B4.5.5.6 Magnesium-Minimum Properties

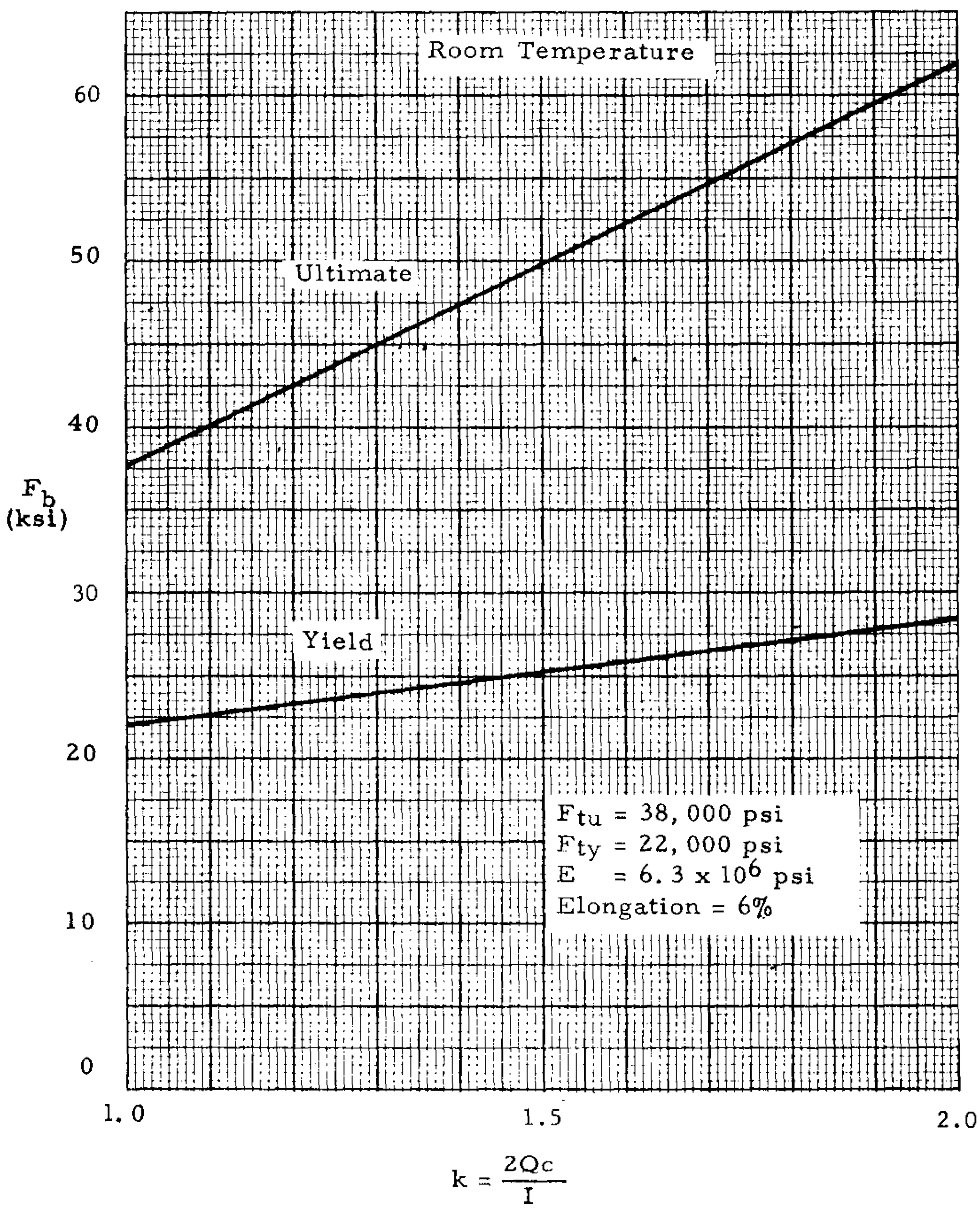


Fig. B4.5.5.6-1 Minimum Bending Modulus of Rupture Curves for Symmetrical Sections AZ61A Magnesium Alloy Forgings (Longitudinal)

B4.5.5.6 Magnesium-Minimum Properties

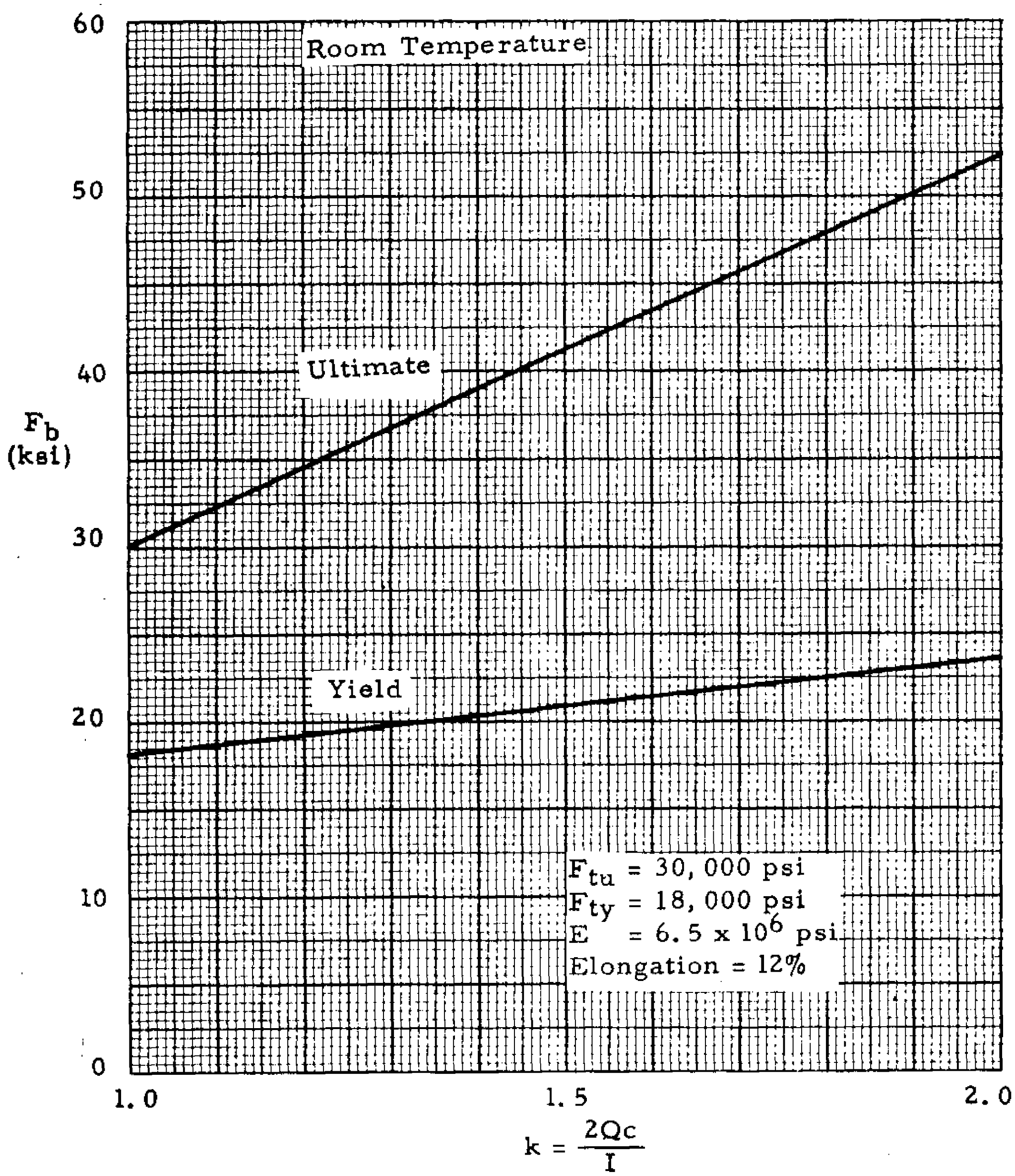


Fig. B4.5.5.6-2 Minimum Bending Modulus of Rupture for Symmetrical Sections HK31A-O Magnesium Alloy Sheet. 0.016 ≥ Thickness ≤ 0.250 in.

Graph to be furnished when available

Graph to be furnished when available

B4.5.5.6 Magnesium-Minimum Properties

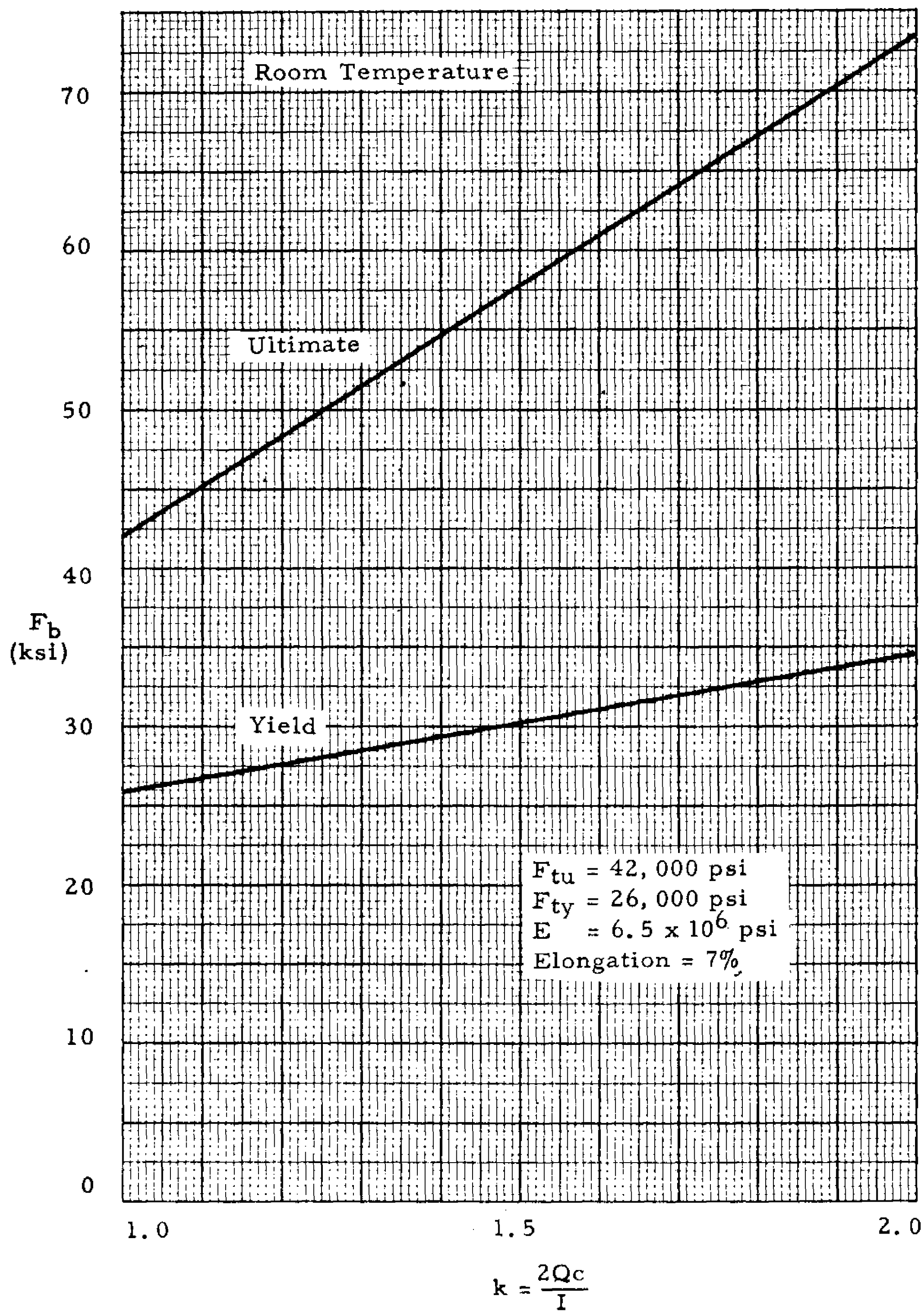


Fig. B4.5.5.6-5 Minimum Bending Modulus of Rupture for Symmetrical Sections ZK60A Magnesium Alloy Forgings (Longitudinal)

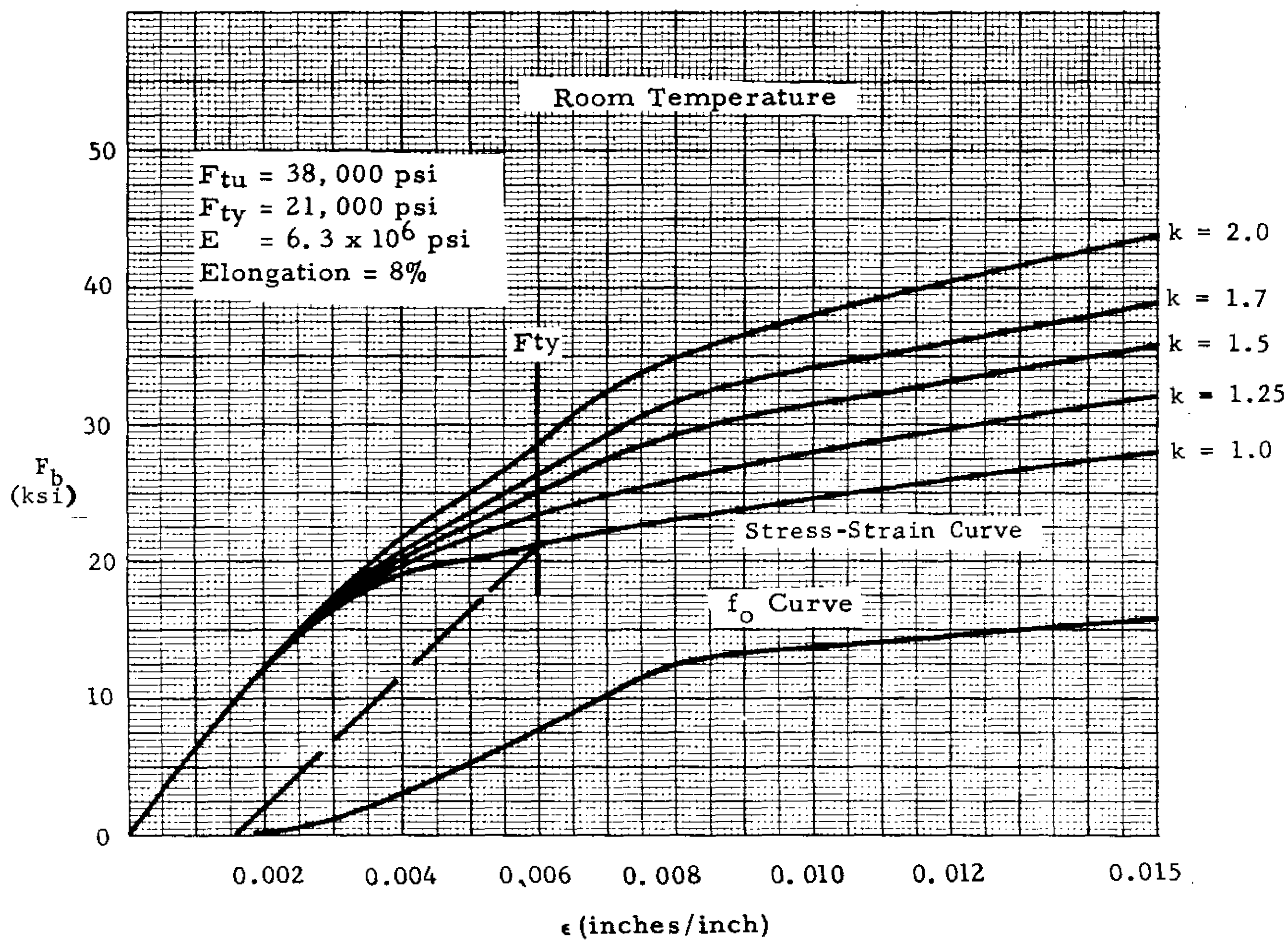


Fig. B4.5.6.6-1 Minimum Plastic Bending Curves A-Z61A Magnesium Alloy Extrusions (Longitudinal) $\leq$ 0.249 in.

Graph to be furnished when available

B4.5.6.6 Magnesium-Minimum Properties

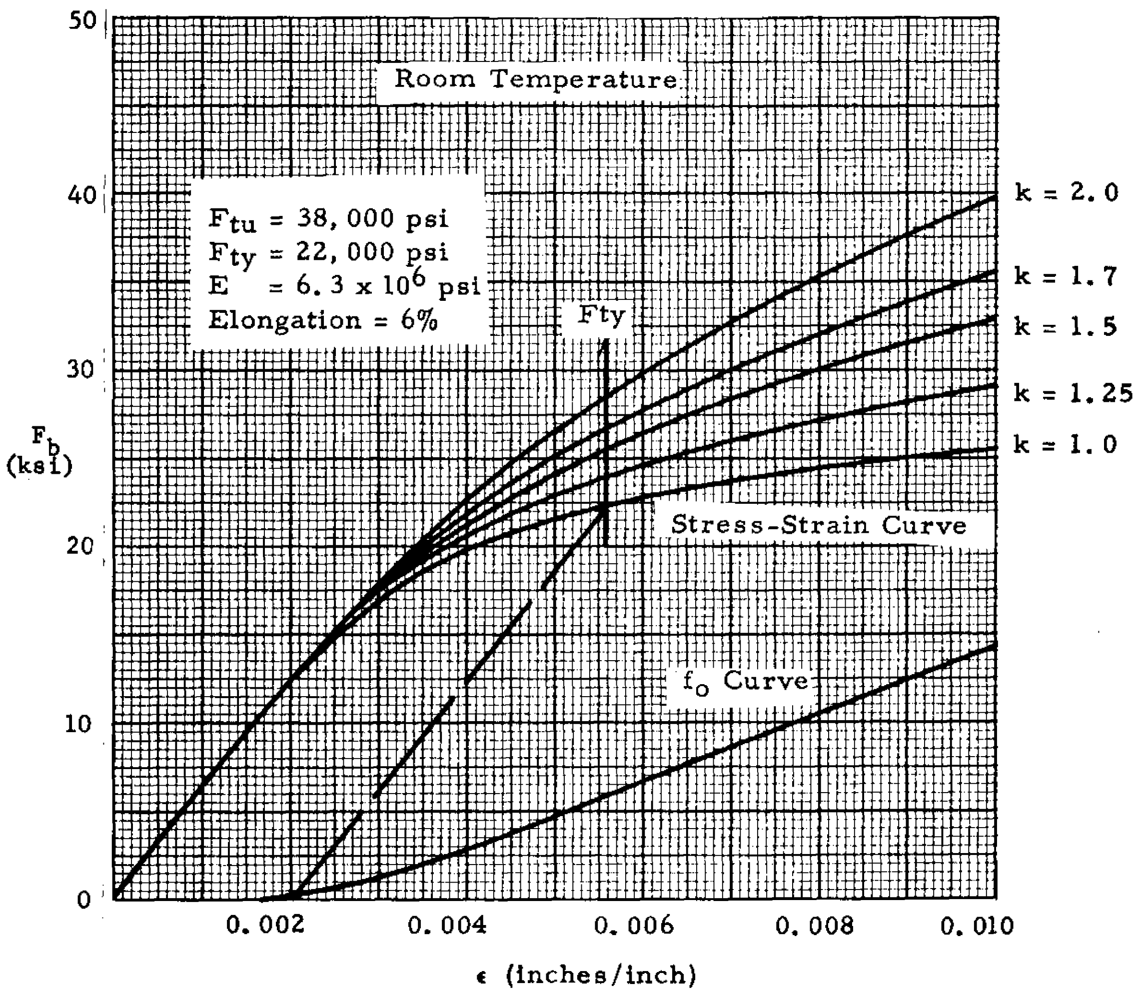


Fig. B4.5.6.6-3 Minimum Plastic Bending Curves AZ61A Magnesium Alloy Forgings (Longitudinal)

B4.5.6.6 Magnesium-Minimum Properties

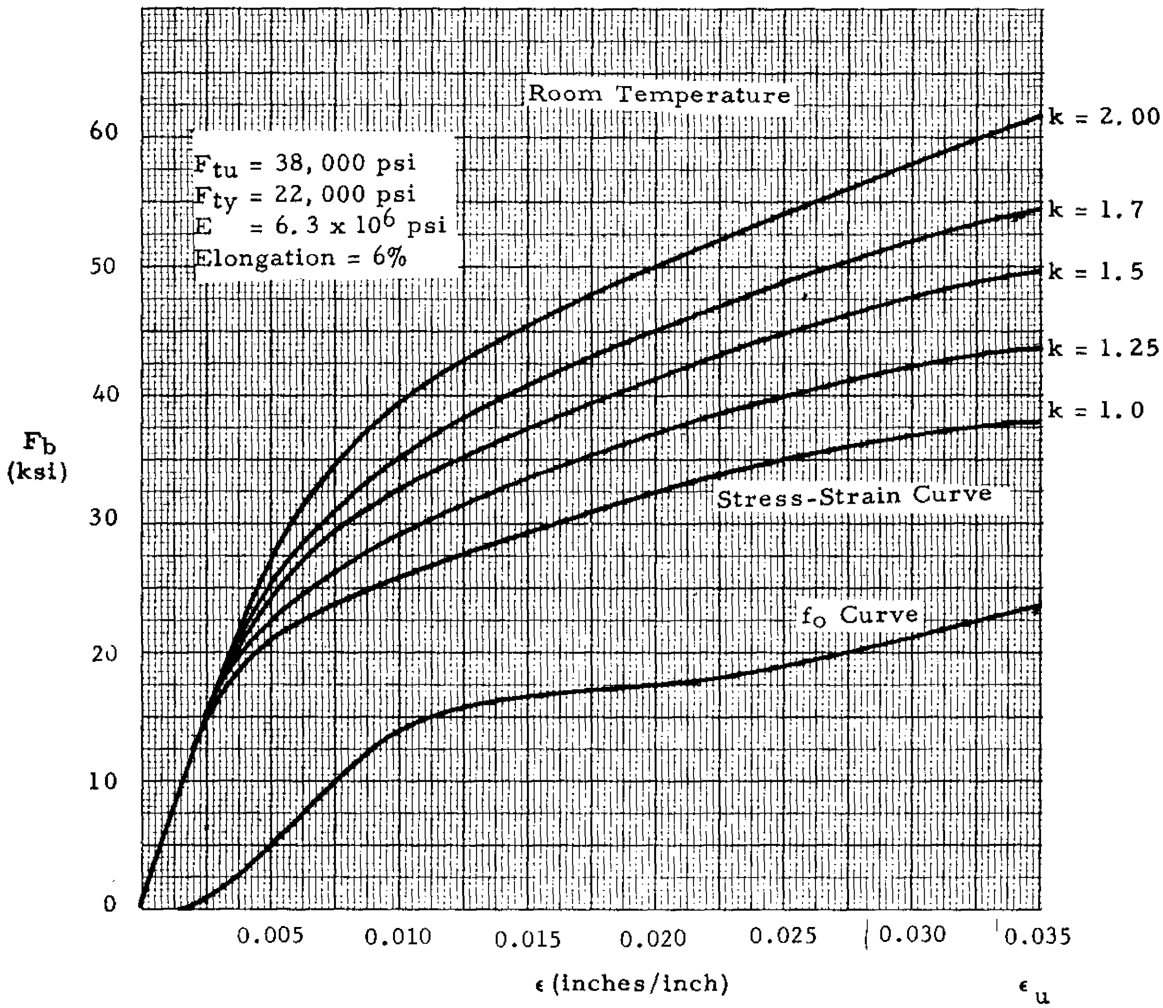


Fig. B4.5.6.6-4 Minimum Plastic Bending Curves AZ61A Magnesium Alloy Forgings (Longitudinal)

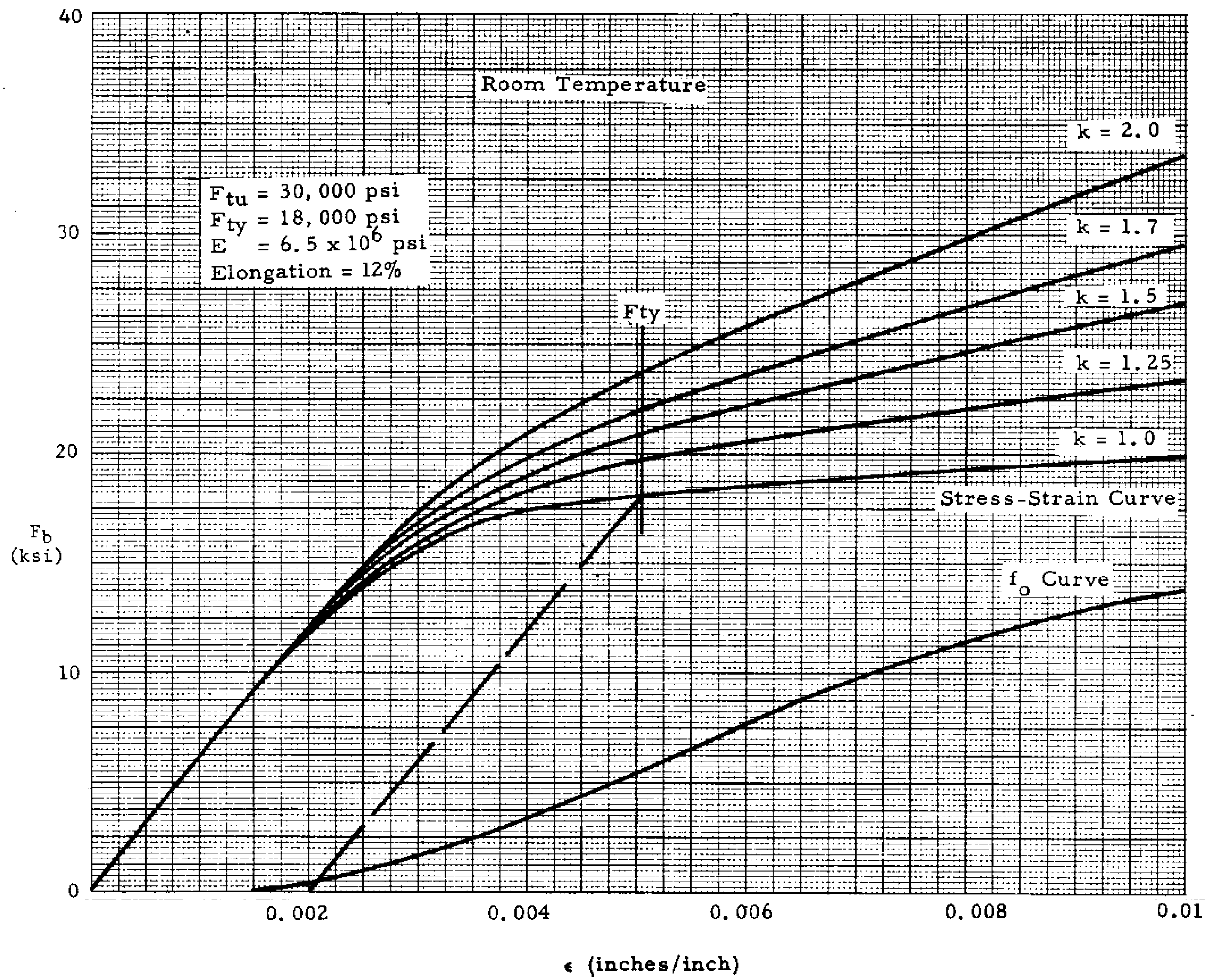


Fig. B4.5.6.6-5 Minimum Plastic Bending Curves HK31A-O Magnesium Alloy Sheet. Thickness ≤ .250

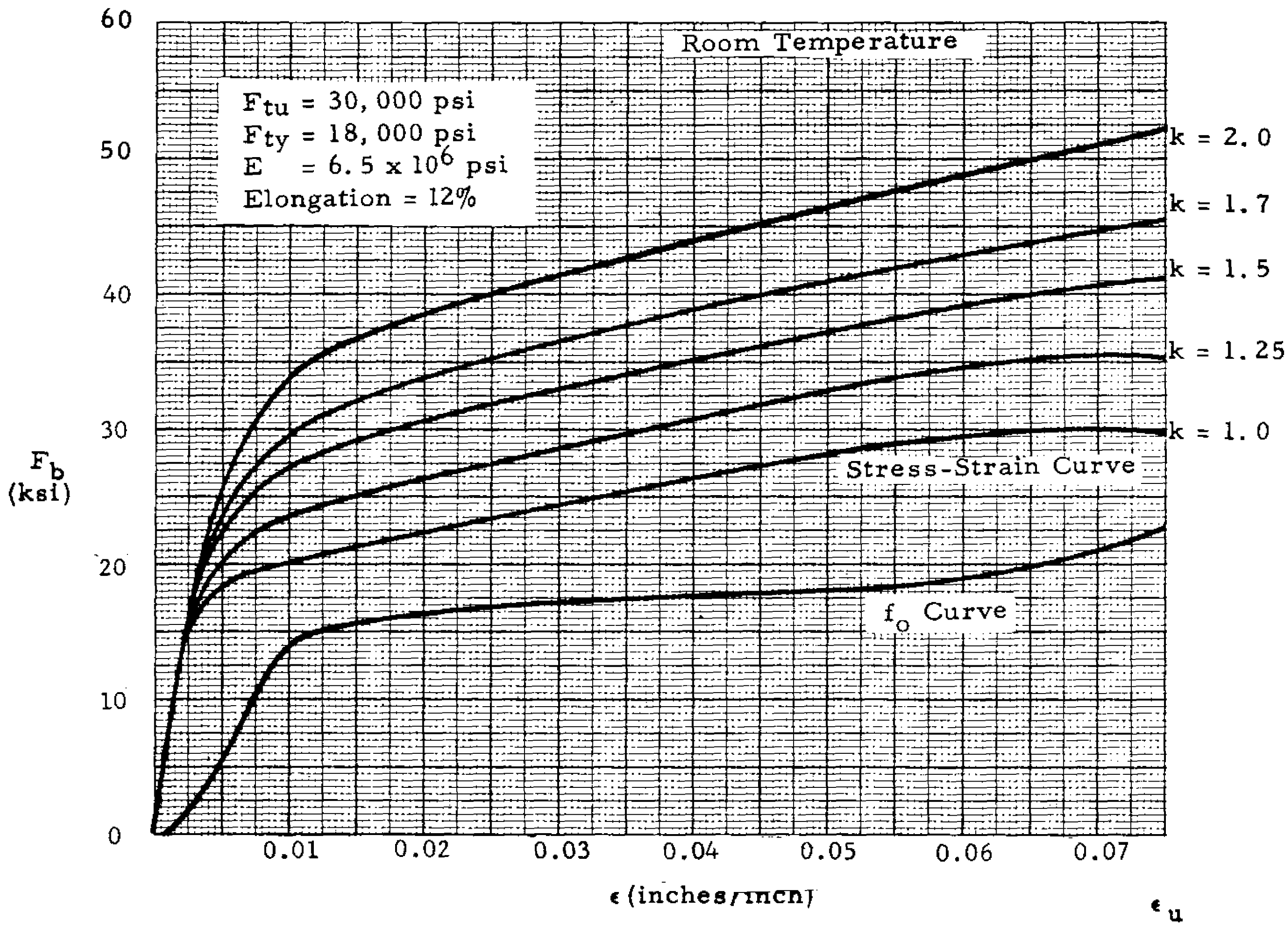


Fig. B4.5.6.6-6 Minimum Plastic Bending Curves for HK31A-O Magnesium Alloy Sheet. Thickness ≥ 016 and ≤ .250

Graph to be furnished when available

Graph to be furnished when available

Graph to be furnished when available

Graph to be furnished when available

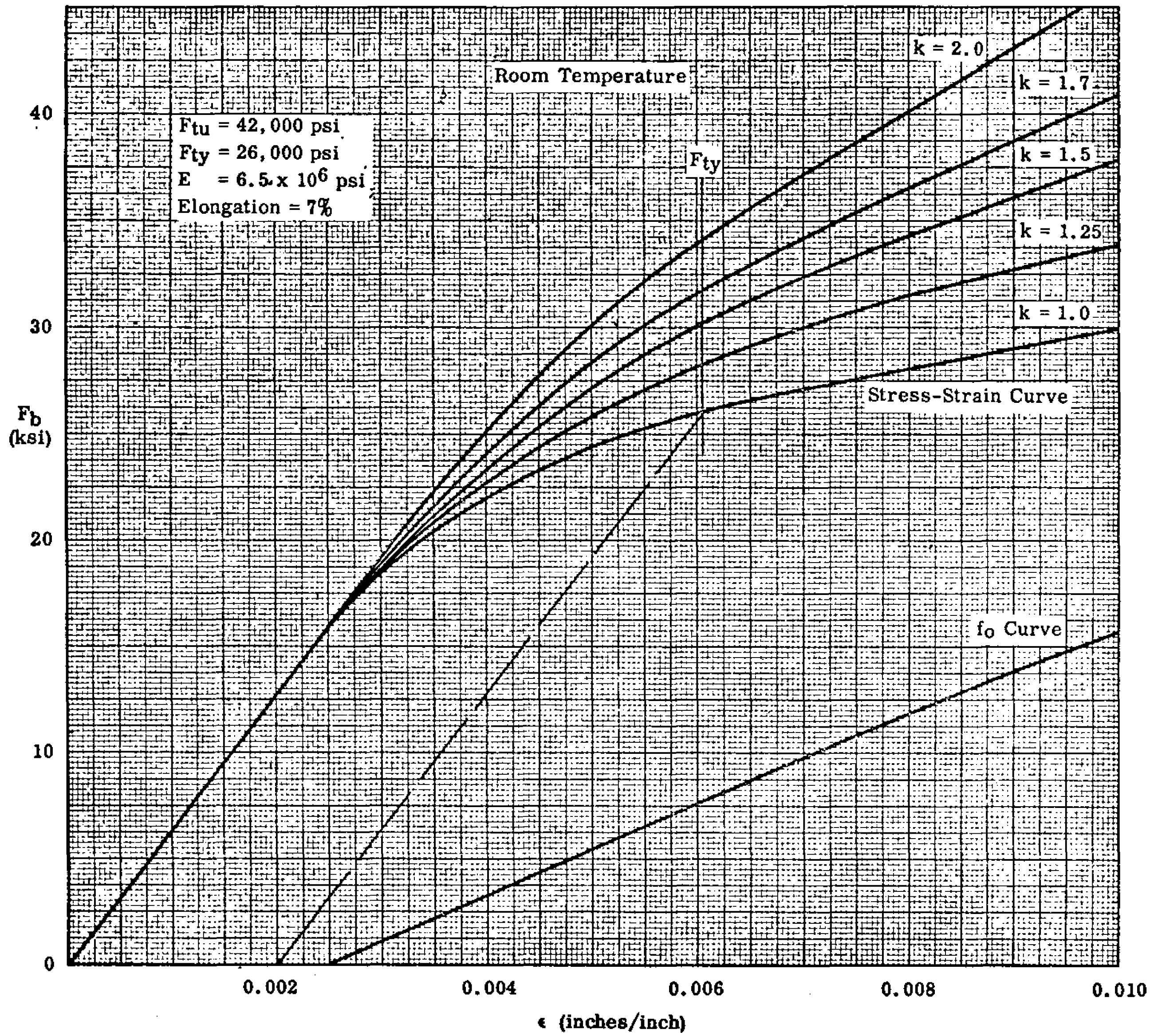


Fig. B4.5.6.6-11 Minimum Plastic Bending Curves ZK60A Magnesium Alloy Forgings (Longitudinal)

B4.5.6.6 Magnesium-Minimum Properties

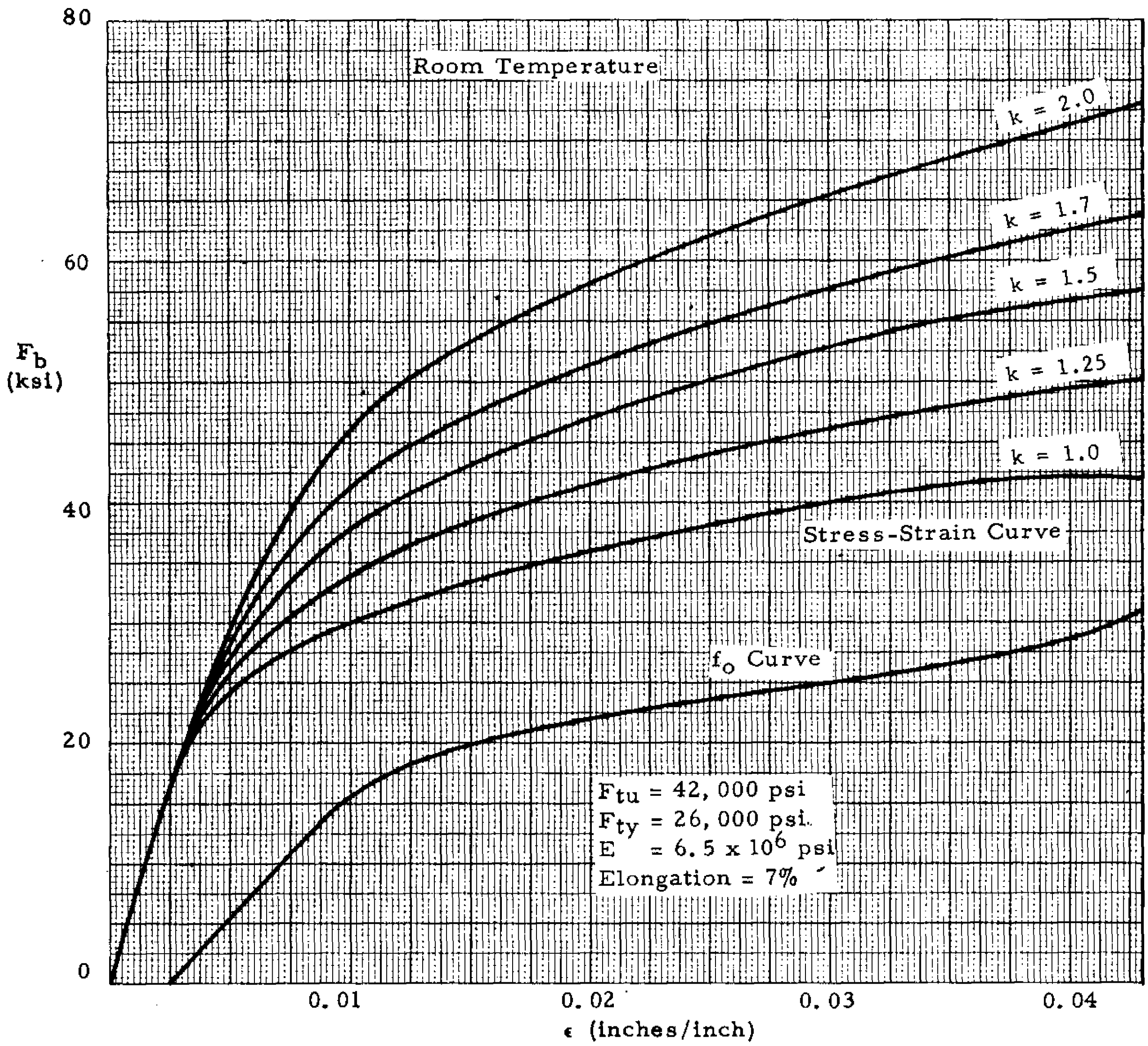


Fig. B4.5.6.6-12 Minimum Plastic Bending Curves ZK60A Magnesium Alloy Forgings (Longitudinal)

B4.5.7 Elastic-Plastic Energy Theory for Bending

B4.5.7.1 General

The Elastic-Plastic Energy Theory is defined as an extension of the Elastic Energy Theory into the plastic range of a material. This section will consider only energy due to bending stresses which may be in the elastic or plastic range, or both. Plastic bending curves found in Section B4.5.6 will be required. Deflections of statically determinate structures due to bending with any or all fiber stresses in the plastic range can be readily determined. Partially or completely plastic statically indeterminate structures can also be solved by procedures similar to those used in the Elastic Energy Theory. Other elastic theories could have been extended as well to include plastic bending effects but the Elastic Energy Theory was chosen due to its simplicity and common usage.

Elastic theories are accurate only if no part of a structure is stressed beyond the proportional limit of a material (no plastic strain). In structures designed to stress levels beyond the proportional limit, the error of an elastic deflection analysis is dependent on the amount of plastic strain involved. In some cases this error may be as much as 100% or more. Therefore when deflection is a limiting factor and plastic strains are involved, an analysis such as the Elastic-Plastic Energy Theory should be used.

B4.5.7.2 Discussion of Margin of Safety

In calculating the margin of safety at yield or ultimate, deflections must be considered as well as loads and/or stress levels, etc.; e.g., although a positive margin of safety for a structural element may be shown on the basis of loads, excessive deflections may occur. If the deflections are then the most critical design condition, the margin of safety becomes

$$\text{M.S.} = \frac{\text{Permissible Load}}{(\text{Safety Factor})\,(\text{Applied Load})} - 1 \qquad (4.5.7.2\text{-}1)$$

where the Permissible Load is the calculated load corresponding to the maximum permissible deflection. Equation (4.5.7.5-11) may be used in obtaining a permissible load level for a maximum permissible deflection by a trial and error process.

B4.5.7.3 Assumptions and Conditions

1. Energy is conserved; i.e., the external work due to a virtual load moving through a real deflection is equal to the internal strain energy developed during that deflection.

2. Plane sections remain plane; i.e., the strain is linearly distributed across any cross-section.

3. Poisson's ratio effects are negligible.

4. The deformations are of a magnitude so small as to not materially affect the geometric relations of various parts of a structure to one another.

B4.5.7.4 Definitions

dA - cross-sectional area of an infinitesimal volume, dV.

c - distance from the neutral axis to the extreme fibers of a cross-section.

δ - real deformation of an infinitesimal volume, dV, in the x-direction.

Δ - real vertical deflection of a beam at the point of application of a virtual load.

ϵ_b - total (elastic plus plastic) strain of an infinitesimal volume, dV, in the x-direction.

$\epsilon_{b_{max}}$ - extreme fiber strain of a cross-section.

F_v - virtual normal force acting on dA.

m - virtual bending moment in a beam due to the application of a virtual load.

W_e - external work equal to a virtual load moving through a real deflection.

W_i - internal strain energy equal to a summation of internal virtual forces times their real deflections.

Q - virtual unit load.

f_{b_v} - virtual bending stress on dA due to m.

B4.5.7.5 Deflection of Statically Determinate Beams

Consider the infinitesimal volume dV of Figure B4.5.7.5-1(a) and (b)

$$f_{b_v} = \frac{my}{I} \tag{4.5.7.5-1}$$

$$F_v = \text{stress} \times \text{area} = f_{b_v}\, dA$$

$$= \frac{my}{I}\, dA \tag{4.5.7.5-2}$$

$$\delta = \epsilon_b\, dx \tag{4.5.7.5-3}$$

Since plane sections remain plane,

$$\epsilon_b = \epsilon_{b_{max}} \frac{y}{c} \tag{4.5.7.5-4}$$

and

$$\delta = \epsilon_{b_{max}} \frac{y}{c}\, dx \tag{4.5.7.5-5}$$

By definition,

$$W_e = Q\,\Delta \tag{4.5.7.5-6}$$

$$W_i = \Sigma\, F_v\, \delta \tag{4.5.7.5-7}$$

Since energy is conserved,

$$W_e = W_i\,, \tag{4.5.7.5-8}$$

or

$$Q\,\Delta = \Sigma\, F_v\, \delta \tag{4.5.7.5-9}$$

Substituting Equations (4.5.7.5-2) and (4.5.7.5-5) into Equation (4.5.7.5-9) since Q is equal to unity,

B4.5.7.5 Deflection of Statically Determinate Beams (Cont'd)

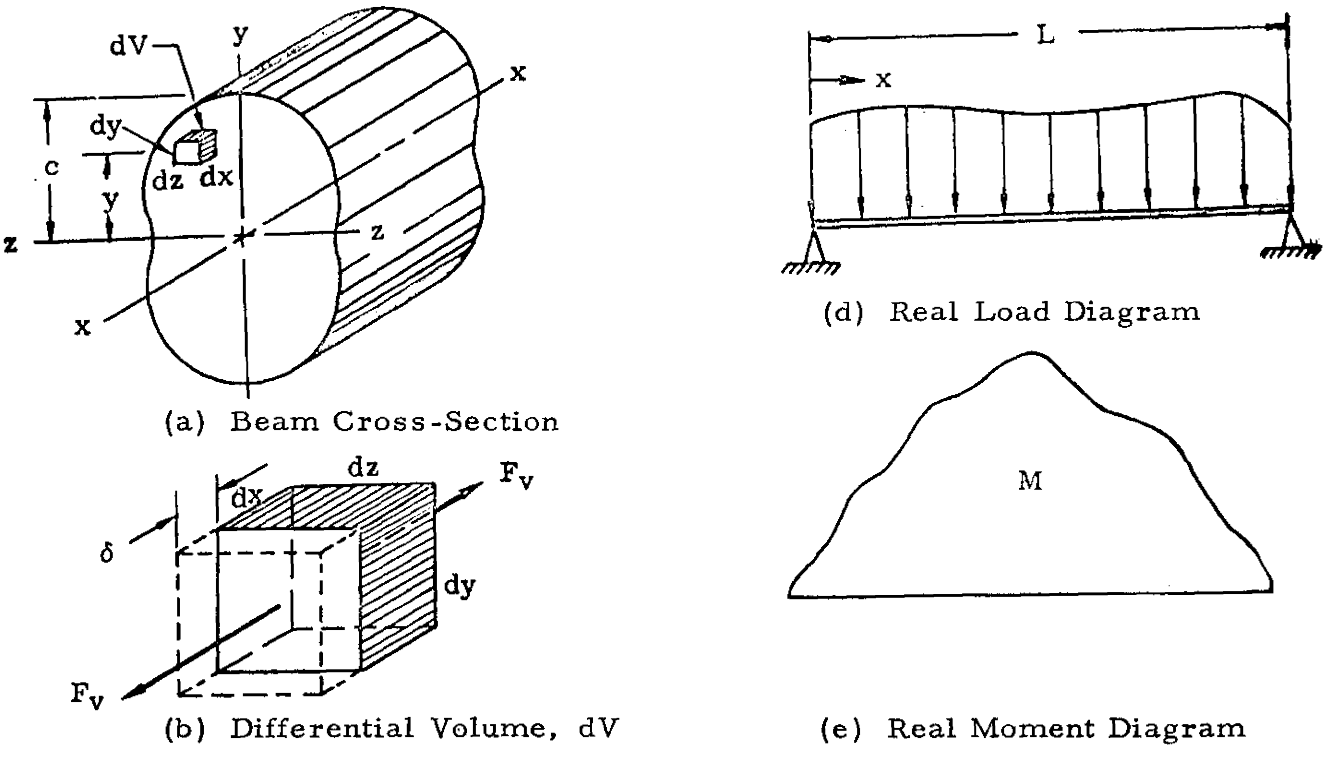
$$\Delta = \int_A \int_0^L \left(\frac{my}{I} \, dA \right) \left(\epsilon_{b_{max}} \quad \frac{y}{c} \, dx \right)$$

$$= \int_A \int_0^L \frac{y^2 \, dA}{I} \quad \frac{m \epsilon_{b_{max}}}{c} \, dx \, , \qquad (4.5.8.5\text{-}10)$$

But, by definition, $I = \int_A y^2 \, dA$,

$$\therefore \Delta = \int_0^L \frac{m \, \epsilon_{b_{max}}}{c} \, dx \qquad (4.5.7.5\text{-}11)$$

Equation (4.5.8.5-11) can now be solved graphically to find Δ. $\epsilon_{b_{max}}$ can be determined from a plastic bending curve for the applicable material as shown in Figure B4.5.7.5-1(c). Enter the F_b (modulus of rupture) scale with Mc/I and move horizontally across to the plastic bending curve for the specific cross-section; this intersection locates the corresponding $\epsilon_{b_{max}}$ on the ϵ (strain) scale. For beams with varying cross-section, c may be a variable.



(a) Beam Cross-Section

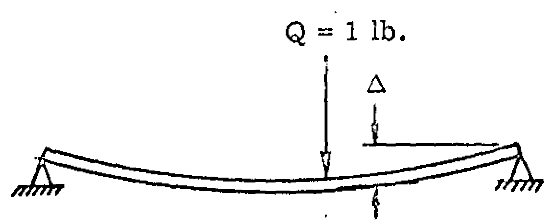
(b) Differential Volume, dV

(d) Real Load Diagram

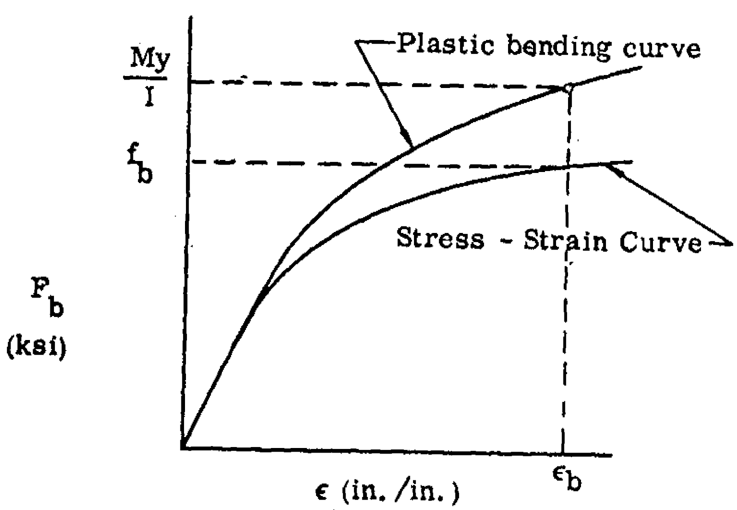
(e) Real Moment Diagram

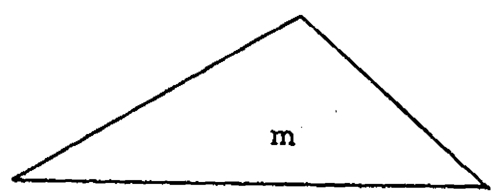
Figure B4.5.7.5-1

B4.5.7.5 Deflection of Statically Determinate Beams (Cont'd)



(f) Virtual Load Diagram
(Real Deflection Shown)





(c) Plastic Bending Diagram for Beam Cross-Section

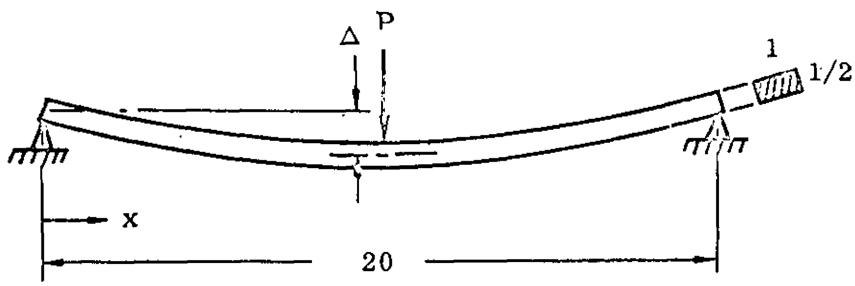
(g) Virtual Moment Diagram

Figure B4.5.7.5-1 (Cont'd)

B4.5.7.6 Example problem

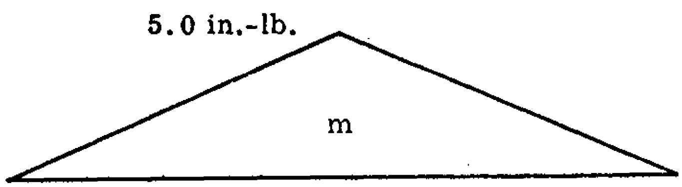
A rectangular beam, simply supported at the ends, is subjected to a concentrated vertical load at its center. Find the vertical deflection of the beam at its center.

Material: 2024-T3 Aluminum Alloy Plate
Beam dimensions: 1/2 in. x 1 in. x 20 in.
Load: P = 400 lb

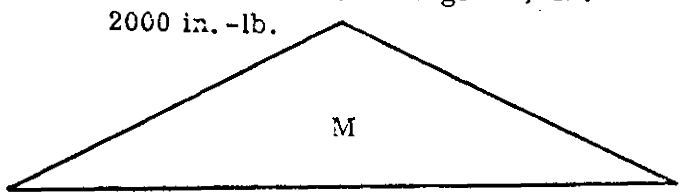


Procedure:

1. Apply a virtual unit load, Q, at the beam center and construct the virtual moment diagram, m.



2. Construct the real moment diagram, M.



3. Calculate the real bending stress, F_b, for each value of x by:

$$F_b = \frac{Mc}{I}$$

4. Enter the plastic bending curves on page 218 (at k=1.5 for a rectangular cross-section) with each value of F_b from Step 3 and determine $\epsilon_{b_{max}}$ for each value of x.

5. Multiply the value of m by the corresponding value of $\epsilon_{b_{max}}$ to obtain a value of $m\,\epsilon_{b_{max}}$ for each value of x.

B4.5.7.6 Example problem (Cont'd)

6. Tabulate the results of the previous steps (see Table B4.5.8.6-1).

7. Construct a plot of $m\,\epsilon_{b_{max}}$ vs. x and determine the area under the curve. This area represents $\int_0^L m\,\epsilon_{b_{max}}\,dx$. (See Figure B4.5.8.6-1)

8. Calculate Δ:

$$\Delta = \int_0^L \frac{m\,\epsilon_{b_{max}}}{c}\,dx \qquad \text{Ref eq (4.5.7.5-11)}$$

$c = 0.250$ in.

$$\int_0^L m\,\epsilon_{b_{max}}\,dx = 0.165 \qquad \left(\text{by graphical integration}\right)$$

$$\therefore \Delta = \frac{0.165}{0.250} = \underline{\underline{0.661 \text{ in.}}}$$

[By an elastic analysis, was found to be 0.6104 in. which is 7.7% in error. Partially plastic fiber stresses existed only over the middle 8 inches in this example. The elastic analysis would be considerable more in error for higher loadings and /or beams in which the plastic stresses exist over greater lengths; e.g., beams of constant moment, etc.]

B4.5.7.6 Example problem (Cont'd)

P = 400 lb.					
x, in.	m in. -lb.	M in. -lb.	Mc/I, ksi	$\epsilon_{b_{max}}$, in./in. x 10^{-3}	$m\,\epsilon_{b_{max}}$, in. -lb. x 10^{-3}
0	0	0	0	0	0
1	.5	200	4.8	.4570	.2285
2	1.0	400	9.6	.9140	.9140
3	1.5	600	14.4	1.3710	2.0565
4	2.0	800	19.2	1.8280	3.6560
5	2.5	1000	24.0	2.2850	5.7125
6	3.0	1200	28.8	2.7420	8.2260
7	3.5	1400	33.6	3.14	10.9900
8	4.0	1600	38.4	3.80	15.2000
9	4.5	1800	43.2	4.64	20.8800
10	5.0	2000	48.0	5.83	29.1500
11	4.5	1800	43.2	4.64	20.8800
12	4.0	1600	38.4	3.80	15.2000
13	3.5	1400	33.6	3.14	10.9900
14	3.0	1200	28.8	2.7420	8.2260
15	2.5	1000	24.0	2.2850	5.7125
16	2.0	800	19.2	1.8280	3.656
17	1.5	600	14.4	1.3710	2.0565
18	1.0	400	9.6	.9140	.9140
19	.5	200	4.8	.4570	.2285
20	0	0	0	0	0

Table B4.5.7.6-1

B4.5.7.6 Example problem (Cont'd)

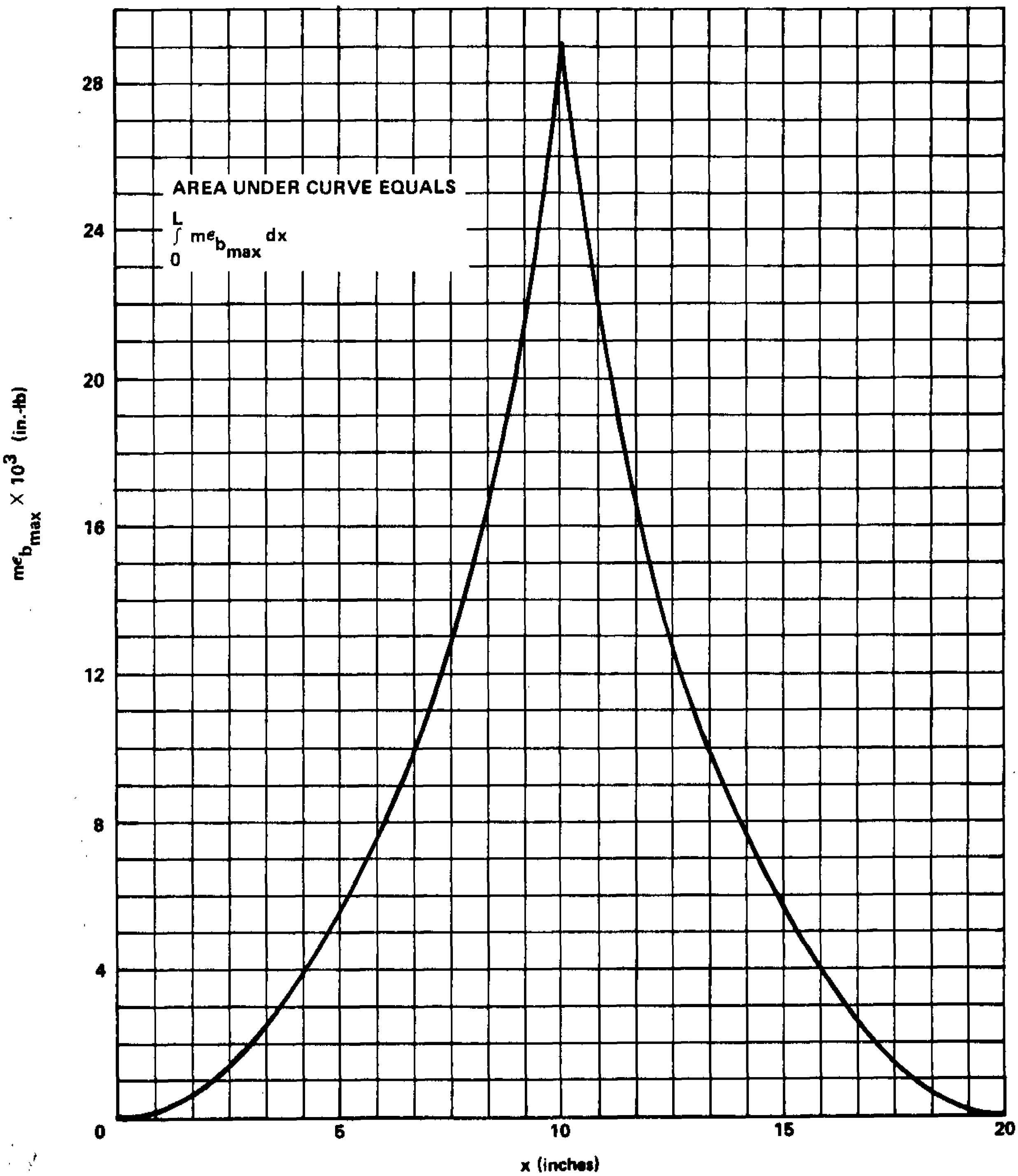


Figure B4.5.7.6-1