SECTION B10

HOLES & CUTOUTS IN PLATES

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DEFINITION OF SYMBOLS

Symbol	Definition
A	Cross-sectional area of plate without hole - $in.^2$
a	Diameter of hole; one-half length of side of rounded rectangular hole; minor diameter of elliptical hole - in.
A_b	Diameter of bead reinforcement - in.
B	Bead factor (Fig. B10-25)
b	Major diameter of elliptical hole; one-half length of side of rounded rectangular hole; one-half length of side of square plate - in.
c	Distance from center of hole to edge of plate; distance between holes; distance between rows in a double row of holes - in.
D	Plate flexural rigidity - psi
e	Displacement of hole from center line of plate - in.
F	Ratio of bead cross-sectional area to hole cross-sectional area
h	Thickness of plate - in.
h_b	Height of bead reinforcement - in.
K	Stress concentration factor
K_t	Theoretical stress concentration factor
K_e	Effective or significant stress concentration factor
K_{tB}	Stress concentration factor (Fig. B10-27)

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
K_{tB}^*	Stress concentration factor for plate with reinforced hole
L	One-half span of a beam - in.
M	Bending moment - in.-lb
M_θ, M_n	Bending moments in plates - in.-lb/in.
P	Axial tensile load - lb
p	One-half distance between holes - multiple hole patterns - in.
q	Uniform normal load intensity on plate or beam - psi or lb/in.
R	Radius of large hole in plate - in.
R_1, R_2	Radius of holes (Fig. B10-15) - in.
r	Radius of hole - in.
s_1	Distance from edge of large hole to center of small circular notch - in.
w	Width of plate - in.
w_{max}	Maximum deflection in plate - in.
β	Angle of obliquity of hole - deg
γ	R_1/R_2 (Fig. B10-16)
η	s_1/R_1 (Fig. B10-15)
θ	Angle of stagger between holes in double row of holes - deg
σ	Stress applied to semi-infinite plate - psi

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
λ	2a/w (Fig. B10-8)
μ	b/a (Fig. B10-8)
ρ	Radius of rounded corner; radius of hole - in.
σ_{max}	Maximum localized stress at edge of hole - psi
σ_{net}	Stress based on net section - psi
σ_o, σ_{nom}	Nominal stress in plate without hole - psi
σ_1	Largest value of stress in a biaxial stress field - psi
σ_2	Smallest value of stress in a biaxial stress field - psi

B10 HOLES AND CUTOUTS IN PLATES

The magnitude of stress concentrations around holes in plates used as components of structures has long been an important design consideration. The localized stress around a hole is usually obtained by multiplying the nominal stress by a factor called a stress concentration factor. For example, the localized stress or stress concentration σ_{max} at the edge of a relatively small hole, at the center of a wide plate that resists an axial tensile load P is

$$\sigma_{max} = K \frac{P}{A} = K\sigma_o \quad , \qquad (1)$$

or

$$K = \frac{\sigma_{max}}{\sigma_o} \quad , \qquad (2)$$

in which K is the stress concentration factor and $\sigma_o = \frac{P}{A}$ is the (nominal) stress that would occur at the same point if the bar did not contain the hole; that is, in this illustration the cross section area is the gross area including the area which is removed at the hole. If the diameter of the hole is relatively large, the net area of cross section is frequently used in calculating the nominal stress σ_o, hence the value of the stress concentration factor for a given discontinuity will depend on the method of calculating the nominal stress.

If σ_{max} in a member is the theoretical value of the localized stress, as found from the mathematical theory of elasticity, or the photoelasticity method, etc., K is given the subscript t, and K_t is called the theoretical stress concentration factor. If, on the other hand, the value of K is found from tests of the actual material under the conditions of use, K is given the subscript e, and K_e is called the effective or significant stress concentration factor.

10.1 SMALL HOLES

Solutions in this subsection will be limited to small holes in plates, that is, holes which are relatively small in comparison to the plate size such that boundary conditions do not affect the results. An exception to this is the case when holes are near a free edge of a plate.

10.1.1 Unreinforced Holes

This paragraph contains information on holes of various shapes in plates with no reinforcement around the hole, such as increased thickness, rings, or doublers.

10.1.1.1 Circular Holes

The case of a circular hole in an infinite plate in tension has been solved by many investigators. The stress concentration factor, based on gross area, is $K_t = \frac{\sigma_{max}}{\sigma} = 3$.

Strictly speaking, $K_t = 3$ applies only for a plate which is very thin relative to the hole diameter. When these dimensions are of the same order, K_t is somewhat greater than 3 at the midplane of the plate and is less than 3 at the surface of the plate [1]. For a ratio of plate thickness to hole diameter of three-fourths, $K_t = 3.1$ at the midplane and 2.8 at the surface.

For the case of a finite-width plate with a transverse hole, a solution has been obtained [2] and the corresponding K_t values are given in Fig. B10-1. These values are in good agreement with photoelastic results.

The case of a hole near the edge of a semi-infinite plate in tension has been solved and results are shown in Fig. B10-2. The load carried by the section between the hole and the edge of the plate has been shown to be

$$P = \sigma c h \sqrt{1 - \left(\frac{r}{c}\right)^2} \quad , \tag{3}$$

where

σ = stress applied to semi-infinite plate.

c = distance from center of hole to edge of plate.

r = radius of hole.

h = thickness of plate.

In Fig. B10-2, the upper curve gives values of $\frac{\sigma_B}{\sigma}$, where σ_B is the maximum stress at the edge of the hole nearest the edge of the plate. Although the factor $\frac{\sigma_B}{\sigma}$ may be used directly in design, it was thought desirable to compute also a "stress concentration factor" on the basis of the load carried by the minimum net section. This K_t factor will then be comparable with the stress concentration factors for other cases; this is important in analysis of experimental data. Based on the actual load carried by the minimum section, the average stress on the net section A-B is:

$$\sigma_{\text{net A-B}} = \frac{\sigma c h \sqrt{1 - \left(\frac{r}{c}\right)^2}}{(c - r)\, h} = \frac{\sigma \sqrt{1 - \left(\frac{r}{c}\right)^2}}{1 - \frac{r}{c}} \quad , \tag{4}$$

$$K_t = \frac{\sigma_B}{\sigma_{\text{net A-B}}} = \frac{\sigma_B \left(1 - \frac{r}{c}\right)}{\sigma \sqrt{1 - \left(\frac{r}{c}\right)^2}} \quad . \tag{5}$$

The case of a tension plate of finite width having an eccentrically located hole is shown in Fig. B10-3. When the hole is centrally located $\left(\frac{e}{c} = 1 \text{ in Fig. B10-3}\right)$, the load carried by section A-B is σch . As $\frac{e}{c}$ is increased to infinity, the load carried by Section A-B is $\sigma ch \sqrt{1 - \left(\frac{r}{c}\right)^2}$. Assuming a linear relation between the foregoing end conditions results in the following expression for the load carried by section A-B :

$$P_{A-B} = \frac{\sigma c h \sqrt{1 - \left(\frac{r}{c}\right)^2}}{1 - \frac{c}{e}\left[1 - \sqrt{1 - \left(\frac{r}{c}\right)^2}\right]} \quad . \tag{6}$$

The stress on the net section A-B is

$$\sigma_{\text{net A-B}} = \frac{\sigma c h \sqrt{1 - \left(\frac{r}{c}\right)^2}}{h\,(c - r)\left\{1 - \frac{c}{e}\left[1 - \sqrt{1 - \left(\frac{r}{c}\right)^2}\right]\right\}} \quad , \tag{7}$$

$$K_t = \frac{\sigma_{max}}{\sigma_{net}} = \frac{\sigma_{max}}{\sigma} \frac{\left(1 - \frac{r}{c}\right)}{\sqrt{1 - \left(\frac{r}{c}\right)^2}} \left\{1 - \frac{c}{e}\left[1 - \sqrt{1 - \left(\frac{r}{c}\right)^2}\right]\right\} . \tag{8}$$

It is seen in the lower part of Fig. B10-3 that this relation brings all the K_t curves rather closely together, so that for all practical purposes the curve for the centrally located hole $\left(\frac{e}{c} = 1\right)$ is, under these circumstances, a reasonable approximation for all eccentricities.

I. <u>Biaxial Tension:</u>

For the case of a hole in an infinite plate stressed biaxially results are given in Fig. B10-4. For a circular hole,

$$\sigma_{max} = 3\,\sigma_1 - \sigma_2 \quad . \tag{9}$$

For σ_2 equal in magnitude to σ_1, $K_t = 2$, when both are of the same sign. When σ_1 and σ_2 are equal but of opposite sign, $K_t = 4$.

II. <u>Bending:</u>

For bending of a plate the following results have been obtained. For a infinitely wide plate with a hole, mathematical results have been obtained in terms of $\frac{a}{h}$ (Fig. B10-5). Values for finite widths and various valves of $\frac{a}{h}$ are shown in Fig. B10-6.

10.1.1.2 Elliptical Holes

I. Axial Loading:

The stress distribution associated with an elliptical hole in an infinitely wide plate subjected to uniformly distributed axial load has been obtained, and the stress concentration factor as a function of the ratio of the major width (b) to the minor width (a) is given in Fig. B10-7.

$$K_t = 1 + \frac{2b}{a} \quad . \tag{10}$$

A photoelastic solution of the distribution of stresses around a centrally located elliptical hole in a plate of finite width and subjected to uniform axial loading has been obtained in Ref. 3, and the stress concentration factors are presented in Fig. B10-8.

For the case of a biaxially stressed plate with an elliptical hole $\left(\frac{b}{a} = 2\right)$, the results are given in Fig. B10-4.

$$\sigma_{max} = \sigma_1 \left(1 + \frac{2b}{a}\right) - \sigma_2 \quad . \tag{11}$$

II. Bending:

Stress concentration values for an elliptical hole in an infinitely wide plate are given in Fig. B10-9.

10.1.1.3 Rectangular Holes with Rounded Corners

Stress concentration factors for an unreinforced rounded rectangular hole in an infinite sheet in tension have been evaluated in Ref. 4. Variation of the stress concentration factor for various values of side length to corner radius is shown in Fig. B10-10.

10.1.1.4 Oblique Holes

An oblique or skew hole may be defined as one having its axis at an angle with respect to the normal to a surface. At the intersection with a plane surface a skew cylindrical hole gives rise to an elliptical trace and produces an acute-angled edge which, for large angles of obliquity with respect to the normal, may be very sharp.

Stress concentration factors have been determined for oblique holes in flat plates by a photoelasticity method in Ref. 5. Results of their analysis are presented in Fig. B10-11 along with theoretical curves for elliptical holes in infinite and finite widths.

The stress-concentration factor based on net area is relatively insensitive to the radius of the acute-angled tip. However, in a relatively narrow plate, the maximum stress may actually be increased by the addition of a radius because of the loss of load-carrying area.

It should be noted that the results of Ref. 5 apply only to plates with a ratio of hole diameter d to plate width w of 0.1. Additional information on oblique holes in plates can also be found in Ref. 6.

10.1.1.5 Multiple Holes

I. Two Holes:

Stress concentration factors for two holes of the same diameter in an infinite plate has been documented in Ref. 2. For the case of uniaxial tension perpendicular to the line of holes, Fig. B10-12 gives the factors; and for the case of biaxial tension, the results are given in Fig. B10-13.

The solution for stress concentration factors of two holes of different diameters in an infinite plate loaded by an equal biaxial stress has been obtained by Ref. 7 and the results are given in Fig. B10-14.

Reference 8 contains the solution for a plate containing a circular hole with a circular notch, as shown in Fig. B10-15. Figure B10-16 shows the stress concentration factor at the bottom of notch (point A) when the tension load is in the y direction. The smaller or deeper the notch is, the greater are the values of maximum stress.

II. Single Row of Holes:

For a single row of holes in an infinite plate, Figures B10-17 and B10-18 give stress concentration factors for tension perpendicular to the line of holes and for biaxially stressed holes respectively.

III. Double Row of Holes:

For a double row of staggered holes, the stress concentration factor is given in Fig. B10-19. For the staggered holes, a problem arises in basing

K_t on net section, since for a given $\frac{b}{a}$ the relation of net sections A-A and B-B depends on θ . For $\theta < 60$ deg , A-A is the minimum section and the following formula is used:

$$K_{t_A} = \frac{\sigma_{max}}{\sigma} \left[1 - 2 \left(\frac{a}{b} \right) \cos \theta \right] \quad , \tag{12}$$

for $\theta > 60$ deg , B-B is the minimum section and the formula is based on the net section in the row:

$$K_{t_B} = \frac{\sigma_{max}}{\sigma} \left[1 - \frac{a}{b} \right] \quad . \tag{13}$$

IV. Arrays of Holes:

Stress concentration factors in a plate containing a large number of uniformly spaced perforations in regular triangular or square arrays under biaxial loading was investigated by photoelastic methods in Ref. 9. Four configurations of perforation were considered as shown in Fig. B10-20.

Stress concentration factors for several combinations of configuration and loading are plotted against $\frac{\rho}{p}$ in Figs. B10-21 through B10-26.

In these figures, the stress concentration factor is defined as the ratio of the peak value of σ_θ at point A , A' , B' on the hole boundary to σ_1 namely $K = \frac{\sigma_{\theta \, max}}{\sigma_1}$. σ_1 is the algebraically larger one of the principal stresses that would be produced in the plate by the combination of the biaxial loads applied if there were no holes. The stress concentrations factors at angular positions on the boundary are shown by subscripts to K .

On these figures, the primary tendency is the rise of K with $\frac{\rho}{p}$. Among the four types of hole configuration, the diagonal-square type is always of disadvantage for it produces the highest stress concentration factor for every type of biaxial loading throughout the range of $\frac{\rho}{p}$, while the parallel-square type gives the lowest factor, and the perpendicular and parallel-triangular types lie in between. For example, a list of K for a comparatively large value of $\frac{\rho}{p} = 0.92$ for each hole configuration and biaxial-load is given in Table B10-1.

Where strength is the main consideration, the above results show that the parallel-square type of hole configuration is most desirable, especially when $\frac{\rho}{p}$ is large. The triangular hole configuration, both perpendicular and parallel, which is usually used is unfavorable contrary to expectation.

10.1.2 Reinforced Holes

This paragraph contains information on holes which have an increased thickness around the hole in order to reduce the stress concentration factors. Information will be divided into two categories: when the reinforcement is of constant cross section and when the reinforcement cross section varies around the hole.

10.1.2.1 Constant Reinforcement

Stress concentration at a hole can be reduced by providing a region of increased thickness around the hole, sometimes called a "boss" or "bead." Values of stress concentration factor for beads of various cross-sectional areas for a plate having a hole diameter one-fifth the plate width $\frac{a}{w} = 0.2$ · are obtained in Ref. 2. Also, the stress concentration factors were obtained on the basis of the radial dimension $\frac{a_b - a}{2}$ being small compared to the hole diameter.

To account for other values of $\frac{a}{w}$, an approximate method has been obtained:

$$K_{tB}^* = B\left(K_t - 1\right) + 1 \quad , \tag{14}$$

where

K_{tB}^* = stress concentration factor for plate with hole and bead, for the particular value of $\frac{a}{w}$ desired.

B = Bead factor = $\frac{K_{tB} - 1}{1.51}$ (Fig. B10-27).

K_t = stress concentration factor for plate with hole and without bead (Fig. B10-1) for the particular value of $\frac{a}{w}$ desired.

It is pointed out in Ref. 10 that the maximum cross-sectional area of a bead, $\frac{a_b - a}{2} h_b$, should be about $\frac{ah}{4}$. Above this value the theory from which the stress concentration values were obtained no longer holds.

Studies by Dhir on stresses around two equal reinforced circular openings in a thin plate [11] have shown that the most effective amount of reinforcement appears to be near 40 percent area replacement (percent of area replaced to area removed by hole). Any additional amount of reinforcement did not produce a proportionate reduction of the stresses. The interaction between the two holes was not significant if the distance between the inner edges of the two holes was one diameter or greater. For tension load perpendicular to the line of the two holes, a 40 percent area replacement lowered the stress concentration factor from 3.0, for an unreinforced hole, to 1.75.

Additional work in the area of reinforced holes has been done in Refs. 12 and 13; however, these references do not give design data in usable form, as a solution must be obtained from a computerized analysis.

I. <u>Asymmetrically Reinforced:</u>

The previous discussion has only concerned holes with symmetrical reinforcement, that is, with reinforcement on both sides of the plate.

In practice, however, these are frequent cases where one surface of the plate must be kept smooth and the reinforcement can be attached to the other surface only. This problem is treated in Refs. 13 and 14; however, because of the interaction between bending and stretching, the problem is highly nonlinear and has only been solved for certain limiting cases. It is found that the asymmetry of the reinforcement introduces bending stresses in the plate and reinforcement because of the eccentricity of the reinforcement. Careful consideration of the parameters must be employed, as in some cases the addition of reinforcement causes a stress concentration factor greater than what would have been present if no reinforcement were added. The work of Ref. 15 shows that for a given loading condition a size of reinforcement can be chosen to minimize the stress concentration factor.

10.1.2.2 Variable Reinforcement

In some cases the design of the reinforcement around a hole may be important when the weight of the structure must be as low as possible. Reinforcement may then be of variable cross section around the hole. Hicks [16, 17] has considered the problem of variably reinforced circular holes and arrived at expressions for stresses for different loading systems. He has shown that when the reinforcement has a given weight, the effect of varying its cross section is to reduce the stress concentration in the plate. One disadvantage of the variable reinforcement is that it may be undesirable from a manufacturing point of view.