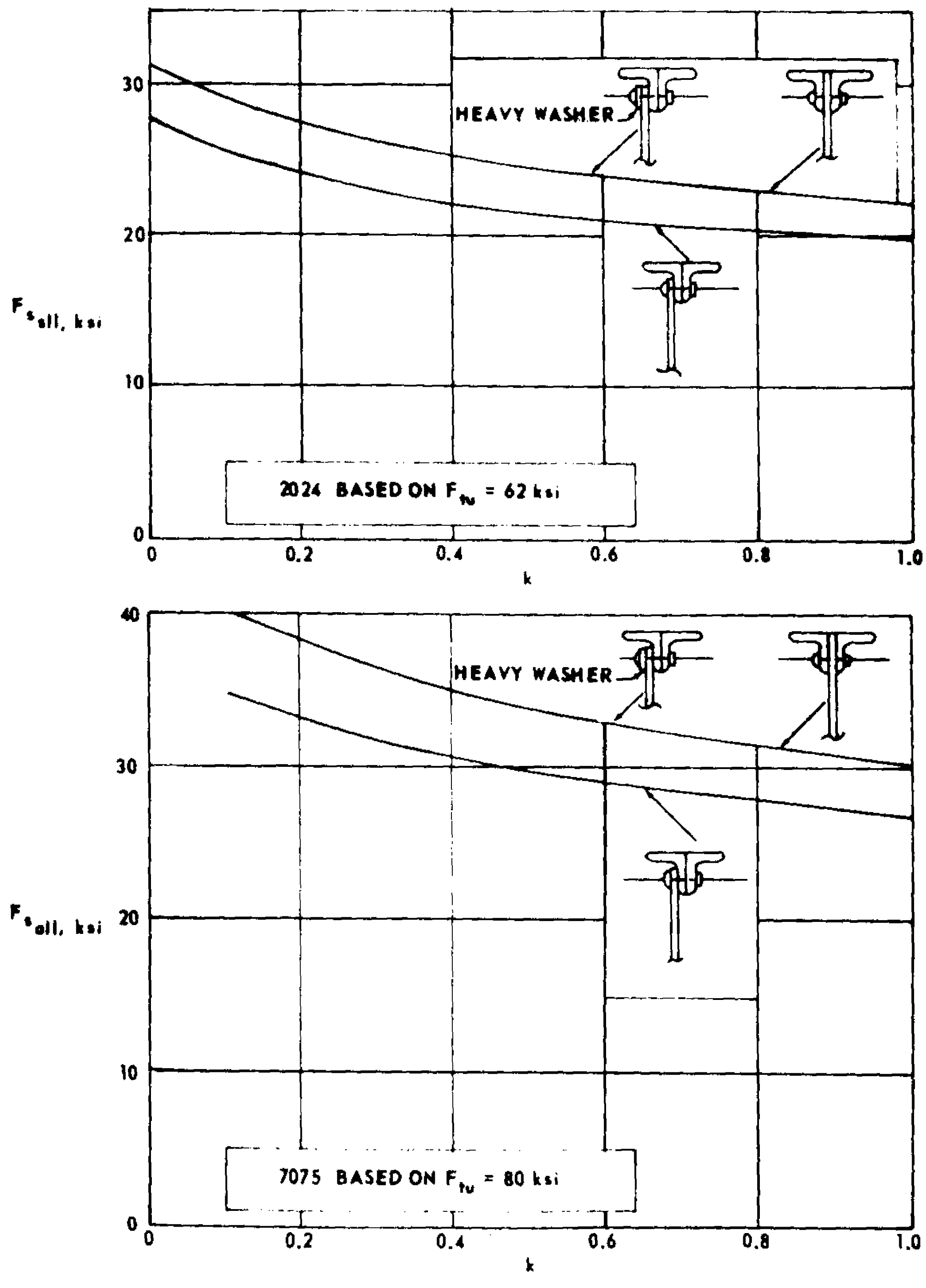


FIGURE 25. ALLOWABLE WEB STRESSES FOR 2024 AND 7075 AT ROOM TEMPERATURE

The average stiffener stress is then

$$f_u = \frac{P_u}{A_{ue} + 0.5(1-k)tb} \quad . \tag{24}$$

The maximum compressive stress in the stiffener occurs near the neutral axis of the beam. The ratio of the maximum stiffener stress to the average stiffener stress, $f_{u_{max}}/f_u$, is obtained from Figure 26.

Stiffeners may fail by column action or by local crippling. Column failure by true elastic instability is possible only in (symmetrical) double stiffeners.

A single stiffener is an eccentrically loaded compression member whose failing stress is a function of the web and stiffener properties. To guard against excessive bowing and column stress, the following must be adhered to:

I. The stress f_u must not exceed the column yield stress.

II. The average stress over the column cross section, $f_{cent} = f_u A_{ue}/A_u$, must not exceed the allowable stress for a column with the slenderness ratio $h_u/2\rho$.

The effective column length for double stiffeners is

$$L_e = \frac{h_u}{\sqrt{1 + k^2(3-2\,b/h_u)}} \qquad b < 1.5h \tag{25}$$

$$L_e = h_u \qquad b \geq 1.5h \quad . \tag{26}$$

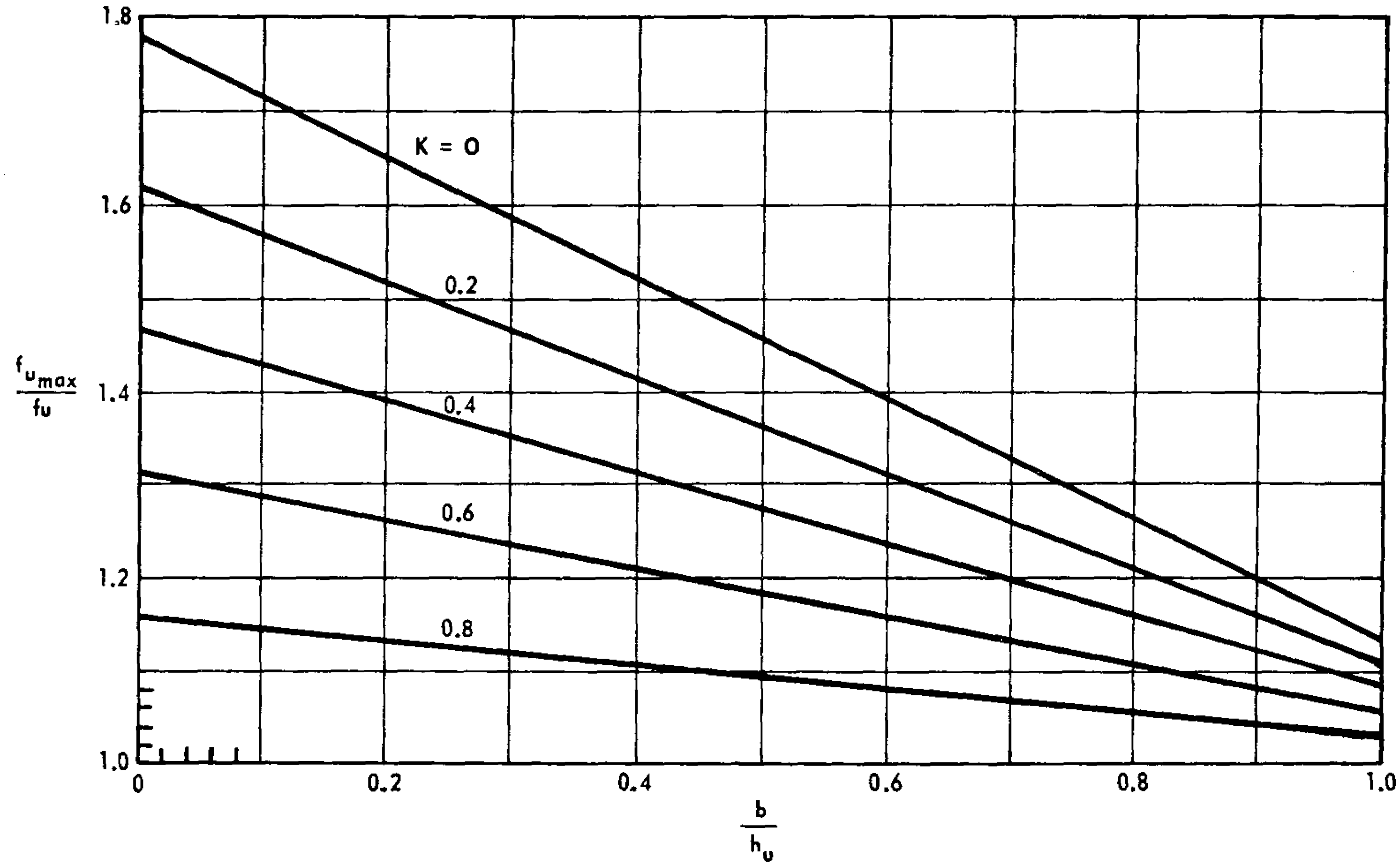


FIGURE 26. RATIO OF MAXIMUM STRESS TO AVERAGE STRESS IN WEB STIFFENER

To avoid column failure of double stiffeners, the average stress, f_u, should be less than the allowable stress taken from the column curve for solid sections of the stiffener material, with the slenderness ratio L_e/ρ.

Forced crippling of stiffeners must be considered. In this mode of failure, the attached leg of the stiffener is deformed by being forced to adapt itself to the web shear wrinkles.

The allowable forced crippling stress is given by the empirical equation

$$F_o = Ck^{2/3}\left(\frac{t_u}{t}\right)^{1/3} , \tag{27}$$

where C is a constant as follows:

	Single Stiffener	Double Stiffener
2024-T (Bare)	C = 26.0	21.0
7075-T (Bare)	C = 32.5	26.0

Nomographs for F_o are given in Figure 27. If F_o exceeds the material proportional limit, a plasticity factor, η, equal to E_{sec}/E_c is used in the equation above.

Torsional stability of single stiffeners is provided by meeting the following criteria:

$$(f_s - F_{s_{cr}})h_e t = 0.23E\left(\frac{J_1 th^2}{b}\right)^{1/3} , \tag{28}$$

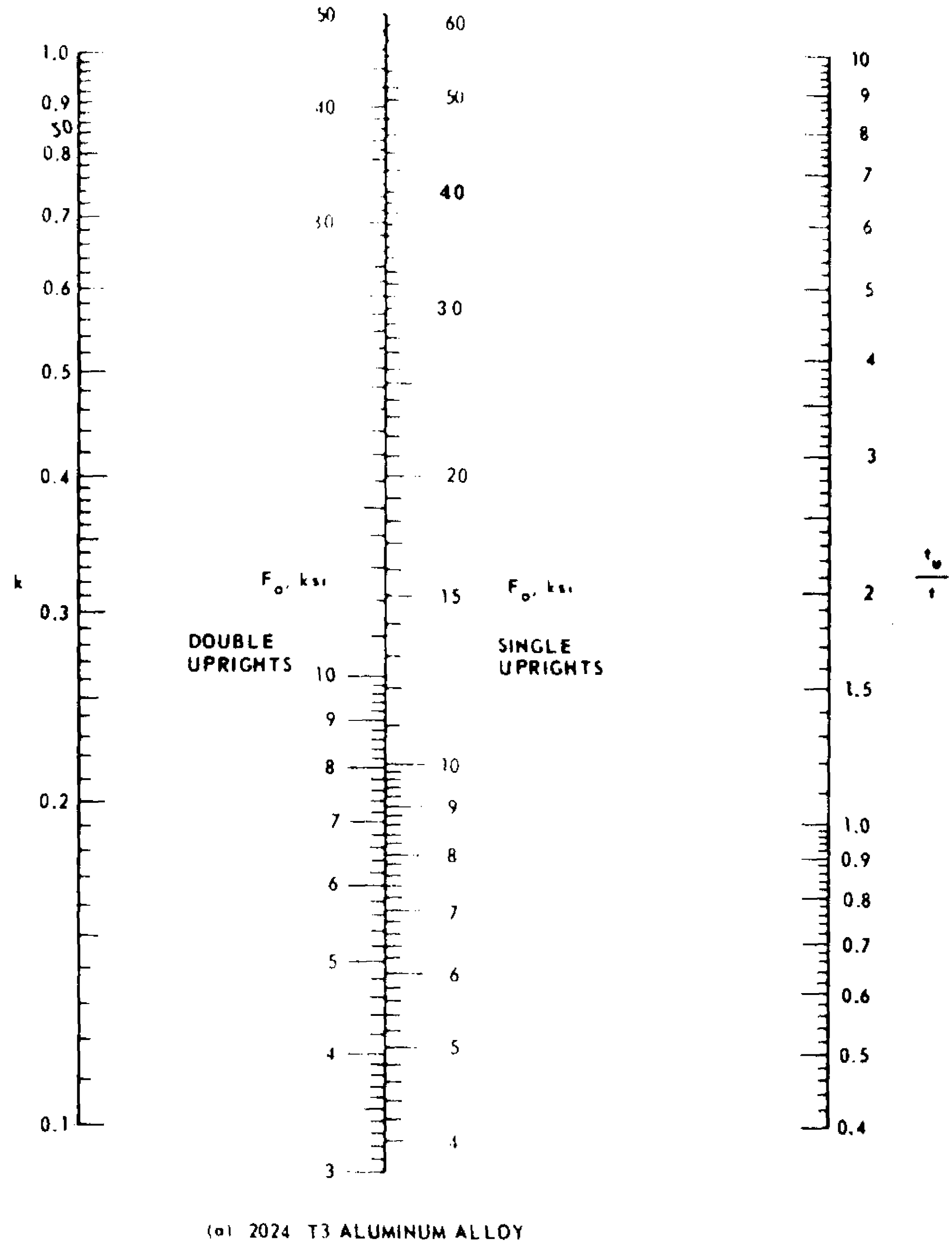
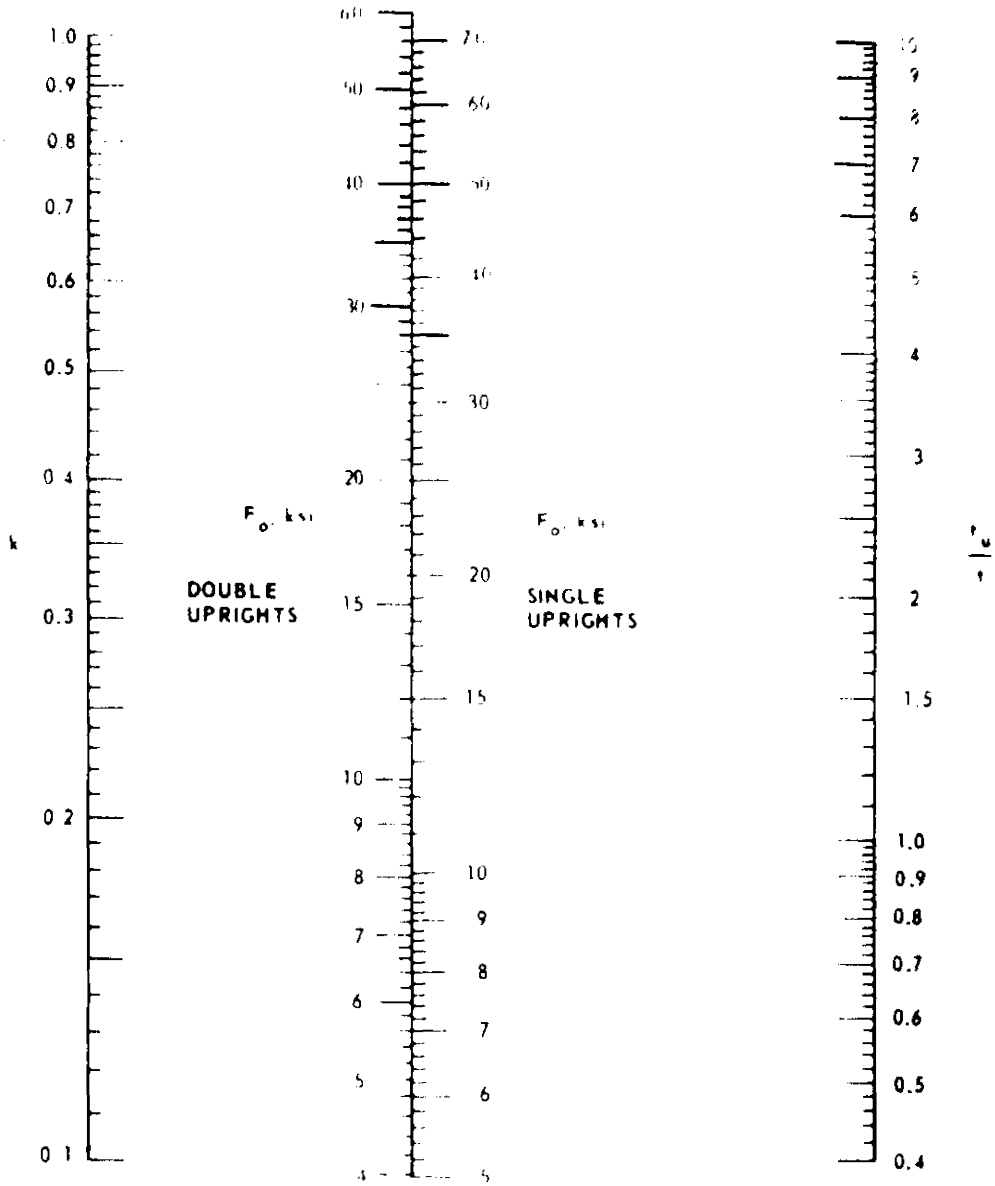


FIGURE 27. NOMOGRAM FOR ALLOWABLE UPRIGHT STRESS (FORCED CRIPPLING)



(b) 7075-T6 ALUMINUM ALLOY

FIGURE 27. (Concluded)

where

$(f_s - F_{s_{cr}}) h_e t$ = total web shear load above buckling which can be carried before the stiffener cripples

J_1 = the effective polar moment of inertia of the stiffener $1/3$ (developed width) t_u^3. This applies for formed sheet stiffeners.

To prevent a forced crippling type of failure when the upright resists an external compressive load in addition to the compressive load resulting from diagonal tension, an interaction equation such as the following must be used to evaluate this effect:

$$\left(\frac{f_{u_{max}}}{F_o}\right)^{1.5} + \left(\frac{f_{ce}}{F_{ce}}\right) = 1 \quad , \tag{29}$$

where $f_{u_{max}}$ and F_o are the maximum upright compressive stress and the allowable forced crippling stress, respectively, for diagonal tension acting alone, and f_{ce} and F_{ce} are the actual and allowable compressive stress, respectively, resulting from external compressive stress acting alone.

An effective area of web plus upright may be used in computing f_{ce}. The allowable crippling stress, F_{max}, may be used for F_{ce}.

The effect of the external load should also be investigated with respect to column failure of the upright. To prevent column failure under combined loading the following criteria should be fulfilled:

$$f_u + f_{ce} \leq F_{co} \tag{30}$$

and

$$f_{cent} + f_{ce} \leq F_c \quad . \tag{31}$$

4.8.2.4 Analysis of Flange

The flange stress is the result of the superposition of three individual stresses: (1) primary bending stresses, (2) axial compression because of the flange-parallel component of the web diagonal tension, and (3) secondary bending stresses because of the stiffener-parallel component of the web diagonal tension.

The primary bending stresses are given by

$$f_{prim} = \frac{M_c}{I_f} \left[1 - \frac{F_{s_{cr}}}{f_s} \left(1 - \frac{I_f}{I}\right) \right] \quad , \tag{32}$$

where

I = moment of inertia of section

and

I_f = moment of inertia of section (web neglected).

The total axial load because of the flange-parallel component of the web diagonal tension and applied axial load is

$$P_{axial} = khtf_s \cot\alpha + P_a \left[1 - \frac{F_{s_{cr}}}{f_s} \left(\frac{th}{A}\right) \right] \quad . \tag{33}$$

P_a is positive for compressive axial load. The axial flange stress is then

$$f_{axial} = \frac{P_{axial}}{A_c + A_t + 0.5(1-k)th} \quad , \tag{34}$$

where A_c and A_t are the area of the compression and tension flange.

The secondary bending stress is given by

$$f_{sec} = M_{sec}\left(\frac{C}{I}\right)_f \tag{35}$$

where

$$M_{sec} = C_3 \frac{P_u b}{12} \quad \text{(over stiffener)}$$

and

$$M_{sec} = C_3 \frac{P_u b}{24} \quad \text{(midway between stiffeners)} \quad .$$

C_3 is an empirical stress concentration factor, given in Figure 23.

The allowable stress for the compression flange can be found by the methods of Section C1.0. The allowable tension stress for a tension flange is given by F_{tu} of the material, modified by the attachment efficiency factor.

4.8.2.5 Analysis of Rivets

Web-to-Flange: The flange-web shear flow at the line of attachment is

$$q = \frac{V}{h'} \; (1 + 0.414k) \quad , \tag{36}$$

where h' = beam depth between attachment line of flange web.

Web-to-Stiffener: The stiffener-web rivets, for double stiffeners, must develop sufficient longitudinal shear strength to make the two stiffeners act as a unit until column failure occurs. The shear strength should be

$$q = \frac{2 F_{cy} Q}{b_s L_e} \quad , \tag{37}$$

where b_s = outstanding stiffener flange width.

The stiffener-web connectors must carry a tension component as follows:

$$N' = 0.15t\, F_{tu} \text{ (double stiffener)} \tag{38}$$

and

$$N' = 0.22t\, F_{tu} \text{ (single stiffener)} \quad . \tag{39}$$

The interaction of shear and tension in the connectors is given in Reference 11.

Stiffener-to-Flange: The stiffener-to-flange connectors are designed with the empirical relationship

$$P_u = f_u A_{ue} \quad , \tag{40}$$

which gives the load in the stiffener. The connection must transfer this load into the cap.

4.8.2.6 Analysis of End of Beam

The previous discussion has been concerned with the "interior" bays of a beam. The vertical stiffeners in these areas are subject, primarily, only to axial compression loads, as presented. The outer, or "end bay," is a special case. Since the diagonal tension effect results in an inward pull on the end stiffener, it produces bending in it, as well as the usual compressive axial load. Obviously, the end stiffener must be considerably heavier than the others, or at least supported by additional members to reduce the stresses resulting from bending.

The component of the running-load-per-inch that produces bending in such edge members is given by the formulas

$$\omega = kq \tan \alpha \tag{41}$$

for edge members parallel to the neutral axis (stringers) and

$$\omega = kq \cot \alpha \tag{42}$$

for members normal to the neutral axis (stiffeners). The longer the unsupported length of the edge member subjected to ω, the greater will be the bending moment it must carry.

There are, in general, three ways of dealing with the edge member subjected to bending, the object being to keep the weight down.

I. Simply "beef-up" or strengthen the edge member so it can carry all of its loads. (This is inefficient for long unsupported lengths.)

II. Increase the thickness of the end bay panel either to make it nonbuckling or to reduce k, and thereby reduce the running load producing bending in the edge member. (This is usually inefficient for large panels.)

III. Provide additional members (stiffeners) to support the edge member and thereby reduce its bending moment because of ω. (This requires additional parts.)

Actually, a combination of these methods might be best.

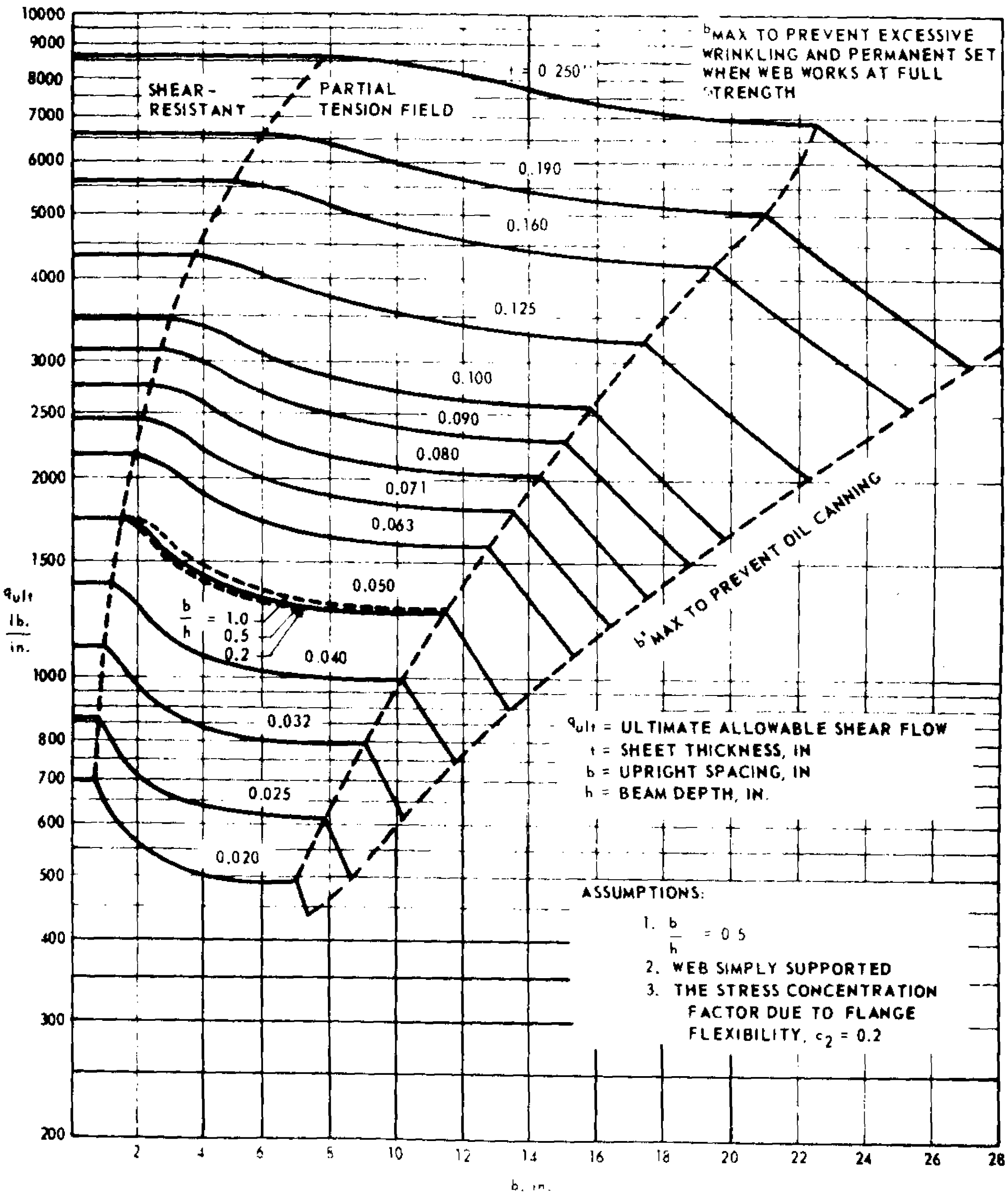
4.8.2.7 Beam Design

This paragraph presents methods to facilitate efficient preliminary design of tension field beams.

Allowable Shear Flow: Figure 28 gives the ultimate allowable shear flow, q, for 7075S-T6 Alclad sheet as a function of the sheet thickness, t, and the stiffener spacing, b. The dashed line on the left is the approximate boundary between shear-resistant and tension-field beams when the sheet is loaded to full strength. The central dashed line is a limitation on b_{max}, the maximum stiffener spacing, in order to minimize the possibility of excessive wrinkling and permanent set when the web works at full strength. Stiffener spacings greater than b_{max} can be used, if necessary, by using shear flow availables which conform to the limitation on the value of k_{max} given in Paragraph 4.8.2.2. The dashed line at the right establishes b'_{max}, the absolute maximum stiffener spacing in order to prevent "oil canning."

One of the assumptions made in the construction of Figure 28 is that the aspect ratio $b/h = 0.5$. Varying b/h has only a small effect on the curves, as can be seen from the curve for 0.050 sheet, where additional curves for $b/h = 0.2$ and $b/h = 1.0$ are plotted. The relationship for other sheet gages is approximately the same.

Stiffener Area Estimation: Figure 29 presents the stiffener area to web area ratio plotted as a function of b/h and $\sqrt{q/h}$, the square root of the structural index. This index is a measure of the loading intensity on the beam. These curves are to be used only as a means of roughly approximating the required area of stiffener for preliminary design and preliminary weight



NOTE: FOR BARE 7075S-T6, MULTIPLY BY 1.07

FIGURE 28. ULTIMATE ALLOWABLE SHEAR FLOW ALCLAD 7075S-T6 SHEET

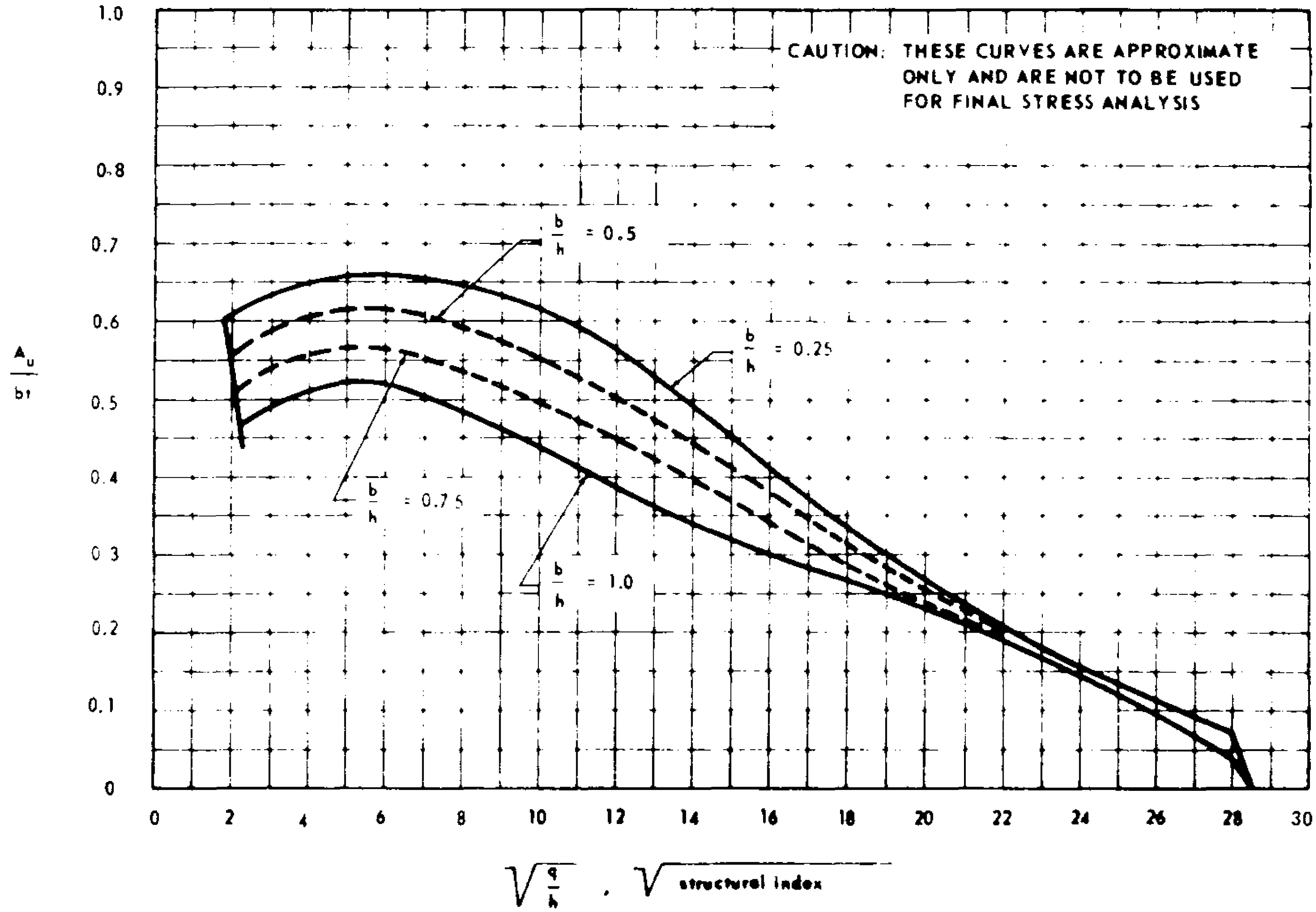


FIGURE 29. CHART FOR ESTIMATING EFFICIENT STIFFENER SIZE ALCLAD 7075S-T6 WEB 70707S-T6 SINGLE ANGLE STIFFENER

estimation. If the stiffener design is limited to standard sections, the required area might be larger than that given in Figure 29 since a zero margin of safety cannot always be obtained. The curves are for 7075S-T single-angle stiffeners. Curves for double stiffeners or 2024S-T material, similar to Figure 29, can be found in Reference 12.

Design Method: The preliminary design of 7075S-T web-stiffener can be arrived at in the following manner:

1. The design shear flow, q, and the depth of beam, h, are usually known. This fixes $\sqrt{q/h}$, the square root of the structural index.
2. The stiffener spacing, b, is often determined by considerations not under the control of the designer. If such is the case, inspection of Figure 29 shows that a stiffener spacing, b, equal to the beam depth, h, is desirable for minimum weight design of the web-stiffener system. However, it is possible that wide stiffener spacing might induce excessive secondary bending in the flange. In general, b/h ratios from 0.5 to around 0.8 are commonly used for tension-field beams.
3. The required web thickness, t, can be obtained from Figure 28 since q and b are known. This figure can also be used to check the stiffener against the maximum allowable spacing, b_{max}.
4. Estimate the required value A_u/bt with the aid of Figure 29.
5. Compute the approximate cross-sectional area of stiffener as follows:

$$A_u = \left(\frac{A_u}{bt}\right) bt \ .$$

6. Choose a stiffener with the proper area. Unless the beam is very deep, or unless there are other design considerations, a single-angle stiffener is an efficient design. Also, a stocky, equal-legged angle gives greater resistance against forced crippling, which is usually the dominant mode of failure.

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